6. BINARY - STAR PERICENTER ADVANCES

(attachment to essay: ***Relativity Replaced- Ether found around Earth***)

By Antonis N. Agathangelidis (Email anagath1943@gmail.com)

Let two stars of masses m_1 and m_2, being at a distance r the one from the other, are revolving around their common center of mass at the corresponding distances r_1 and r_2, it is clear ($r_1 + r_2 = r$), we also have:

$$r_1 \cdot m_1 = r_2 \cdot m_2 \ , \qquad r_1 = r \cdot \frac{m_2}{(m_1 + m_2)}, \qquad r_2 = r \cdot \frac{m_1}{(m_1 + m_2)} \tag{1}$$

$$r_1 \cdot m_1 = r_2 \cdot m_2 = \mu \cdot r \qquad \text{where} \qquad \mu = \frac{m_1 \cdot m_2}{(m_1 + m_2)} \quad (\text{'}reduced \ \ mass\text{'}) \tag{2}$$

We will describe the Newtonian motion of the body-1 (and also of body-2) around the common center of mass. Since the gravitational fields are conservative it means that the momentary masses m_1 and m_2 are depended from their distances from the center of mass only, we have for body-1:

$$-\frac{G \cdot m_2 \cdot m_1}{r^2} \cdot \hat{r} = \frac{d(m_1 \cdot \dot{\vec{r}}_1)}{dt} = \frac{dm_1}{dt} \cdot \dot{\vec{r}}_1 + m_1 \cdot \frac{d\dot{\vec{r}}_1}{dt} \tag{3}$$

Now we have $\vec{r}_1 \equiv r_1 \cdot \hat{r}$ and since $\dot{\hat{r}} \equiv \dot{\varphi} \cdot \hat{\varphi}$, $\dot{\hat{\varphi}} \equiv -\dot{\varphi} \cdot \hat{r}$ ($\hat{r}, \hat{\varphi}$ are unit vectors

–Fig. 8- of essay) the velocity vector of body-1 is written as:

$$\dot{\vec{r}}_1 = \dot{r}_1 \cdot \hat{r} + r_1 \cdot \dot{\varphi} \cdot \hat{\varphi} \tag{4}$$

it gives by differentiation:

$$\frac{d(\dot{\vec{r}}_1)}{dt} = (\ddot{r}_1 - r_1 \cdot \dot{\varphi}^2) \cdot \hat{r} + (2 \cdot \dot{r}_1 \cdot \dot{\varphi} + r_1 \cdot \ddot{\varphi}) \cdot \hat{\varphi} \tag{5}$$

Inserting (4) and (5) relations into (3) we get the vector equation for body-1:

$$-\frac{G \cdot m_2 \cdot m_1}{r^2} \cdot \hat{r} = \left[\frac{dm_1}{dt} \cdot \dot{r}_1 + m_1 \cdot (\ddot{r} - r_1 \cdot \dot{\varphi}^2)\right] \cdot \hat{r} + \left(\frac{dm_1}{dt} \cdot r_1 \cdot \dot{\varphi} + 2m_1 \cdot \dot{r}_1 \cdot \dot{\varphi} + m_1 r_1 \cdot \ddot{\varphi}\right) \cdot \hat{\varphi}$$

From above relation we get the well known relations:

$$-\frac{G \cdot m_2 \cdot m_1}{r^2} = \frac{dm_1}{dr_1} \cdot \dot{r}_1^{\,2} + m_1 \cdot (\ddot{r} - r_1 \cdot \dot{\varphi}^2) \tag{6}$$

(as the gravitational field is conservative even if mass m_1 is a function of υ_1^2 it must finally depended from r_1) , and

$$\frac{dm_1}{dt} \cdot r_1 \cdot \dot{\varphi} + 2m_1 \cdot \dot{r}_1 \cdot \dot{\varphi} + m_1 r_1 \cdot \ddot{\varphi} = \frac{1}{r_1} \cdot \frac{d}{dt}\left(m_1 \cdot r_1^{\,2} \cdot \dot{\varphi}\right) = 0 \tag{7}$$

From (7) we conclude:

$$m_1 \cdot r_1^{\,2} \cdot \dot{\varphi} \equiv M_1 (body-1..angular..momentum - cons\tan t-) \tag{8}$$

From (8) we get :

$$\dot{\varphi} = \frac{M_1}{m_1 \cdot r_1^2} \tag{9}$$

$$\dot{r}_1 = \frac{dr_1}{d\varphi} \cdot \frac{M_1}{m_1 \cdot r_1^2} \tag{10}$$

and

$$\ddot{r}_1 = \frac{d^2 r_1}{d\varphi^2} \cdot \frac{1}{r_1^4} \cdot \left(\frac{M_1}{m_1}\right)^2 - \left(\frac{dr_1}{d\varphi}\right)^2 \cdot \frac{2}{r_1^5} \cdot \left(\frac{M_1}{m_1}\right)^2 - \frac{dm_1}{m_1 \cdot dr_1} \cdot \frac{1}{r_1^4} \cdot \left(\frac{dr}{d\varphi}\right)^2 \cdot \left(\frac{M_1}{m_1}\right)^2 \tag{11}$$

Substituting (9), (10), and (11) into (6) relation we get:

$$-\frac{G \cdot m_2}{r^2} = \frac{d^2 r_1}{d\varphi^2} \cdot \frac{1}{r_1^4} \cdot \left(\frac{M_1}{m_1}\right)^2 - \left(\frac{dr_1}{d\varphi}\right)^2 \cdot \frac{2}{r_1^5} \cdot \left(\frac{M_1}{m_1}\right)^2 - \frac{1}{r_1^3} \cdot \left(\frac{M_1}{m_1}\right)^2 \tag{12}$$

Now we can do the well known transformation:

$r_1 = \frac{1}{u_1}$ then we have $\frac{dr_1}{d\varphi} = -\frac{1}{u_1^2} \cdot \frac{du_1}{d\varphi}$ and $\frac{d^2 r_1}{d\varphi^2} = -\frac{1}{u_1^2} \cdot \frac{d^2 u_1}{d\varphi^2} + \frac{2}{u_1^3} \cdot \left(\frac{du_1}{d\varphi}\right)^2$

The last transformation relations make (12) to receive the form:

$$\frac{G \cdot m_2}{r^2} = \frac{d^2 u_1}{d\varphi^2} \cdot u_1^2 \cdot \left(\frac{M_1}{m_1}\right)^2 + u_1^3 \cdot \left(\frac{M_1}{m_1}\right)^2$$

Which can retransformed back into the

$$\frac{G \cdot m_2}{r^2} = \frac{d^2\left(\frac{1}{r_1}\right)}{d\varphi^2} \cdot \left(\frac{M_1}{m_1 \cdot r_1}\right)^2 + \left(\frac{1}{r_1}\right) \cdot \left(\frac{M_1}{m_1 \cdot r_1}\right)^2$$

after the relations (2) the last relation becomes:

$$G \cdot m_2 = \frac{d^2\left(\frac{1}{r_1}\right)}{d\varphi^2} \cdot \frac{M_1^2}{\mu^2} + \left(\frac{1}{r_1}\right) \cdot \frac{M_1^2}{\mu^2} \tag{13}$$

Now the corresponding –to (13)- relation for the body-2 (orbiting around the common center of mass) takes the form

$$G \cdot m_1 = \frac{d^2\left(\frac{1}{r_2}\right)}{d\varphi^2} \cdot \frac{M_2^2}{\mu^2} + \left(\frac{1}{r_2}\right) \cdot \frac{M_2^2}{\mu^2} \tag{14}$$

Where

$$m_2 \cdot r_2^2 \cdot \dot{\varphi} \equiv M_2 (body-2..angular..momentum-cons\tan t-) \tag{15}$$

By adding in members (13) and (14) we get

$$G \cdot (m_1 + m_2) = \frac{d^2\left(\frac{1}{r_1}\right)}{d\varphi^2} \cdot \frac{M_1^2}{\mu^2} + \frac{d^2\left(\frac{1}{r_2}\right)}{d\varphi^2} \cdot \frac{M_2^2}{\mu^2} + \left(\frac{1}{r_1}\right) \cdot \frac{M_1^2}{\mu^2} + \left(\frac{1}{r_2}\right) \cdot \frac{M_2^2}{\mu^2} \tag{16}$$

By taking in mind (1)-(2) relations and that total angular momentum M is the sum of the two partial angular moments $M = M_1 + M_2$ we get:

$$M = M_1 + M_2 = (m_1 \cdot r_1^2 + m_2 \cdot r_2^2) \cdot \dot{\varphi} = \left(\mu \cdot r \cdot (r_1 + r_2)\right) \cdot \dot{\varphi} = \mu \cdot r^2 \cdot \dot{\varphi}$$

Finally equation (16) with the help of (1)-(2) relations, offers the equation of the relative orbit of the two masses m_1 and m_2, it takes finally the form:

$$\frac{d^2\left(\frac{1}{r}\right)}{d\varphi^2} + \frac{1}{r} = \frac{G}{M^2} \cdot \frac{(m_1 \cdot m_2)^2}{(m_1 + m_2)} \qquad (17)$$

Since below it will be assumed the masses of the two stars as being very slightly variables (depending from *r*), and since the relative orbit of the two masses is also an ellipse for this reason, the above relative ellipse it has a parameter P:

$$\frac{1}{P} \equiv \frac{G}{M^2} \cdot \frac{(< m_1 > \cdot < m_2 >)^2}{(< m_1 > + < m_2 >)} \qquad (18)$$

Where *P* the parameter of the relative elliptic orbit $P = a \cdot (1 - \varepsilon^2)$, *α the maximum semi axis* of the relative orbit, ε the eccentricity and the <*m*> s are to denote the mean values of rotating *masses* along the relative orbit.

Now because of the assumed behavior of Newtonian gravity to act with equal forces on each one of the two stars, creating and offering mutually ‘gravitational work’ (as their distances are increased around their center of mass), we have to write down the “complementary” equations of the “Conservation of the Energy” into the gravitational field for the two separate masses in two separate and directional ways –around the center of mass-:

$$-\frac{G \cdot m_2 (attracting.mass)}{r^2} \cdot m_1 (attracted) \cdot dr_1 + d(m_1 \cdot C^2_{1 \leftarrow (2)}) + \Delta W_1 = 0 \qquad (19)$$

$$. -\frac{G \cdot m_1 (attracting.mass)}{r^2} \cdot m_2 (attracted) \cdot dr_2 + d(m_2 \cdot C^2_{2 \leftarrow (1)}) + \Delta W_2 = 0 \qquad (20)$$

The quantities ΔW_1 and ΔW_2 don’t correspond necessarily to any “radiation of gravity” (we don’t know if such a “radiation” is produced at all), but we are sure that new ether can be inserted in the rotating system, (along the axis passing through the center of mass, -from the “North” and “South” poles of the axis-, of the rotating system and be finally centrifuged). The incoming fluid ether initially must flows close-around the limits of rotating Roche lobes and then it must be mixed with the resting ether being in the further wide space. This centrifuging of the fluid ether can be the cause for a perennial loss of the kinetic energy of the rotating system (i.e. the slight increase of the period of rotation of the system).

In the equation (19) the attraction of mass-2 produces a work on mass-1 and the quantity $C_{1 \leftarrow (2)}$ is the speed of light in the position of mass-1, due of the presence of mass-2 (at the distance r), and symmetrically in equation (20) the attraction of mass-1 produces a work on mass-2 and the quantity $C_{2\leftarrow(1)}$ is the speed of light in the position of mass-2, due of the presence of mass-1 (at the distance r).

By adding in members the two equations (19) and (20) (and by putting for simplicity $\Delta W_1 = \Delta W_2 = 0$) we get the "totally Complementary equation" of the energy in gravitational field.

$$-\frac{G \cdot m_2 \cdot m_1}{r^2} \cdot dr + d(m_1 \cdot C_{1\leftarrow(2)}^2) + d(m_2 \cdot C_{2\leftarrow(1)}^2) = 0 \qquad (21)$$

We will not use this total-energy equation (21) to avoid the great perplexing if the two masses but we will use the (19) and (20) Complementary equations separately:

From the (19) relation we get (we assume $\Delta W_1 = \Delta W_2 = 0$):

$$\frac{G \cdot m_2}{r^2} \cdot \left(\frac{m_2}{m_1 + m_2} \right) = \frac{dm_1}{m_1 \cdot dr} \cdot C_{1\leftarrow(2)}^2 + \frac{dC_{1\leftarrow(2)}^2}{dr} \qquad (19\text{-a})$$

from (2.76)* we have $\frac{dC_{1\leftarrow(2)}^2}{dr} = \frac{4G \cdot m_2}{r^2}$ and from (2.77)* it is $C_{1\leftarrow(2)}^2 = C_\infty^2 \cdot \left[1 - \frac{4 \cdot G \cdot m_2}{r \cdot C_\infty^2} \right]$

and thus we get from equation (19-a):

$$\frac{dm_1}{m_1} = \frac{-\frac{4Gm_2}{C_\infty^2} \cdot \left(1 - \frac{m_2}{4 \cdot (m_1 + m_2)} \right) dr}{\left[r^2 - \frac{4 \cdot G \cdot m_2}{C_\infty^2} \cdot r \right]} = -4 \cdot \beta_2 \cdot \left(1 - \frac{m_2}{4 \cdot (m_1 + m_2)} \right) \cdot \frac{dr}{\left[r^2 - 4 \cdot \beta_2 \cdot r \right]}$$

(19-b)

where $\beta_2 \equiv \frac{G \cdot m_2}{C_\infty^2}$

Similarly from (20) relation we get (again is assumed $\Delta W_1 = \Delta W_2 = 0$):

$$\frac{G \cdot m_1}{r^2} \cdot \left(\frac{m_1}{(m_1 + m_2)} \right) = \frac{dm_2}{m_2 \cdot dr} \cdot C_{2\leftarrow(1)}^2 + \frac{dC_{2\leftarrow(1)}^2}{dr} \qquad (20\text{-a})$$

From (2.76)* we have $\frac{dC_{2\leftarrow(1)}^2}{dr} = \frac{4G \cdot m_1}{r^2}$ and from (2.77)* it is $C_{2\leftarrow(1)}^2 = C_\infty^2 \cdot \left[1 - \frac{4 \cdot G \cdot m_1}{r \cdot C_\infty^2} \right]$

and thus we get from the complementary equation (20-a):

$$\frac{dm_2}{m_2} = \frac{-\frac{4Gm_1}{C_\infty^2} \cdot \left(1 - \frac{m_1}{4 \cdot (m_1 + m_2)} \right) dr}{\left[r^2 - \frac{4 \cdot G \cdot m_1}{C_\infty^2} \cdot r \right]} = -4 \cdot \beta_1 \cdot \left(1 - \frac{m_1}{4 \cdot (m_1 + m_2)} \right) \cdot \frac{dr}{\left[r^2 - 4 \cdot \beta_1 \cdot r \right]}$$

(20-b)

where $\beta_1 \equiv \frac{G \cdot m_1}{C_\infty^2}$

In the above relations (19-b) [and (20-b)] we assume (successively) and for the moment, the terms and factors containing masses in their second members, as constants and variable (in the second member) only the r ; then we obtain easily by integration, –from r to infinite-, the following relations:

$$m_{1,r} \approx m_{1,\infty} \left\{ 1 + \frac{4\beta_2}{r} \cdot \left[1 - \frac{m_2}{4 \cdot (m_1 + m_2)} \right] \right\} \quad \text{and}$$

$$m_{2,r} \approx m_{2,\infty} \cdot \left\{ 1 + \frac{4\beta_1}{r} \cdot \left[1 - \frac{m_1}{4 \cdot (m_1 + m_2)} \right] \right\} \quad (22)$$

Evidently, the mean values of the two masses $m_{1,r}$ and $m_{2,r}$ along the relative ellipse, are given respectively by:

$$< m_{1,r} > \approx m_{1,\infty} \cdot \left\{ 1 + \frac{4\beta_2}{P} \cdot \left[1 - \frac{m_2}{4 \cdot (m_1 + m_2)} \right] \right\} \text{and}$$

$$< m_{2,r} > \approx m_{2,\infty} \left\{ 1 + \frac{4\beta_1}{P} \cdot \left[1 - \frac{m_1}{4 \cdot (m_1 + m_2)} \right] \right\} \quad (23)$$

Now by eliminating the masses $m_{1,\infty}$ and $m_{2,\infty}$ between the corresponding pairs of equations (22) and (23) we do find:

$$m_{1,r} \approx . < m_{1,r} > \left\{ 1 + 4\beta_2 \left(\frac{1}{r} - \frac{1}{P} \right) \cdot \left[1 - \frac{m_2}{4 \cdot (m_1 + m_2)} \right] \right\} \quad (24)$$

and $$m_{2,r} \approx . < m_{2,r} > \cdot \left\{ 1 + 4\beta_1 \left(\frac{1}{r} - \frac{1}{P} \right) \cdot \left[1 - \frac{m_1}{4 \cdot (m_1 + m_2)} \right] \right\} \quad (25)$$

Replacing the masses m_1 and m_2 from (24)-(25) into the product $(m_1 \cdot m_2)^2$ we get

$$(m_1 \cdot m_2)^2 \approx (< m_1 >< m_2 >)^2 \left\{ 1 + 8\beta_2 \cdot \left(\frac{1}{r} - \frac{1}{\mathrm{P}} \right) \cdot \left(1 - \frac{m_2}{4 \cdot (m_1 + m_2)} \right) + 8\beta_1 \cdot \left(\frac{1}{r} - \frac{1}{\mathrm{P}} \right) \cdot \left(1 - \frac{m_1}{4 \cdot (m_1 + m_2)} \right) \right\}$$

(26)

while replacing the masses from (24)-(25) into the sum $(m_1 + m_2)$ we should get

$$m_1 + m_2 \approx (< m_1 > + < m_2 >) \cdot \left\{ 1 + \frac{7 \cdot G < \mu >}{C_\infty^2} \cdot \left(\frac{1}{r} - \frac{1}{P} \right) \right\} \quad (27)$$

Now by replacing (26) and (27) into the 2[nd] part of (17) and after (18) we get for the 2[nd] member of the relation (17):

$$\frac{G}{M^2} \frac{(m_1 \cdot m_2)^2}{(m_1 + m_2)} \approx \frac{1}{P} \cdot \left\{ 1 + 8\beta_2 \cdot \left(\frac{1}{r} - \frac{1}{\mathrm{P}} \right) \cdot \left(1 - \frac{m_2}{4 \cdot (m_1 + m_2)} \right) + 8\beta_1 \cdot \left(\frac{1}{r} - \frac{1}{\mathrm{P}} \right) \cdot \left(1 - \frac{m_1}{4 \cdot (m_1 + m_2)} \right) - \frac{7 \cdot G < \mu >}{C_\infty^2} \cdot \left(\frac{1}{r} - \frac{1}{P} \right) \right\}$$

After the above we can write down the relation (17):

$$\frac{d^2\left(\frac{1}{r}\right)}{d\varphi^2}+\left[\frac{1}{r}-\frac{1}{\mathrm{P}}\right]\cdot\left(1-\frac{8G<m_2>\left(1-\frac{m_2}{4\cdot(m_1+m_2)}\right)+8G<m_1>\left(1-\frac{m_1}{4\cdot(m_1+.m_2)}\right)-7G<\mu>}{P\cdot C_\infty^2}\right)=0$$

(28)

The solution of the above differential equation is the following

$$\frac{1}{r}=\frac{1}{P}+\aleph\cdot\cos(\psi-\psi_0)$$

where $\aleph$ is the integration constant and the angle ψ is:

$$\psi=\varphi\cdot\sqrt{1-\frac{8G<m_2>\left(1-\frac{m_2}{4\cdot(m_1+m_2)}\right)+8G<m_1>\left(1-\frac{m_1}{4\cdot(m_1+.m_2)}\right)-7G<\mu>}{\mathrm{P}\cdot C_\infty^2}}$$

(ψ_0 is the value of ψ at the moment $t=0$)

It is clear that between two consecutive arrivals of the two masses m_1 and m_2 at their closest position ψ must change by 2π and in order for this to be happen the angle φ must changed by

$$\Delta\varphi=\frac{2\pi}{\sqrt{1-\frac{8G<m_2>\left(1-\frac{m_2}{4\cdot(m_1+m_2)}\right)+8G<m_1>\left(1-\frac{m_1}{4\cdot(m_1+.m_2)}\right)-7G<\mu>}{\mathrm{P}\cdot C_\infty^2}}}$$

or

$$\Delta\varphi\approx 2\pi\cdot\left[1+\frac{4G<m_2>\left(1-\frac{m_2}{4\cdot(m_1+m_2)}\right)+4G<m_1>\left(1-\frac{m_1}{4\cdot(m_1+.m_2)}\right)-3{,}5\cdot G<\mu>}{P\cdot C_\infty^2}\right]$$

i.e. the big axis of the relative ellipse revolves in the sense of revolution of the system at a rate:

$$(\Delta\varphi-2\pi)\approx 2\pi\cdot\left[\frac{4\cdot G<m_2>\left(1-\frac{m_2}{4\cdot(m_1+m_2)}\right)+4\cdot G<m_1>\left(1-\frac{m_1}{4\cdot(m_1+.m_2)}\right)-3{,}5\cdot G<\mu>}{\mathrm{P}\cdot C_\infty^2}\right]$$

(2.113-b)*

(rads/period of revolution)

This relation (2.113-b)* can be applied succefully to Mercury's perihelion advance (here –calculated- 42.96 degree seconds per century), as well as to the famous double pulsar PSR 1913+16 where $m_1 = 1.4$ and $m_2 = 1.42$ (solar masses), $\alpha = 6.5$ light seconds, eccentricity $\varepsilon = 0.617127$, and period of revolution of system $T = 27907$ seconds. Then the relation (2.113-b)* gives

$$(\Delta\varphi - 2\pi) \approx 3.6887^0 \text{ degrees per year}$$

This theoretically calculated result for the PSR 1913+16 system appears to be quite satisfactory since it must be referred to a rotating system of bodies appearing of course spherical symmetries, (-exactly like the relativistic consideration-calculation-); The "measured mean advance of 4.226^0 degrees per year" is sufficiently close to the calculated one (of 3.6887^0 degrees per year) and the small difference of: 0.537^0 degrees per year have necessarily to be owed in the deviations from spherical symmetry of the two masses due of their proximity of members or their fast rotations about their axes. In other words, due of the expected lack of spherical symmetry of the two masses of the tight system PSR 1913+16, it does not permitted the calculation, -by the relation (2.113-b)*, to reach the value of the observed rate of the pericenter advance of the system PSR 1913+16.

NOTE. The measured reduction of the orbital period of the said double pulsar PSR 1913+16 does not necessarily means the emission of "gravitational waves" but instead it easily can be ascribed to the following phenomenon: "As the ether, -or a dark matter- is attracted by the gravity along the axis of that rapidly rotating binary system, it –the fluid ether-, can be set in motion undergoing *centrifugal motion outwards* while new ether –or dark matter- comes into this centrifuge along the *north* and *south poles* of the axis of rotation of the fast binary system".

---

*This numbering of relations is referred in the Text of the essay: "***Relativity Replaced- Ether found around Earth***"