A Possible Approach to Relativistic Thermodynamics  
Bernhard Rothenstein and G. Spix  
brothenstein@gmail.com

Abstract. Considering a system of non-interacting particles characterized by the number $N$ of its constituents and by its Kelvin temperature $T$, we reduce the transformation of the Kelvin temperature and heat to the transformation of mass (energy).

Many authors present approaches to relativistic thermodynamics, with different final results, deriving transformation equations for absolute temperature $T$ and heat $Q$. [1], [2], [3], [4], [5]. The purpose of our Note is to derive transformation equations for $T$ and $Q$, reducing the problem to the transformation of internal energy, a physical quantity proportional to $T$ via a relativistic invariant factor.

Consider an ideal mono-atomic gas consisting of $N$ identical non-interacting molecules at rest in the inertial reference frame $I_0$ which moves with constant speed $u$ relative to the inertial reference frame $I$ and with speed $u'$ relative to the inertial reference frame $I'$, $I'$ moving with speed $V$ relative to $I$. The three inertial reference frames are in the standard configuration, $u$, $u'$ and $V$ showing in the positive direction of the permanently overlapped $x,x',x_0$ axes. In order to characterize the studied system of molecules, observers from $I_0$ measure the proper absolute temperature $T_0$ and the proper internal energy

$$U_0 = \frac{3}{2} kN T_0$$  \hspace{1cm} (1)

where $k$ stands for the Boltzmann constant. In accordance with the principle of relativity $\frac{3}{2} kN$ is a relativistic invariant. Measuring the absolute temperature of the same ensemble of molecules observers from $I$ obtain $T$ expressing its internal energy as $U = \frac{3}{2} NkT$ the energies $U$ and $U_0$ being related by [4]

$$U = \frac{3}{2} NkT = \frac{U_0}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{3}{2} Nk \frac{T_0}{\sqrt{1-\frac{u^2}{c^2}}}. \tag{2}$$

For the same reasons observers from $I'$ consider that (2) reads

$$U' = \frac{3}{2} NkT' = \frac{U_0}{\sqrt{1-\frac{u'^2}{c^2}}} = \frac{3}{2} kN \frac{T_0}{\sqrt{1-\frac{u'^2}{c^2}}}. \tag{3}$$
Eliminating $T_0$ between (2) and (3) the result is that $T$ and $T'$ are related by

$$T = T' \sqrt{\frac{1-u'^2}{c^2}}. \quad (4)$$

Expressing the right side of (4) as a function of $u'$ via the addition law of parallel speeds [5]

$$u = \frac{u'+V}{1+\frac{Vu'}{c^2}} \quad (5)$$

(4) becomes

$$T = T' \sqrt{\frac{1+\frac{Vu'}{c^2}}{1-\frac{V^2}{c^2}}} \quad (6)$$

The invariance of entropy ($S=Q/T; S'=Q'/T'$) makes that heat $(Q,Q')$ transforms as temperature does, i.e.

$$Q = Q' \frac{1+\frac{Vu'}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{Q'+V \frac{u'Q'}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \quad (9)$$

As we see the transformation of heat has as a byproduct the physical quantity $p'_Q = u \frac{Q'}{c^2}$ which reads in $p_q = u \frac{Q}{c^2}$, which we call **heat momentum**, transforming as

$$p'_Q = \frac{p'_Q + V \frac{Q}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \quad (10)$$

Expressed as a function of $p'_Q$ (9) becomes

$$Q = Q' + Vp'_Q \quad (11)$$
Equations (10) and (11) tell us that cp and Q (c \( cp'_{Q}, Q' \)) are in an one space dimensions approach the components of a two vector.

Introductory physics textbooks, presently in use, do not treat the relativistic aspects of thermodynamics. Our approach offers an easy way to relativistic thermodynamics for the introductory physics lectures.

References