Generalization of Galilean Transformation

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Abstract:

In the article, a generalized Galilean transformation was derived. Obtained transformation is the basis for development of new physical theory, which was called the Special Theory of Ether.

The generalized Galilean transformation can be expressed by relative speeds (26)-(27) or by the parameter $\delta(v)$ (37)-(38). Based on conclusions of the Michelson-Morley's and Kennedy-Thorndike's experiments, the parameter $\delta(v)$ was determined. This allows the transformation to take a special form (81)-(82), which is consistent with experiments in which velocity of light is measured.

On the basis of obtained transformation, the formulas for summing speed and relative speed were also determined.

The entire article includes only original research conducted by its author.

Keywords: kinematics of bodies, universal frame of reference, coordinate and time transformation, one-way speed of light, summing speed, relative speed

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1. Introduction

The article explains results of Michelson-Morley's [3] and Kennedy-Thorndike's experiments [1], assuming that there is a universal frame of reference (ether), in which velocity of light has a constant value. In moving inertial frame of reference, the velocity of light may vary.

In the article, transformations between inertial systems were derived with analytical method. Derived transformation is a generalization of Galilean transformation, because she becomes Galilean transformation in a particular case. Thus it has been shown that it is not true that Michelson-Morley's and Kennedy-Thorndike's experiments prove that there is no universal frame of reference and that velocity of light in vacuum is constant.

The reasoning presented in this article is based on observation that one-way speed of light has never been measured accurately. In all accurate laboratory experiments, as in Michelson-Morley's and Kennedy-Thorndike's experiment, the average velocity of light on a closed trajectory that returns to its starting point was only measured. Therefore, assumption of a constant velocity of light in vacuum (instantaneous velocity) adopted in the Special Theory of Relativity has no strict experimental justification. In works [6]-[11] we have shown that Michelson-Morley's and Kennedy-Thorndike's experiments can be explained by the theory with a universal frame of reference. In the work [12] we have shown that there is infinite number of such theories. Thus it is not true that these experiments have shown that there is no ether in which light propagates.

Derivation presented in this article is based on these findings, i.e. assumptions that for each observer the average velocity of light moving forth and back is constant and that there is a universal frame of reference.

2. Adopted assumptions

In presented analysis, the following assumptions were adopted:

I. There is a frame of reference in relation to which the velocity of light in vacuum has the same value in each direction. This universal frame of reference is called ether.

II. Average velocity of light on the light path forth and back is for every observer independent from the direction of light propagation. This results from Michelson-Morley's experiment.

III. Average velocity of light on the light path forth and back does not depend on the observer's velocity in relation to a universal frame of reference. This results from Kennedy-Thorndike's experiment.

IV. In perpendicular direction to the velocity direction of body in relation to ether, its contraction or extension does not occur.

V. «Inertial system – inertial system» transformation is linear.

VI. Between inertial systems, there is a symmetry of the following form (when inertial systems U_1 and U_2 move in relation to universal frame of reference along their axes x_1 and x_2, which are parallel to each other).

$$\left.\frac{dx_1}{dt_2}\right|_{\frac{dx_2}{dt_2}=0} = -\left.\frac{dx_2}{dt_1}\right|_{\frac{dx_1}{dt_1}=0} \tag{1}$$

Assumption VI indicates that in coordinate transformation, the module coefficient at t is the same in primary and reverse transformation (coefficient e in transformations (15)).

Derived transformation presented in this article differs from derivation of Lorentz's transformation on which STR is based. In STR, in derived Lorentz's transformation, it is assumed that reverse transformation has the same form as the primary transformation. This assumption is based on a belief that all inertial systems are equivalent. In derivation presented in this article we do not assume what form the whole reverse transformation takes. We only assume what form one reverse transformation factor has (assumption VI).

Adopted assumptions in this article on the velocity of light are also weaker than those adopted in STR. The STR assumes that velocity of light is absolutely constant, even though no experiment has proved it. In this article, the assumption was made resulting from experiments that the average velocity of light on a path forth and back to the mirror is constant (assumption II and III). In presented dissertations, light velocity is assumed to be constant in only one universal frame of reference – ether (assumption I).

Assumptions IV and V are identical to those on which STR is based.

In works [6]-[12] an identical transformation was derived as (83)-(84), but in a different way, using the geometric method.

3. Derived transformation between inertial systems

An aim is to determine coordinate and time transformation between inertial systems U_1 and U_2, Figure 1. Systems move in relation to each other parallel to axis x. The U_1 system moves relative to U_2 system with velocity $v_{1/2}$. The U_2 system moves relative to U_1 system with velocity $v_{2/1}$ ($v_{1/2} \cdot v_{2/1} \leq 0$).

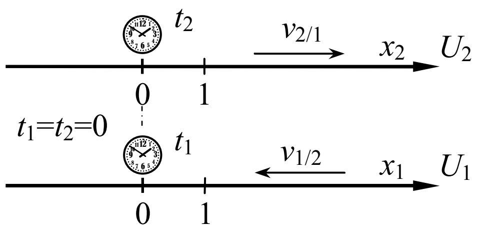

Fig. 1. Two inertial systems U_1 and U_2 move relative to each other with relative speeds $v_{1/2}$ and $v_{2/1}$.

Generalization of Galilean transformation is to allow the possibility that modules of velocity value $v_{1/2}$ and $v_{2/1}$ can be different.

In considered inertial systems, clocks are synchronized. Now we are only establishing that in a moment, when beginnings of systems overlap (coordinate $x_1 = 0$ from U_1 system is next to coordinate $x_2 = 0$ from U_2 system), then clocks found at these coordinates are reset. Thanks to such an establishment, there are no constant terms in transformations (2) and (3).

Assumption V guarantees that the Newton's first law is applicable in every inertial frame of reference, i.e. if a body moves uniformly in one inertial frame of reference, then its motion observed from another inertial frame of reference will also be uniform. This means that coordinate and time transformation between inertial systems U_1 and U_2 has a form of

$$\begin{aligned} t_1 &= a \cdot t_2 + b' \cdot x_2 \\ x_1 &= e' \cdot t_2 + g \cdot x_2 \end{aligned} \tag{2}$$

Coefficient $a > 0$, as in no system the time cannot flow backwards.

Now we will write the reverse transformation. If in U_2 system, the time flows quicker, thus in U_1 system it is slower. Therefore, in reverse transformation, the coefficient must be replaced by $1/a$. Similarly, if in one system a length contraction occurs, in the second is an extension. Hence in the reverse transformation, it is necessary to replace coefficient g by $1/g$. This method to determine values of two coefficients in reverse transformation on $1/a$ and $1/g$, we call the natural way of determining coefficients in the reverse transformation.

There are no assumptions for coefficient b', and therefore in the reverse transformation any coefficient b'' was accepted.

The reverse transformation has a form of

$$\begin{aligned} t_2 &= \frac{1}{a} t_1 - b'' \cdot x_1 \\ x_2 &= -e'' \cdot t_1 + \frac{1}{g} x_1 \end{aligned} \tag{3}$$

If the velocity of U_2 system relative to U_1 is positive, the velocity of U_1 system relative to U_2 is negative. Hence coefficients e' and $-e''$ are opposite signs. Assumption VI regards values of these coefficients. It is possible to calculate differentials appearing in this assumption from (2) and (3). They have a form of

$$dx_1 = e' \cdot dt_2 + g \cdot dx_2 \quad \Rightarrow \quad \frac{dx_1}{dt_2} = e' + g \frac{dx_2}{dt_2} \tag{4}$$

$$dx_2 = -e'' \cdot dt_1 + \frac{1}{g} dx_1 \quad \Rightarrow \quad \frac{dx_2}{dt_1} = -e'' + \frac{1}{g} \frac{dx_1}{dt_1} \tag{5}$$

i.e.

$$\frac{dx_2}{dt_2} = 0 \quad \Rightarrow \quad e' = \frac{dx_1}{dt_2} \tag{6}$$

$$\frac{dx_1}{dt_1} = 0 \quad \Rightarrow \quad e'' = -\frac{dx_2}{dt_1} \tag{7}$$

Due to assumption VI we obtain

$$e' = e'' = e \tag{8}$$

Placing t_2, x_2 from the reverse transformation (3) to transformation (2) we will obtain

$$\begin{aligned} t_1 &= a(\frac{1}{a}t_1 - b''x_1) + b'(-et_1 + \frac{1}{g}x_1) = t_1(1 - b'e) + x_1(\frac{b'}{g} - ab'') \\ x_1 &= e(\frac{1}{a}t_1 - b''x_1) + g(-et_1 + \frac{1}{g}x_1) = t_1(\frac{e}{a} - eg) + x_1(1 - b''e) \end{aligned} \tag{9}$$

Since formula (9) should be real for all t_1, x_1, the equations must be fulfilled

$$1 - b'e = 1 \tag{10}$$

$$\frac{b'}{g} = ab'' \tag{11}$$

$$\frac{e}{a} = eg \tag{12}$$

$$1 - b''e = 1 \tag{13}$$

As from the assumption, systems move in relation to each other, thus $e \neq 0$. On this basis from (10) results that $b' = 0$. By analogy from (13) results that $b'' = 0$. From (12) results

$$g = \frac{1}{a} \tag{14}$$

Searched transformations can be written in a form of

$$\begin{cases} t_1 = at_2 \\ x_1 = et_2 + \frac{1}{a}x_2 \end{cases} \qquad \begin{cases} t_2 = \frac{1}{a}t_1 \\ x_2 = -et_1 + ax_1 \end{cases} \tag{15}$$

We will determine the differentials from these transformations

$$\begin{cases} dt_1 = a \cdot dt_2 \\ dx_1 = e \cdot dt_2 + \frac{1}{a}dx_2 \end{cases} \qquad \begin{cases} dt_2 = \frac{1}{a}dt_1 \\ dx_2 = -e \cdot dt_1 + a \cdot dx_1 \end{cases} \tag{16}$$

On the basis of these differentials, it is possible to determine relative velocities of U_1 and U_2 systems. If we consider any point with a fixed coordination in U_2 system, then from the first transformation (16) we obtain velocity $v_{2/1}$ of U_2 system in relation to U_1 system

$$\frac{dx_2}{dt_2} = 0 \quad \Rightarrow \quad v_{2/1} = \frac{dx_1}{dt_1} = \frac{e \cdot dt_2 + \frac{1}{a}dx_2}{a \cdot dt_2} = \frac{e}{a} + \frac{1}{a^2}\frac{dx_2}{dt_2} = \frac{e}{a} \tag{17}$$

If we will consider any point with a fixed coordination in U_1 system, then the second transformation (16) we obtain velocity of $v_{1/2}$ of U_1 system in relation to U_2 system

$$\frac{dx_1}{dt_1}=0 \quad \Rightarrow \quad v_{1/2}=\frac{dx_2}{dt_2}=\frac{-e\cdot dt_1+a\cdot dx_1}{\frac{1}{a}dt_1}=-ea+a^2\frac{dx_1}{dt_1}=-ea \tag{18}$$

We divide the equation (18) by equation (17) and we will obtain

$$\frac{v_{1/2}}{v_{2/1}}=-a^2 \tag{19}$$

From the relation (19) and on the basis of (17) and (18), it is possible to determine unknown coefficients

$$a=\sqrt{-\frac{v_{1/2}}{v_{2/1}}} \tag{20}$$

$$e=-v_{1/2}/a=-v_{1/2}\sqrt{-\frac{v_{2/1}}{v_{1/2}}} \tag{21}$$

$$e=v_{2/1}\cdot a=v_{2/1}\sqrt{-\frac{v_{1/2}}{v_{2/1}}} \tag{22}$$

Since velocity of $v_{1/2}$ and $v_{2/1}$ have different signs, and therefore it is possible to show that relations (21) and (22) are equivalent (below, in '±', character '+' is appears when $v_{1/2} < 0$, while character '–' appears when $v_{1/2} > 0$)

$$\begin{aligned} e&=-v_{1/2}\sqrt{-\frac{v_{2/1}}{v_{1/2}}}=\pm\sqrt{v_{1/2}^2}\sqrt{-\frac{v_{2/1}}{v_{1/2}}}=\pm\sqrt{-v_{1/2}^2\frac{v_{2/1}}{v_{1/2}}}= \\ &=\pm\sqrt{-v_{1/2}\cdot v_{2/1}}=\pm\sqrt{-\frac{v_{1/2}}{v_{2/1}}v_{2/1}^2}=\pm\sqrt{v_{2/1}^2}\sqrt{-\frac{v_{1/2}}{v_{2/1}}}=v_{2/1}\sqrt{-\frac{v_{1/2}}{v_{2/1}}}=e \end{aligned} \tag{23}$$

If we multiply (21) and (22), we will obtain

$$e^2=-v_{1/2}v_{2/1} \tag{24}$$

and thus the same as from (23) we will obtain

$$e=+\sqrt{-v_{1/2}v_{2/1}} \quad \vee \quad e=-\sqrt{-v_{1/2}v_{2/1}} \tag{25}$$

Coefficient e may have a different sign. From (23) results that coefficient $e > 0$, when velocity $v_{2/1} > 0$, while $e < 0$, when velocity $v_{2/1} < 0$.

On the basis of (20), (21) and (22), transformations (15) can be expressed from relative speeds and can be written in a form of

$$\begin{cases} t_1=\sqrt{-\frac{v_{1/2}}{v_{2/1}}}\cdot t_2 \\ x_1=v_{2/1}\sqrt{-\frac{v_{1/2}}{v_{2/1}}}\cdot t_2+\sqrt{-\frac{v_{2/1}}{v_{1/2}}}\cdot x_2 \end{cases} \tag{26}$$

$$\begin{cases} t_2 = \sqrt{-\frac{v_{2/1}}{v_{1/2}}} \cdot t_1 \\ x_2 = v_{1/2}\sqrt{-\frac{v_{2/1}}{v_{1/2}}} \cdot t_1 + \sqrt{-\frac{v_{1/2}}{v_{2/1}}} \cdot x_1 \end{cases} \quad (27)$$

We have obtained completely symmetrical transformations. In transformation (26), we may just convert indexes 1 into 2 and 2 into 1 in order to obtain transformation (27). This is despite the fact that apparently non-symmetry was introduced in derived transformation (formula (2) and (3)).

Assumption V and VI was enough to obtain transformation (26)-(27), as well as a natural way of determining the value of coefficients in reverse transformation.

Transformation (26)-(27) is a generalized Galilean transformation, expressed from relative speeds. If $v_{2/1} \approx -v_{1/2}$ occurs for U_2 and U_1 systems, then these transformations came down to Galilean transformation.

From time transformation (26)-(27) results that if in some inertial system the clock indicates time $t_2 = 0$, then in every inertial system the clock found by this clock also indicates time $t_1 = 0$. This means that clocks in inertial systems are synchronized with the external method, proposed in the article [2]. It results that this method of clock synchronization is a consequence of assumptions on the basis of which the transformation (26)-(27) was derived (foundations V and VI) and the natural method of determining values of coefficients in reverse transformation.

Synchronization of clocks with the external method consists in setting all clocks on the basis of clocks indications of one distinguished inertial system (let it be U_1 system). Clocks in U_2 system are reset when beginnings of U_1 and U_2 systems overlap. If the clock of U_1 system indicates time $t_1 = 0$, then clock next to it of U_2 system is also reset, i.e. $t_2 = 0$. This way of clocks synchronization enables to synchronize clocks in all inertial systems, if there is a possibility to synchronize clocks in some first inertial system. At this stage we do not resolve how the synchronized clocks in U_1 system have been synchronized. The problem of clocks synchronization in the first system will be solved in Chapter 5.

4. Implementation of a universal frame of reference

To transformation (26) and (27) we will implement a universal frame of reference (ether). By v_1, v_2 were indicated velocities of U_1 and U_2 system relative to universal frame of reference (absolute speeds). Since there is a universal frame of reference, every movement in the space can be described by absolute speeds in relation to that system. Therefore relative speeds $v_{1/2}$ and $v_{2/1}$ depend explicitly on absolute speeds v_1, v_2. We assume that function $F(\cdot,\cdot)$ combines relative speeds of systems and their absolute speeds in the following way

$$\begin{cases} v_{1/2} = -v_{2/1}F(v_1, v_2) \\ v_{2/1} = -v_{1/2}F(v_2, v_1) \end{cases} \quad (28)$$

From equations (28), after multiplying them by sides, results that function $F(\cdot,\cdot)$ has a form of

$$F(v_1, v_2) = \frac{1}{F(v_2, v_1)} \quad (29)$$

Trivial solutions of this functional equation are

$$F(v_1, v_2) = 1 \quad (30)$$

and

$$F(v_1, v_2) = -1 \tag{31}$$

The first of these solutions gives Galilean transformation. The second leads to contradiction. Nontrivial solution of this functional equation is function $F(\cdot,\cdot)$ in a form of

$$F(v_1, v_2) = \frac{G(v_1, v_2)}{G(v_2, v_1)} = \frac{1}{\frac{G(v_2, v_1)}{G(v_1, v_2)}} = \frac{1}{F(v_2, v_1)} \tag{32}$$

We assume that for our needs a function $F(\cdot,\cdot)$ is sufficient with divided variables, then it is possible to write it with quotient of certain functions $M(\cdot)$ and $N(\cdot)$

$$F(v_1, v_2) = \frac{G'(v_1)\cdot G''(v_2)}{G'(v_2)\cdot G''(v_1)} = \frac{G'(v_1)/G''(v_1)}{G'(v_2)/G''(v_2)} = \frac{M(v_1)}{N(v_2)} = \frac{1}{\frac{M(v_2)}{N(v_1)}} = \frac{N(v_1)}{M(v_2)} \tag{33}$$

From the equation (33) results that $M(v) = N(v)$. Now it can be written in a form of

$$F(v_1, v_2) = \frac{M(v_1)}{M(v_2)} = \frac{\frac{M(v_1)}{M(0)}}{\frac{M(v_2)}{M(0)}} = \frac{\delta(v_1)}{\delta(v_2)} \tag{34}$$

Function $\delta(v)$ at this stage is unknown. Based on (34), it is known to be dimensionless. Without a loss of generality, it can be assumed that it is a positive function and in zero assumes value one, because

$$\delta(0) = \frac{M(0)}{M(0)} = 1 \tag{35}$$

On the basis of (28) and (34) we will obtain

$$-\frac{v_{2/1}}{v_{1/2}} = \frac{\delta(v_2)}{\delta(v_1)} \tag{36}$$

On this basis, transformation (26)-(27) can be written in the form expressed from parameter $\delta(v)$

$$\begin{cases} t_1 = \sqrt{\frac{\delta(v_1)}{\delta(v_2)}}\cdot t_2 \\ x_1 = v_{2/1}\sqrt{\frac{\delta(v_1)}{\delta(v_2)}}\cdot t_2 + \sqrt{\frac{\delta(v_2)}{\delta(v_1)}}\cdot x_2 \end{cases} \tag{37}$$

$$\begin{cases} t_2 = \sqrt{\frac{\delta(v_2)}{\delta(v_1)}}\cdot t_1 \\ x_2 = v_{1/2}\sqrt{\frac{\delta(v_2)}{\delta(v_1)}}\cdot t_1 + \sqrt{\frac{\delta(v_1)}{\delta(v_2)}}\cdot x_1 \end{cases} \tag{38}$$

This transformation form required one additional assumption in relation to assumptions on which transformations (26) and (27) are based. This is assumption on the existence of a universal frame of reference.

* * *

If $v_1 = -v_2 = v$, then there is a full symmetry, for the observer related to ether, between U_1 and U_2 systems. If the space is supposed to be isotropic, i.e. all directions in ether are supposed to be equivalent, then $v_{2/1} = -v_{1/2}$ must occur. On the basis of (37) and (38) we will obtain

$$x_1 = v_{2/1}\sqrt{\frac{\delta(v_1)}{\delta(v_2)}}\cdot\left(\sqrt{\frac{\delta(v_2)}{\delta(v_1)}}\cdot t_1\right)+\sqrt{\frac{\delta(v_2)}{\delta(v_1)}}\cdot\left(-v_{2/1}\sqrt{\frac{\delta(v_2)}{\delta(v_1)}}\cdot t_1+\sqrt{\frac{\delta(v_1)}{\delta(v_2)}}\cdot x_1\right) \tag{39}$$

$$0 = v_{2/1}\cdot t_1 - v_{2/1}\frac{\delta(-v)}{\delta(v)}\cdot t_1 \tag{40}$$

On this basis we will obtain another, after (35), a universal property of function $\delta(v)$

$$\delta(v) = \delta(-v) \tag{41}$$

5. Designation of function $\delta(v)$ based on Michelson-Morley's experiment

Function $\delta(v)$ was determined in subsection, assuming that results of Michelson-Morley's and Kennedy-Thorndike's experiments are fulfilled. Experiments show that measured average velocity of light $c_{śr}$, on the path forth and back, is constant in each inertial frame of reference U' and is the same in each direction (assumption II and III). We assume that in U system, i.e. ether, the velocity of light c is constant in each direction (assumption I).

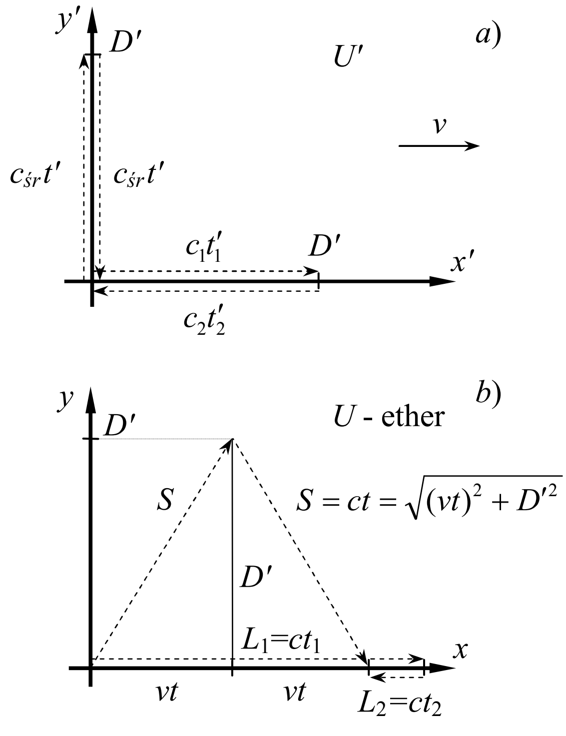

Fig. 2. Light flow paths in two systems moving relative to each other:
a) inertial system U' the flow parallel to axis x' and y',
b) light flows seen from U system (ether).

From assumption II and III results that average velocity of light $c_{śr}$ in inertial frame of reference is the same as velocity of light c in ether. It will be sufficient to notice that light signal has the same average velocity of light $c_{śr}$ in U' system, when U' system does not move in relation to U system (i.e. $v = 0$). Since then velocity of light $c_{śr}$ is exactly the same as velocity c, and therefore for each velocity v occurs $c_{śr} = c$.

Paths of light flow are shown in Figure 2. U system lies in ether, while U' system moves in relation to ether at a constant velocity v. Axes x and x' lie on one straight.

Distance D' which is perpendicular to velocity v, is the same from a point of view of both frames of reference (assumption IV). Therefore on Figure is the same length D' in part a) and parts b).

In U' system, the measured average velocity is constant in each direction, which can be written in a form of

$$c_{śr} = c = \frac{D'}{t'} = \frac{2D'}{2t'} = \frac{2D'}{t'_1 + t'_2} \tag{42}$$

Similar dependencies can be written for U system (ether)

$$c = \frac{2\sqrt{(vt)^2 + D'^2}}{2t} = \frac{L_1 + L_2}{t_1 + t_2} \tag{43}$$

If for transformation (37), the following new determinations will be adopted: $U_2 \equiv U'$ and $U_1 \equiv U$ (ether), then according to

$$\begin{aligned} &v_1 = 0 \\ &v_{2/1} = v_2 = v \\ &\delta(v_1) = \delta(0) = 1 \end{aligned} \tag{44}$$

Then time transformation (37) will take the form of

$$t = \frac{1}{\sqrt{\delta(v)}} \cdot t' \tag{45}$$

On the basis of equation (42) and equation (43) we will obtain the relation of

$$\frac{2D'}{2t'} = \frac{2\sqrt{(vt)^2 + D'^2}}{2t} \tag{46}$$

After reduction by 2 and applying determined time transformation (45) we will obtain

$$\frac{D'}{t'} = \frac{\sqrt{\left(v\frac{t'}{\sqrt{\delta(v)}}\right)^2 + D'^2}}{\frac{1}{\sqrt{\delta(v)}} \cdot t'} \tag{47}$$

i.e.

$$D'\frac{1}{\sqrt{\delta(v)}} = \sqrt{\frac{v^2 t'^2}{\delta(v)} + D'^2} \tag{48}$$

$$D'^2 \frac{1}{\delta(v)} = \frac{v^2 t'^2}{\delta(v)} + D'^2 \tag{49}$$

$$D'^2 (\frac{1}{\delta(v)} - 1) = \frac{v^2 t'^2}{\delta(v)} \tag{50}$$

$$\frac{1-\delta(v)}{\delta(v)} = \frac{v^2}{\delta(v)} \left(\frac{t'}{D'} \right)^2 \tag{51}$$

$$1-\delta(v) = v^2 \left(\frac{t'}{D'} \right)^2 \tag{52}$$

On the basis of (42) we will obtain

$$1-\delta(v) = v^2 \left(\frac{1}{c} \right)^2 \tag{53}$$

Finally, function $\delta(v)$, for which the transformation meets conditions of Michelson-Morley's experiment takes the form of

$$\delta(v) = 1-(v/c)^2 = \frac{c^2 - v^2}{c^2} \tag{54}$$

Transformations (37) and (38) with a function (54) required additional assumptions I, II, III and IV.

By introducing into the theory of a universal frame of reference, in which one-way speed of light is constant, it is possible to solve mentioned above problem of clocks synchronization. In a universal frame of reference, the clocks can be synchronized by means of light (internal method). It will be a system to which clocks in all inertial systems (external method) will be synchronized.

6. Summing speed and relative speed

Let us consider a situation presented in Figure 3. All considered velocities are parallel to each other.

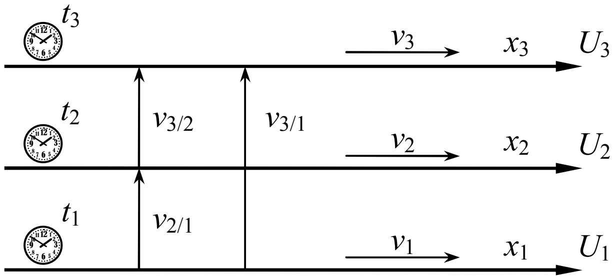

Fig. 3. Inertial systems U_1, U_2, U_3 moving relative to ether with velocities v_1, v_2, v_3.

On the basis of (37) and (38), transformations from U_2 system to U_3 system and from U_1 system to U_2 system will have a form of

$$\begin{cases} t_3 = \sqrt{\dfrac{\delta(v_3)}{\delta(v_2)}} \cdot t_2 \\ x_3 = v_{2/3}\sqrt{\dfrac{\delta(v_3)}{\delta(v_2)}} \cdot t_2 + \sqrt{\dfrac{\delta(v_2)}{\delta(v_3)}} \cdot x_2 \end{cases} \qquad \begin{cases} t_2 = \sqrt{\dfrac{\delta(v_2)}{\delta(v_1)}} \cdot t_1 \\ x_2 = v_{1/2}\sqrt{\dfrac{\delta(v_2)}{\delta(v_1)}} \cdot t_1 + \sqrt{\dfrac{\delta(v_1)}{\delta(v_2)}} \cdot x_1 \end{cases} \tag{55}$$

Combining these two transformations by putting t_2, x_2 from the second to the first one, we will obtain a transformation from U_1 system to U_3 system

$$\begin{cases} t_3 = \sqrt{\dfrac{\delta(v_3)}{\delta(v_2)}}\sqrt{\dfrac{\delta(v_2)}{\delta(v_1)}} \cdot t_1 \\ x_3 = v_{2/3}\sqrt{\dfrac{\delta(v_3)}{\delta(v_2)}}\sqrt{\dfrac{\delta(v_2)}{\delta(v_1)}} \cdot t_1 + \sqrt{\dfrac{\delta(v_2)}{\delta(v_3)}} \cdot \left[v_{1/2}\sqrt{\dfrac{\delta(v_2)}{\delta(v_1)}} \cdot t_1 + \sqrt{\dfrac{\delta(v_1)}{\delta(v_2)}} \cdot x_1 \right] \end{cases} \tag{56}$$

After reduction we will obtain

$$\begin{cases} t_3 = \sqrt{\dfrac{\delta(v_3)}{\delta(v_1)}} \cdot t_1 \\ x_3 = \left[v_{2/3}\sqrt{\dfrac{\delta(v_3)}{\delta(v_1)}} + v_{1/2}\dfrac{\delta(v_2)}{\sqrt{\delta(v_1)\delta(v_3)}} \right] \cdot t_1 + \sqrt{\dfrac{\delta(v_1)}{\delta(v_3)}} \cdot x_1 \end{cases} \tag{57}$$

Transformation from U_1 system to U_3 system can also be obtained directly from (38)

$$\begin{cases} t_3 = \sqrt{\dfrac{\delta(v_3)}{\delta(v_1)}} \cdot t_1 \\ x_3 = v_{1/3}\sqrt{\dfrac{\delta(v_3)}{\delta(v_1)}} \cdot t_1 + \sqrt{\dfrac{\delta(v_1)}{\delta(v_3)}} \cdot x_1 \end{cases} \tag{58}$$

Combined transformation presented in (57) must have the same form as transformation (58). Hence we will obtain

$$v_{1/3}\sqrt{\frac{\delta(v_3)}{\delta(v_1)}} = v_{2/3}\sqrt{\frac{\delta(v_3)}{\delta(v_1)}} + v_{1/2}\frac{\delta(v_2)}{\sqrt{\delta(v_1)\delta(v_3)}} \tag{59}$$

After reduction, the equation takes the form of

$$v_{1/3}\delta(v_3) = v_{2/3}\delta(v_3) + v_{1/2}\delta(v_2) \tag{60}$$

On this basis, we obtain the formula for summing parallel relative speeds

$$v_{1/3} = v_{1/2}\frac{\delta(v_2)}{\delta(v_3)} + v_{2/3} \tag{61}$$

An analogous equation as (60) can be written between other systems by changing indexes in (60). For three systems there are six such equations. For example, after replacing indexes 2→1 and 1→2, we will obtain

$$v_{2/3}\delta(v_3) = v_{1/3}\delta(v_3) + v_{2/1}\delta(v_1) \tag{62}$$

If we will assume that U_3 system is ether (a universal frame of reference), then velocity $v_3 = 0$. On this basis we have $v_{2/3} = v_2$, $v_{1/3} = v_1$ and $\delta(v_3) = \delta(0) = 1$. From equations (60) and (62) we will obtain equations

$$\begin{aligned} v_1 &= v_2 + v_{1/2} \cdot \delta(v_2) \\ v_2 &= v_1 + v_{2/1} \cdot \delta(v_1) \end{aligned} \tag{63}$$

After conversion we will obtain relations

$$\begin{aligned} v_{2/1} &= (v_2 - v_1) / \delta(v_1) \\ v_{1/2} &= (v_1 - v_2) / \delta(v_2) \end{aligned} \tag{64}$$

After taking into account (54), formulas (63) for summing parallel speeds take the form of

$$\begin{aligned} v_1 &= v_2 + v_{1/2} \cdot (1 - (v_2 / c)^2) \\ v_2 &= v_1 + v_{2/1} \cdot (1 - (v_1 / c)^2) \end{aligned} \tag{65}$$

After taking into account (54), formulas (64) for relative speeds take the form of

$$\begin{aligned} v_{2/1} &= \frac{v_2 - v_1}{1 - (v_1 / c)^2} \\ v_{1/2} &= \frac{v_1 - v_2}{1 - (v_2 / c)^2} \end{aligned} \tag{66}$$

* * *

In the analogous way, it is possible to put transformations between systems, expressed with relative speeds (26) and (27). Transformations from U_2 system to U_1 system and from U_3 system to U_2 system have a form of

$$\begin{cases} t_1 = \sqrt{-\dfrac{v_{1/2}}{v_{2/1}}} \cdot t_2 \\ x_1 = v_{2/1}\sqrt{-\dfrac{v_{1/2}}{v_{2/1}}} \cdot t_2 + \sqrt{-\dfrac{v_{2/1}}{v_{1/2}}} \cdot x_2 \end{cases} \qquad \begin{cases} t_2 = \sqrt{-\dfrac{v_{2/3}}{v_{3/2}}} \cdot t_3 \\ x_2 = v_{3/2}\sqrt{-\dfrac{v_{2/3}}{v_{3/2}}} \cdot t_3 + \sqrt{-\dfrac{v_{3/2}}{v_{2/3}}} \cdot x_3 \end{cases} \tag{67}$$

Making these transformations by putting t_2, x_2 from the second to the first one, we will obtain transformation from U_3 system to U_1 system

$$\begin{cases} t_1 = \sqrt{-\dfrac{v_{1/2}}{v_{2/1}}} \cdot \sqrt{-\dfrac{v_{2/3}}{v_{3/2}}} \cdot t_3 \\ x_1 = v_{2/1}\sqrt{-\dfrac{v_{1/2}}{v_{2/1}}} \cdot \sqrt{-\dfrac{v_{2/3}}{v_{3/2}}} \cdot t_3 + \sqrt{-\dfrac{v_{2/1}}{v_{1/2}}} \cdot \left[v_{3/2}\sqrt{-\dfrac{v_{2/3}}{v_{3/2}}} \cdot t_3 + \sqrt{-\dfrac{v_{3/2}}{v_{2/3}}} \cdot x_3 \right] \end{cases} \tag{68}$$

On this basis we will obtain

$$\begin{cases} t_1 = \sqrt{-\dfrac{v_{1/2}}{v_{2/1}}} \cdot \sqrt{-\dfrac{v_{2/3}}{v_{3/2}}} \cdot t_3 \\ x_1 = \left[v_{3/2}\sqrt{-\dfrac{v_{2/1}}{v_{1/2}}} \cdot \sqrt{-\dfrac{v_{2/3}}{v_{3/2}}} + v_{2/1}\sqrt{-\dfrac{v_{1/2}}{v_{2/1}}} \cdot \sqrt{-\dfrac{v_{2/3}}{v_{3/2}}} \right] t_3 + \sqrt{-\dfrac{v_{2/1}}{v_{1/2}}}\sqrt{-\dfrac{v_{3/2}}{v_{2/3}}} x_3 \end{cases} \tag{69}$$

Transformation from U_3 system to U_1 system can also be obtained directly from (37)

$$\begin{cases} t_1 = \sqrt{-\dfrac{v_{1/3}}{v_{3/1}}} \cdot t_3 \\ x_1 = v_{3/1}\sqrt{-\dfrac{v_{1/3}}{v_{3/1}}} \cdot t_3 + \sqrt{-\dfrac{v_{3/1}}{v_{1/3}}} \cdot x_3 \end{cases} \tag{70}$$

Putting transformation presented in (69) must have the same form as transformation (70). Hence we will obtain

$$\sqrt{-\frac{v_{1/3}}{v_{3/1}}} = \sqrt{-\frac{v_{1/2}}{v_{2/1}}} \cdot \sqrt{-\frac{v_{2/3}}{v_{3/2}}} \tag{71}$$

$$\sqrt{-\frac{v_{3/1}}{v_{1/3}}} = \sqrt{-\frac{v_{2/1}}{v_{1/2}}}\sqrt{-\frac{v_{3/2}}{v_{2/3}}} \tag{72}$$

$$v_{3/1}\sqrt{-\frac{v_{1/3}}{v_{3/1}}} = v_{3/2}\sqrt{-\frac{v_{2/1}}{v_{1/2}}} \cdot \sqrt{-\frac{v_{2/3}}{v_{3/2}}} + v_{2/1}\sqrt{-\frac{v_{1/2}}{v_{2/1}}} \cdot \sqrt{-\frac{v_{2/3}}{v_{3/2}}} \tag{73}$$

From the relation (71) and (72), after increasing to square, an identical equation is obtained

$$-\frac{v_{1/2}}{v_{2/1}}\frac{v_{2/3}}{v_{3/2}}\frac{v_{3/1}}{v_{1/3}} = 1 \tag{74}$$

From the relation (73) after conversion we will obtain

$$v_{3/1} = v_{3/2}\sqrt{-\frac{v_{2/1}}{v_{1/2}}}\sqrt{-\frac{v_{2/3}}{v_{3/2}}}\sqrt{-\frac{v_{3/1}}{v_{1/3}}} + v_{2/1}\sqrt{-\frac{v_{1/2}}{v_{2/1}}}\sqrt{-\frac{v_{2/3}}{v_{3/2}}}\sqrt{-\frac{v_{3/1}}{v_{1/3}}} \tag{75}$$

From the equation (74) it is known that factor at $v_{2/1}$ is equal 1, hence

$$v_{3/1} = v_{3/2}\sqrt{-\frac{v_{2/1}}{v_{1/2}}}\sqrt{-\frac{v_{2/3}}{v_{3/2}}}\sqrt{-\frac{v_{3/1}}{v_{1/3}}} + v_{2/1} \tag{76}$$

i.e.

$$v_{3/1} = v_{3/2}\sqrt{-\frac{v_{1/2}}{v_{2/1}}}\sqrt{-\frac{v_{2/3}}{v_{3/2}}}\sqrt{-\frac{v_{3/1}}{v_{1/3}}} \cdot \left(-\frac{v_{2/1}}{v_{1/2}}\right) + v_{2/1} \tag{77}$$

Using (74) we will obtain the formula for summing relative speeds ($v_{1/2} \cdot v_{2/1} \le 0$)

$$v_{3/1} = -v_{3/2}\frac{v_{2/1}}{v_{1/2}} + v_{2/1} \tag{78}$$

On the basis of (36) and (54) we will obtain

$$-\frac{v_{2/1}}{v_{1/2}} = \frac{\delta(v_2)}{\delta(v_1)} = \frac{1-(v_2/c)^2}{1-(v_1/c)^2} = \frac{c^2 - v_2^2}{c^2 - v_1^2} \tag{79}$$

Now the formula (78) for summing relative speeds has a form of

$$v_{3/1} = v_{3/2}\frac{1-(v_2/c)^2}{1-(v_1/c)^2} + v_{2/1} = v_{3/2}\frac{c^2-v_2^2}{c^2-v_1^2} + v_{2/1} \tag{80}$$

7. Transformation expressed from absolute speed

On the basis of (54) and (66), transformation (37)-(38) can be expressed from absolute speed v_1 and v_2. Then a general form (26)-(27) and (37)-(38) is lost, but we will obtain its special form, which is consistent with experiments in which the velocity of light was measured.

$$\begin{cases} t_1 = \sqrt{\dfrac{1-(v_1/c)^2}{1-(v_2/c)^2}} \cdot t_2 \\ x_1 = \dfrac{v_2 - v_1}{\sqrt{1-(v_1/c)^2}\sqrt{1-(v_2/c)^2}} \cdot t_2 + \sqrt{\dfrac{1-(v_2/c)^2}{1-(v_1/c)^2}} \cdot x_2 \end{cases} \tag{81}$$

$$\begin{cases} t_2 = \sqrt{\dfrac{1-(v_2/c)^2}{1-(v_1/c)^2}} \cdot t_1 \\ x_2 = \dfrac{v_1 - v_2}{\sqrt{1-(v_1/c)^2}\sqrt{1-(v_2/c)^2}} \cdot t_1 + \sqrt{\dfrac{1-(v_1/c)^2}{1-(v_2/c)^2}} \cdot x_1 \end{cases} \tag{82}$$

8. Transformation between ether and inertial system

We adopt the following determinations: $U_2 \equiv U'$ and $U_1 \equiv U$ (ether). Then relations occur (44). We also adopt the following determinations: $x = x_1$, $t = t_1$, $x' = x_2$ and $t' = t_2$. With such determinations, on the basis of (81) and (82), we obtain transformations from the inertial system U' to ether U and ether U to inertial system U' in a form of

$$\begin{cases} t = \dfrac{1}{\sqrt{1-(v/c)^2}} \cdot t' \\ x = \dfrac{v}{\sqrt{1-(v/c)^2}} \cdot t' + \sqrt{1-(v/c)^2} \cdot x' \end{cases} \tag{83}$$

$$\begin{cases} t' = \sqrt{1-(v/c)^2} \cdot t \\ x' = \dfrac{-v}{\sqrt{1-(v/c)^2}} \cdot t + \dfrac{1}{\sqrt{1-(v/c)^2}} \cdot x \end{cases} \tag{84}$$

This transformation is identical as transformation derived in works [6]-[12], in which it was derived with other method based on geometrical analysis of Michelson-Morley's and Kennedy-Thorndike's experiment. In monograph [6], on the basis of this transformation, a new theory of kinematics and dynamics of bodies was derived, called the Special Theory of Ether.

Transformation (83)-(84) was also derived, but with other method, in articles [2] and [13]. In work [2], the author obtained this transformation from Lorentz's transformation thanks to clocks synchronization in inertial systems with the external method. The transformation obtained in work [2] is a differently written Lorentz's transformation after the change of the way of measuring time in the inertial frame of reference, and therefore the authors have assigned it the properties of

Lorentz's transformation. Transformation derived in this article has a different physical meaning than Lorentz's transformation, because according to the theory presented here, it is possible to determine the velocity in relation to a universal frame of reference by means of local measurement. This means that a universal frame of reference is real, and is not an arbitrarily chosen inertial system.

9. One-way speed of light

In works [6], [10] and [12] based on transformation (83)-(84), a formula for one-way speed of light in vacuum was derived, which is measured by the observer from inertial frame of reference

$$c'_{\alpha'} = \frac{c^2}{c + v\cos\alpha'} \tag{85}$$

In the work [6], a formula for one-way speed of light in the material medium *s* was derived, which is measured by the observer from inertial frame of reference

$$c'_{s\alpha'} = \frac{c^2 c_s}{c^2 + c_s v\cos\alpha'} \tag{86}$$

In these two relations, angle α', measured by the observer, is an angle between vector of its velocity in relation to ether and vector of the velocity of light. The velocity c_s is a velocity of light in the motionless material medium in relation to ether, seen by motionless observer in relation to ether.

Although, the velocity of light expressed by formula (86) depends on angle α' and velocity v, the average velocity of light on the path forth and back to the mirror is always constant. It is sufficient to verify that for the velocity of light expressed by formula (86), the average velocity on path L forth and back to the mirror is as follows

$$c'_{sr} = \frac{2L}{t'_{s\alpha'} + t'_{s(\pi-\alpha')}} = \frac{2L}{\dfrac{L}{\dfrac{c^2 c_s}{c^2 + c_s v\cos\alpha'}} + \dfrac{L}{\dfrac{c^2 c_s}{c^2 + c_s v\cos(\pi-\alpha')}}} \tag{87}$$

$$c'_{sr} = \frac{2}{\dfrac{c^2 + c_s v\cos\alpha'}{c^2 c_s} + \dfrac{c^2 - c_s v\cos\alpha'}{c^2 c_s}} = \frac{2}{\dfrac{2c^2}{c^2 c_s}} = c_s \tag{88}$$

From the relation (88) results that c_s is also an average velocity of light on the path forth and back to the mirror in the motionless material medium relative to the observer.

10. Conclusions

Determined transformations (81)-(82) and (83)-(84) are consistent with Michelson-Morley's and Kennedy-Thorndike's experiment. It results from above transformations that measurement of the velocity of light in vacuum with so far used methods, will always give an average value equal to c. This is despite the fact that for a moving observer the velocity of light has different values in different directions. The average velocity of light is always constant and independent from the velocity of an inertial frame of reference. Because of this property the velocity of light, Michelson-Morley's and Kennedy-Thorndike's experiments could not detect ether.

The analysis shows that it is possible to explain the results of Michelson-Morley's experiment on the basis of ether. A statement is false that Michelson-Morley's experiment has shown that velocity of light is absolutely constant. It is also false that Michelson-Morley's experiment has proved that there is no ether in which light propagates and moves at a constant velocity.

Assumption that velocity of light can depend on the direction of its emission, does not distinguish any direction in space. It is about the velocity of light measured by moving observer. It is a velocity, at which the observer moves in relation to universal frame of reference (ether), that distinguishes in space the characteristic direction, but only for this observer. For motionless observer in relation to universal frame of reference, the velocity of light is always constant and does not depend on the direction of its emission. If the observer moves in relation to a universal frame of reference, then the space for observer is not symmetrical. In this case, it will be like for an observer sailing on water and measuring the velocity of wave on the water. Despite the fact that the wave propagates at a constant velocity in each direction, for sailing observer the wave velocity will vary in different directions.

Currently it is believed that STR is the only theory that explains the Michelson-Morley's and Kennedy-Thorndike's experiments. This article shows that other theories are possible according to these experiments. In works [6] and [12], based on determined here transformation, the new physical theory of kinematics and dynamics of bodies was derived, called by authors the Special Theory of Ether. The work [12] shows that there is infinite number of theories with ether that correctly explain Michelson-Morley's and Kennedy-Thorndike's experiments. Even the theory with ether is possible, in which time is absolute.

In the work [6], it is shown that within each such kinematics, an infinite number of dynamics can be derived. In order to derive dynamics, it is necessary to adopt an additional assumption, which enables to introduce the concept of mass, kinetic energy and momentum in the theory.

Predictions of the Special Theory of Ether and Special Theory of Relativity are very similar. However, there are differences which may allow for experimental falsification of these theories in the future. In STR, all inertial systems are equivalent, i.e. there is no universal frame of reference. For this reason, according to STR, it is not possible to measure absolute speed using local measurement. This means that for each observer the space is completely isotropic (the same properties in each direction). However, according to STE, the observer can use local measurements (i.e. when is completely isolated from the environment) to determine the direction of its movement in relation to ether. This means that for observers moving in relation to ether, the space is not isotropic (has different properties in different directions). This is the most important difference between the Special Theory of Ether and Special Theory of Relativity. Confirmation of this by experiment is not easy due to the low speed of the Solar System relative to ether. For a small velocity, the effects of non-isotropic space are very slight.

On the basis of presented kinematics, it is possible in a natural way to explain the anisotropy of cosmic microwave background, which in detail is discussed in the article [5]. This enables to determine the velocity at which the Solar System moves in relation to universal frame of reference, i.e. 369,3 km/s = 0,0012 c. This was presents in works [8], [9] and [12].

Michelson-Morley's and Kennedy-Thorndike's experiments were conducted repeatedly by different teams. Modified and improved versions of this experiment were also carried out, such as experiment with sapphire crystals in 2015 [4]. Each of these experiments only confirmed that the average velocity of light is constant. Therefore, assumptions on which presented derivations are based are justified experimentally.

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