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Relativistic Time Dilation or Contraction?

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Abstract
We consider the time intervals of the half cycles of the classical light clock aligned along the direction of its motion. When the photon travels in this direction the light clock shows time dilation which is different from the Lorentz-Einstein type. If the photon moves opposite the clock shows time contraction.

Introduction

One of the tenets of Special relativity is its concept of time dilation which depends upon the second postulate of Special relativity that the speed of light c is the same in all inertial frames of reference [1].

According to Special relativity, if ΔT_0 is the time interval of an event that occurs at the same position in an inertial frame of reference, then the time interval of this event has a longer duration ΔT as measured by an observer in an inertial frame of reference that is in uniform motion relative to the first frame. Thus, both of the time intervals ΔT_0 (so-called proper time) and ΔT (so-called improper time) refer to the time of the same event occurring in the two different inertial frames of reference moving with a relative speed v. According to the Lorentz-Einstein transformations the two time intervals ΔT and ΔT_0 are related by the formula: $\Delta T = \Delta T_0/\sqrt{1-v^2/c^2}$ where $1/\sqrt{1-v^2/c^2}$ is the Lorentz factor or time dilation factor. Apparently, time dilation been demonstrated in many experiments, including muon experiment [2 - 4].

In this note, we especially focus our attention on the time intervals of the half cycles of the classical light clock colinear to the direction of its motion.

Discussion and Conclusions

The light clock consists of two plane parallel mirrors M_1 and M_2 facing each other at a distance d apart as in Fig. 1a. The lower mirror M_1 has a light source at the center that emits a photon at 90 degrees in the direction of mirror M_2. For the sake of simplicity, we will firstly consider time for the photon to travel from mirror M_1 to mirror M_2. Of course, you may find all following derivations in many elementary physics texts.

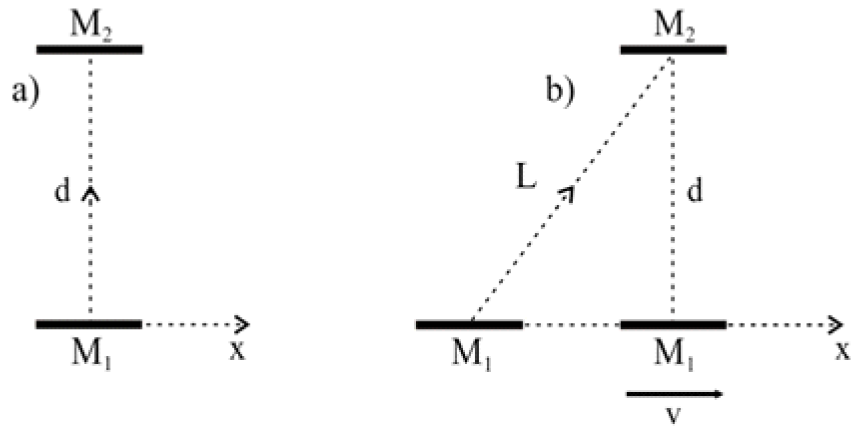

Fig. 1. Measurements and analysis made in different frames. The light clock positioned perpendicular to the x-axis. (a): No relative motion. (b): light clock moving at speed *v*.

For the light clock at rest the proper time interval is then $\Delta T_0 = d/c$. Now allow the same light clock to be moving with a certain relative speed ***v*** horizontally in the direction of positive x-axis. An observer in the rest frame who is watching this clock could then design a diagram associated with a simple Pythagorean triangle, Fig. 1b. Clearly, a photon will now travel the larger distance *L* and thus it will take a longer (improper) time $\Delta T = L/c$. Of course, ΔT_0 and ΔT are related by the relativistic time dilation expression $\Delta T = \Delta T_0/\sqrt{1-v^2/c^2}$.

Let us now performed the experiments using the same light clock as in Fig 1a but now aligned along the positive x-axis, as shown in Fig. 2a. Obviously, if this clock is resting the time interval is $\Delta T_0 = d/c$.

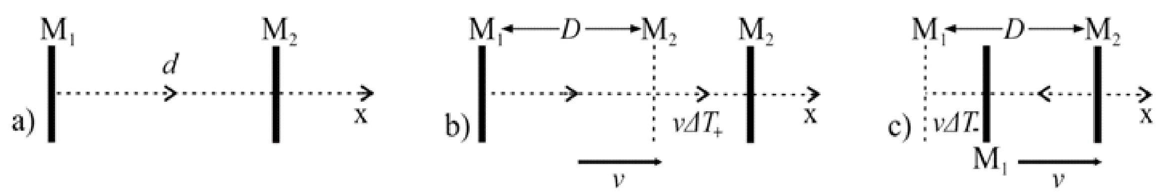

Fig. 2. Measurements and analysis for the light clock positioned along the positive x-axis. (a): No relative motion; (b) and (c): the light-clock moving at speed *v*.

Allow now this clock to move in the direction of the positive x-axis with the speed *v*, Fig. 2b. To reach now the mirror M_2 the photon will travel the distance

$$c\Delta T_+ = D + v\Delta T_+$$

in the time interval

$$\Delta T_+ = D/(c - v)$$

where *D* is the length of light clock measured by a stationary observer.

To acquire a relationship between ΔT_0 and ΔT_+ one may invoke the relativistic length contraction $D = d\sqrt{1-v^2/c^2}$ and then write

$$\Delta T_+ = d[(\sqrt{1 - v^2/c^2})/(c - v)]$$

Factoring out c in the dominator

$$\Delta T_+ = d/c[(\sqrt{1 - v^2/c^2})/(1 - v/c)]$$

or

$$\Delta T_+ = \Delta T_0[(\sqrt{1 - v^2/c^2})/(1 - v/c)]$$

After simplification, we get

$$\Delta T_+ = \Delta T_0(\sqrt{(1 + v/c/1 - v/c)}$$

ΔT_+ represents time dilation as $1 + v/c$ greater than $1 - v/c$. Of course, this dilation is different than Lorentz-Einstein type of time dilation.

For the back-reflected photon the traveling time interval to the mirror M_1 is

$$c\Delta T_- = D - v\Delta T_-$$

After a simple rearrangment

$$\Delta T_- = D/(c + v)$$

Applying same mathematics as above we get

$$\Delta T_- = \Delta T_0\sqrt{(1 - v/c)/(1 + v/c)}$$

Clearly, ΔT_- describes time contraction. However, we must be aware that any photon detections at mirror M_1 or M_2 would involve its annihilation.

Thus, the same light clock shows time contraction ΔT_+ if its photon moving in the direction of motion and time dilation ΔT_- if it moves opposite to that direction. The question is now: which of these two time intervals is "relativistically" right?

Finally, in the classical light clock experiment we usually consider the total time needed for photon to complete cycle

$$\Delta T = \Delta T_+ + \Delta T_- = 2cD/(c^2 - v^2)$$

Factoring out c^2 in the denominator

$$\Delta T = 2D/c(1 - v^2/c^2)$$

After simplification including length contraction, we get the relativistic time dilation

$$\Delta T = 2\Delta T_0 / \sqrt{1 - v^2/c^2}$$

We see, however, no reason why the complete time interval of the classical light clock aligned along the direction of motion is more "relativistically speaking" appropriate than other two time intervals.

References

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