The Equivalence of Magnetic and Kinetic Energy

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An electron is a mass with a charge that can move relative to an observer, and when doing, so represents both kinetic energy and magnetic energy. This paper investigates some obvious questions about the relationship between the two forms of energy.

1. Introduction

An electric charge placed in vacuum produces an electrostatic field that surrounds the charge. When an observer moves relative to the charge, the electrostatic field observed changes in time, and in addition, the observer will measure a magnetic field. The presence of the magnetic field indicates magnetic energy.

For an observer moving relative to a mass, the relative speed of the mass represents kinetic energy. Like magnetic energy, kinetic energy exists only if there is relative movement – in this case, relative motion between observer and mass. Kinetic and magnetic energy are thus quite comparable: both forms of energy exist only when there is motion relative to an observer.

The questions that I want to answer are these:
1) How much energy does the magnetic field of a moving charge represent?
2) What is the relation between the magnetic and kinetic energy of a charged mass?

In addressing these questions, only non-relativistic velocities need be considered, because relativistic conditions unnecessarily complicate the situation without adding any additional insight.

2. Magnetic Energy of a Single Moving Charge

An electric current induces a magnetic field in the surrounding space. The magnetic energy of an electric current is described by the formula \( \mathcal{W}_m = LI^2 \) (Joule), where \( L \) is the magnetic induction coefficient of the electric circuit and \( I \) is the electric current.

The magnetic energy \( \mathcal{W}_m \) of an electric current tends to conserve the electric current. Only when there is electric or magnetic resistance will the current \( I \) decline over time and the magnetic energy \( \mathcal{W}_m \) decrease, with both eventually disappearing completely.

In his electron theory, H.A. Lorentz [1] uses the same representation for a magnetic field \( d\mathbf{H} \) (amp/m) at distance \( R \) (m) from an electric current element \( IdS \) (amp \( \times \) m) as Biot & Savart:

\[
d\mathbf{H}(x,y,z) = \left( \frac{IdS}{4\pi R^2} \right) \mathbf{e}_x \times \mathbf{e}_r \text{ [amp/m]} \tag{1}
\]

Figure 1 illustrates Eq. (1). The total magnetic field that an electric current induces at point \( \mathbf{P}(x,y,z) \) is the summation (integration) of all the magnetic fields \( d\mathbf{H} \) each moving individual electron in the electric circuit induces at \( \mathbf{P} \).

An electric current normally consists of an essentially infinite number of moving electrons, but there exists, however, no theoretical objection to letting an electric current consist of one single moving charge. So theoretically the current element \( IdS \) can consist of only one moving charge \( Q_e \).

If the current \( IdS \) consists of only one moving charge, then

\[
IdS = Q_e \mathbf{V}_e \text{ [charge \( \times \) m/sec], } \int \mathbf{V}_e \, dt = Q_e \mathbf{V}_e \text{ Idt = } Q_e \text{ [charge]}
\]

In the case of a single moving charge \( Idt = Q_e \), where \( Q_e \) is the charge of that single electron. The current \( I \) is not divisible any further, so \( IdS = Q_e \mathbf{V}_e \) is the differential limit of an electric current element.

When the electric current element \( IdS \) consists of a single charge \( Q_e \), that moves relative to \( \mathbf{P}(x,y,z) \) with velocity \( \mathbf{V}_e \), the magnetic field \( \mathbf{H} \) at \( \mathbf{P} \) due to current \( IdS = Q_e \mathbf{V}_e \), is according Eq. (1):

\[
\mathbf{H}(x,y,z) = \left( \frac{Q_e \mathbf{V}_e}{4\pi R^2} \right) \mathbf{e}_x \times \mathbf{e}_r \text{ [amp/m]}
\]

Let us consider a sphere-shaped charge \( Q_e \) with radius \( R_e \). Because nature always seeks the way to minimize the energy level, the charge \( Q_e \) will be distributed over the surface of the sphere. Fig. 2 illustrates the situation where charge \( Q_e \) is at rest and the movement of the charge is revealed by the relative speed \( \mathbf{V} \).

When the observer, moving relative to \( Q_e \) with velocity \( \mathbf{V}_e \), wants to determine the magnetic field that \( Q_e \) is inducing in the surrounding space, the observer can choose any coordinate \( \mathbf{P}(x,y,z) \) compared to the position of charge \( Q_e \) (0,0,0) to determine the magnetic field at \( \mathbf{P} \), as long as particle \( Q_e \) keeps the velocity \( \mathbf{V}_e \) relative to the observer.
Therefore be verified in an experiment according to the equation (3):

\[ W = \frac{Q_e^2}{8\pi\varepsilon_0 R_e} \]

For the observer moving relative to charge \( Q_e \) with speed \( V_e \), the total energy \( W_t \) the charge represents is the sum of magnetic energy \( W_m \) (2) and electrostatic energy \( W_e \) (3):

\[ W_t = W_m + W_e = \mu_0 Q_e^2 V_e^2 / 8\pi R_e + Q_e^2 / 8\pi\varepsilon_0 R_e \]

Considering \( c^2 = 1 / \varepsilon_0\mu_0 \), we derive:

\[ W_t = \left( \frac{\mu_0 Q_e^2}{8\pi R_e} \right) \left( V_e^2 + c^2 \right) \] (4)

The \( W_t \) is the total energy the moving charge presents to an observer; the electrostatic energy plus the dynamic energy. Using Einstein’s equivalence of mass and energy \( E = mc^2 \) for the electrostatic energy \( W_e \), the equivalent mass that \( W_e \) represents must be:

\[ M_p = W_e / c^2 = (Q_e^2 / c^2) / 8\pi\varepsilon_0 R_e = \mu_0 Q_e^2 / 8\pi R_e \] (5)

Substituting the equation for the electrostatic mass \( M_p \) [Eq. (5)] in the formula for the total energy \( W_t \) of the moving charge (formula 4) we derive:

\[ W_t = W_m + W_e = M_p (V_e^2 + c^2) \]

The magnetic energy \( W_m \) of the moving charge, expressed in the mass equivalence \( M_p \) of the electrostatic energy, becomes:

\[ W_m = M_p V_e^2 \] (6)

This derived formula for the magnetic energy of a moving charge is remarkable, considering the kinetic energy \( W_k \) of a ‘normal’ mass \( M_p \), moving with relative velocity \( V_e \), is \( W_k = \frac{1}{2} M_p V_e^2 \).

3. The Moving Electron and Magnetic Energy

The classical radius, or Compton radius \( R_c \), of an electron is calculated by means of the Compton equation [2]:

\[ \mathbf{B}(x, y, z) = \left( \mu_0 Q_e V_e / 4\pi R^2 \right) \mathbf{d}e \times \mathbf{e}_x \] [tesla]

\( Q_e \), in the above equation is then the total charge of the sphere, \( V_e \) the speed of the charge relative to the magnetometer, and \( R \) the distance to the center of the charge.

To be able to relate the magnetic energy \( W_m \) [Eq. (2)] of the moving charge to the electrostatic energy of \( Q_e \), we have to consider the potential electrostatic energy of a sphere of radius \( R_e \) and charge \( Q_e \). The electrostatic energy of a charge \( Q_e \) distributed over a sphere of radius \( R_e \) in vacuum is given by:

\[ W_e = \frac{Q_e^2}{8\pi\varepsilon_0 R_e} \] (3)
\[ M_e c^2 = \frac{Q_e^2}{4\pi \varepsilon_0 R_C} \]  

(7)

where \( M_e \) is the rest mass of the electron, \( Q_e \) is the electron charge, and \( c^2 = 1/\varepsilon_0 \mu_0 \). This equation yields \( R_C = \frac{\mu_0 Q_e^2}{4\pi M_e} = 2.81794 \times 10^{-15} \) meter.

The equation for the magnetic energy of a charge \( Q_e \) on a sphere of radius \( R_C \), moving with a relative velocity \( \mathbf{V}_e \), is from the formula for the magnetic energy \( W_m \) [Eq. (2)],

\[ W_m = \mu_0 Q_e^2 V_e^2 / 8\pi R_C \]

When substituting the mass equivalence, \( M_e \), of the Compton-equation (7) into (2) and considering \( c^2 = 1/\varepsilon_0 \mu_0 \), we get:

\[ W_m = \frac{1}{2} M_e V_e^2 \]

So when we assume the electron has the radius \( R_C \), derived with the Compton equation, the magnetic energy of the moving electron presents energy equal to the kinetic energy of that same electron.

A moving ‘pure electrostatic’ mass represents magnetic energy according to \( W_m = M_p V_e^2 \) [Eq. (6)], while the magnetic energy of a moving electron with the Compton radius \( R_C \) has a magnetic energy of \( W_m = \frac{1}{2} M_e V_e^2 \). What causes the factor-of-two difference? Considering the Compton equation \( M_e c^2 = \frac{Q_e^2}{4\pi \varepsilon_0 R_C} \) and the electrostatic energy of a charged sphere \( W_e = \frac{Q_e^2}{8\pi \varepsilon_0 R_e} \), we observe a similar factor two difference. The Compton equation ‘stores’ twice as much energy as a charged sphere with the same charge and radius. This difference explains exactly the difference between the equations for the magnetic energy \( W_m = M_p V_e^2 \) for the moving charged sphere, and \( W_m = \frac{1}{2} M_e V_e^2 \), for the moving electron with the Compton radius.

We know that apart from a charge, the electron also has a spin. The magnetic spin of the electron is not considered in the Compton equation.

In the book From Paradox to Paradigm [3], the chapter “The Electron” gives the total energy of an electron \( W_e \), at rest, by the formula:

\[ M_e c^2 = \frac{Q_e^2}{8\pi \varepsilon_0 R_e} + \mu_0 Q_e c^2 / 8\pi R_e \]  

(8)

In this presentation for the total energy of an electron at rest, half the energy is represented by electrostatic energy \( \frac{Q_e^2}{8\pi \varepsilon_0 R_e} \), consistent with the energy of a charged sphere, and the other half of the energy by \( \mu_0 Q_e c^2 / 8\pi R_e \), the magnetic spin energy of the electron. Calculating the radius \( R_e \) of the electron from Eq. (8) yields \( R_e = 1.40897 \times 10^{-15} \) meter: exactly the Compton radius.

When we consider that half the intrinsic energy of the electron at rest is electrostatic energy and the other half is magnetic spin energy [Eq. (8)], the magnetic energy of a moving electron, becomes:

\[ W_m = \frac{1}{8} \mu_0 Q_e^2 V_e^2 / \pi R_C = \frac{1}{2} M_e V_e^2 \]  

(9)

That is, when half of the intrinsic energy of the mass of the electron is represented by the electrostatic energy, and the other half by the magnetic spin energy, the calculated magnetic energy of the moving electron is equal to the kinetic energy of that electron.

Because the kinetic energy of an electron is \( W_e = \frac{1}{2} M_e V_e^2 \) and at the same time the magnetic energy \( W_m \) is also \( \frac{1}{2} M_e V_e^2 \), the kinetic energy of the electron must be same energy as the magnetic energy. Otherwise the conservation law for energy would be violated every time an electron was accelerated or slowed down.

4. The Electromagnetic Mass

In Section 2 we showed, with the help of the Electron Theory of Lorentz, that the magnetic energy \( W_m \) of a single moving spherical charge is by Eq. (6) equal to \( W_m = M_p V_e^2 \), where \( M_p \) is the mass equivalence of the electrostatic energy of the charged sphere according to \( M_p = \mu_0 Q_e^2 V_e^2 / 8\pi R_e \). In Section 3, we calculated the theoretical magnetic energy of the ‘classical electron’. Eq. (9) evaluated the magnetic energy of the moving electron \( W_m = \frac{1}{2} M_e V_e^2 \). Let us suggest that the electron, a charged mass, can be represented by the energy/mass of a charged sphere and a not-yet-identified mechanical part of the mass. Eq. (7) becomes:

\[ M_e c^2 = \frac{1}{8} \mu_0 Q_e^2 / \pi \varepsilon_0 R_e + \frac{1}{2} M_e c^2 \]  

(10)

The electrostatic mass of a charged sphere with the Compton radius explains exactly half the energy/mass of the electron. The magnetic energy of the moving electron equals the kinetic energy of the moving electron. The question to be answered is: “What kind of energy/mass represents the other half of the energy in Eq. (10)?” In the chapter “The Electron” in the book From Paradox to Paradigm [3], the magnetic energy of the spin of the electron is calculated as exactly half the intrinsic energy of the electron. The magnetic spin energy is responsible, and explains the physics why the charge of an electron is confined. The expanding force of the charged sphere of the electron is compensated by means of the contracting force of the spinning magnetic field surrounding the electron. The charge of the electron is trapped.

The above presented E&M physics for the electron is however inconsistent with the QM perspectives, which must be addressed. We refer to The Feynman Lectures on Physics [4], part II, Chapter 28, “The Electromagnetic Mass”. In this chapter the electromagnetic mass of the electron is derived by means of the momen-
The momentum density \( \mathbf{g} = \varepsilon_0 \mathbf{E} \times \mathbf{B} \) (Eq. 27.21 in [4]). According to the QM approach in (28-2), \( \mathbf{g} = (\varepsilon_0 v/c^2)E^2 \sin(\theta) \) because the momentum density vector is directed obliquely toward the line of motion. (*) Furthermore, I quote (28-2): “The fields are symmetric about the line of motion, so when we integrate over space, the transverse components will sum to zero, giving a resultant momentum parallel to \( \mathbf{v} \). The component of \( \mathbf{g} \) in this direction is \( \mathbf{g} \sin(\theta) \), which we must integrate all over space.” (**)

In the above argumentation (*) and (**), vector \( \mathbf{p} \) is thought to be partly compensated by the opposite vector \( \mathbf{p} \). Vector summation is allowed in static situations, where the vectors for example express the magnitude and direction of a static force. The \( \mathbf{p} \) is however a dynamic vector, presenting the impulse of the moving mass/energy density at a certain point. The total momentum \( \mathbf{p} \), according to the QM approach is then:

\[
\mathbf{p} = \int (\varepsilon_0 v / c^2)E^2 \sin(\theta)2\pi r^2 \sin(\theta)d\theta dr
\]

where \( 2\pi r^2 \sin(\theta)d\theta dr \) is the volume element. The integration over all space gives:

\[
\mathbf{p} = \frac{2}{3}(e^2 / ac^2)\mathbf{v} \quad \text{(Eq. 28.3 in [4])}
\]

This equation, expressed in the symbols used in this article, gives the impulse:

\[
\mathbf{p} = \left[ \frac{4}{3} / 8\pi \varepsilon_0 c^2 R_C \right] \mathbf{v}
\]

The electromagnetic mass calculated according to Eq. (28.3) is

\[
m_e = \frac{4}{3}Q_e^2 / 8\pi \varepsilon_0 c^2 R_C,
\]

which is 4/3 the mass equivalence of the electric field, or 2/3 the mass of the electron \( M_e \).

Although the QM approach differs from the approach in this article, the outcomes should be consistent with each other. Because there is no consistency between the outcomes of the approaches, there must be an error. Recall that in the QM approach the momentum density magnitude of vector \( \mathbf{g} \) is diminished with the factor \( \sin^2(\theta) \), because of the previous mentioned and marked arguments (*) and (**). To comprehend the effect of the correction of the magnitude of the momentum density \( \mathbf{g} \) with the factor \( \sin^2(\theta) \) we have to consider the QM equation for the total momentum; the integration of the momentum density \( \mathbf{g} \) over space to \( \mathbf{p} \), in more detail [Ref.4, Eq. (28.2)]:

\[
\mathbf{p} = \int (\varepsilon_0 v / c^2)E^2(r) \sin^2(\theta)2\pi r^2 \sin(\theta)d\theta dr \quad \text{(11)}
\]

In the above equation, the motion of the charge is independent of any variable in the equation, so we are allowed to abstract motion out of the integral. Considering that the electrostatic field \( \mathbf{E} \) is a function of \( r \), we get:

\[
\mathbf{p} = \mathbf{v} \int (\varepsilon_0 / c^2)E(r)^2 \sin^2(\theta)2\pi r^2 \sin(\theta)d\theta dr \quad \text{(11a)}
\]

Because \( (\varepsilon_0 / c^2)E(r)^2 \) in (11a) presents the mass density \( (\text{kg/m}^3) \) of the energy of the electrostatic field \( E(r) \) and \( 2\pi r^2 \sin(\theta)d\theta dr = dV \) is the volume element, the integration in Eqs. (11) and (11a) represents the calculation of the mass of the electrostatic field surrounding the charge. The corrections mentioned in (*) and (***) do not alter the implied physics of the integration. By integrating the momentum density \( \mathbf{g} \) around the charge all over space, the physical interpretation of the integration is the calculation of the mass of the electrostatic field surrounding the charge. A mass is a scalar and therefore the correction of the magnitude of the momentum density vector with factor \( \sin^2(\theta) \) becomes an omission, by which the conservation law for energy is violated. Part of the mass of the electrostatic field is unjustly ignored.

The magnitude of \( \mathbf{g} \) presents the magnitude of the mass density impulse of the moving electrostatic field \( \mathbf{E} \) in the direction of \( \mathbf{v} \). The direction of vector \( \mathbf{g} \) does not represent the direction of the mass/energy density movement of the electric field, and therefore not the direction of the impulse. Compensation of the magnitude of \( \mathbf{g} \) with factor \( \sin^2(\theta) \) violates the energy conservation law because part of the mass/energy of the electrostatic field is then ignored.

The momentum density \( \mathbf{g} \) at \( \mathbf{p} \) as a result of the electrostatic field/energy \( E(r) \) moving with speed \( v \) at \( r \) should be:

\[
\mathbf{g} = M_e \mathbf{v}
\]

where \( M_e = (\varepsilon_0 / c^2)E(r)^2 \) represents the mass density \( (\text{kg/m}^3) \) of the electrostatic field. Integration of \( \mathbf{g} \) over space gives:

\[
\mathbf{p} = \mathbf{v} \int 2\pi r^2 \sin(\theta)M_e d\theta dr = \mathbf{v} \int (\varepsilon_0 / c^2)E(r)^2 2\pi r^2 \sin(\theta)d\theta dr
\]

Feynman’s Eq. (28.3) becomes now \( \mathbf{p} = (e^2 / ac^2)\mathbf{v} \). This equation, expressed in the symbols used in this article, is:

\[
\mathbf{p} = (\theta_e ^2 / 4\pi \varepsilon_0 c^2 R_C) \mathbf{v} \quad \text{or} \quad \mathbf{p} = M_e \mathbf{v} \quad \text{or} \quad \mathbf{p} = 2M_p \mathbf{v}
\]

\( M_e \) is the mass of the electron and \( M_p \) is the mass equivalence of the electrostatic field). The corrected total momentum of the moving charge \( \mathbf{p} \) is now completely consistent with the derived equations in the previous Sections of this article. The correctly derived electromagnetic mass \( m_{elec} \) according to the QM approach, equals now the mass of the electron \( M_e \) and the dynamic energy of the electron becomes fully magnetic.

5. Discussion

How can the magnetic energy of an electron be at the same time the kinetic energy? The answer to that question must be that both forms of energy are different presentations of the same ‘dynamic’ energy. If an electron moves and collides with another particle, the change in kinetic energy is transferred from one particle to the other. The kinetic energy of the electron changes, because the electron moves now with a different speed. The magnetic energy also has to change, because the charge of the electron now also moves with a changed velocity.
The same argument is valid when an electron (electric current) loses magnetic energy through magnetic induction. The electron(s) slow down and lose kinetic energy.

When we consider the formulas for magnetic energy of an electric current and the kinetic energy of a mass, there are similarities. The magnetic energy of a current is \( W_m = LI^2 \) (joule), while the kinetic energy of a moving mass is \( W_k = \frac{1}{2} MV^2 \) (joule).

Consider an electric current, where the electrons move twice as fast, then the induced magnetic energy \( W_m \) becomes four times as large and so is the kinetic energy of the moving electrons in the current. Both formulas, for the magnetic and kinetic energy, are consistent with the presentation of equivalence for kinetic and magnetic energy.

The logical consequence of the equivalence for magnetic and kinetic energy is, that every mass that moves possesses kinetic energy also must have magnetic energy!

**The moving proton and magnetic energy**

The proton is, like the electron, a single particle, charged \( +Q_e \). The observable difference between the proton and electron is the opposite sign of the charge and the difference in mass. According to the chapter “The Proton and Neutron” in From Paradox to Paradigm [3], the intrinsic energy of the mass of a proton can be expressed by the equation:

\[
M_p c^2 = Q_e^2 / 8\varepsilon_0 R_p + \mu_0 Q_e^2 c^2 / 8\pi R_p
\]

(8)

where \( M_p \) is the mass of the proton and \( R_p \) is the radius of the proton sphere. The proton mass energy is half electrostatic energy, and half magnetic spin energy. The magnetic energy \( W_m \) of the magnetic field induced by the moving proton becomes accordingly:

\[
W_m = \frac{1}{8} \mu_0 Q_e^2 V_e^2 / \pi R_p = \frac{1}{2} M_p V_e^2
\]

**The moving hydrogen atom and the neutron**

The proton is a positive charged particle. The moving proton must possess magnetic energy. But if kinetic energy is the same as magnetic energy, any mass possessing kinetic energy must have magnetic energy!

The hydrogen atom possesses no electric field outside the radius of the molecule. All the electrostatic energy and therefore all the magnetic/kinetic energy of the moving hydrogen atom concentrate in the atom, between electron and proton.

The charge of a separated moving electron and proton is not shielded, as it is in the hydrogen atom. The separated electron and proton therefore have a much larger range in which the electric fields are present and can interact with other charged particles when there is relative movement. The electrostatic field in the hydrogen atom, and therefore the kinetic/magnetic energy, is contained in the space between the proton and the electron.

For the moving hydrogen atom the same arguments as for the moving electron and proton are valid. The kinetic energy of the hydrogen atom is the induced magnetic energy by the moving electrostatic field between proton and electron. Outside the hydrogen atom there is no electrostatic field or dielectric displacement, so outside the atom there is no magnetic field. The magnetic/kinetic energy is confined to the area of the electric field, between proton and electron, in the hydrogen atom.

When a proton and an electron fuse to a neutron, during the fusion process, the potential electrostatic energy of proton and electron is transferred into kinetic energy (magnetic energy). Although fused, the positive and negative charges of proton and electron still oscillate in the neutron, so the electrostatic field between both charges still exists, only now concentrated and therefore confined in the neutron.

The neutron does not possess an electrostatic field we can observe, because the oscillation frequency of the neutron is far too high (approx. \( 2 \times 10^{26} \text{ Hz} \) “From Paradox to Paradigm”, chapter “The Proton and Neutron”) to be detected. Not being detectable does not mean that for a neutron the equivalence of magnetic and kinetic energy is no longer valid.

**Bibliography**