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DEPENDENCE OF ACCELERATING FORCE ON VELOCITY OF A CHARGED PARTICLE MOVING IN AN ELECTRIC FIELD

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Abstract

According to classical and relativistic electrodynamics, accelerating force **F** on a particle of charge q moving with velocity **v** of magnitude v, at angle θ to an electric field of intensity **E** and magnitude E, is a vector **F** = q**E**, independent of velocity in the field. It is proposed here that the force is **F** = qE(**c** – **v**)/c, where (**c** - **v**) is the relative velocity between the particle and the force of the field transmitted with velocity of light **c** of magnitude c, at aberration angle α such that $\sin\alpha = (v/c)\sin\theta$. It is shown that for motion in the direction of the field, with $\theta = 0$, the force is **F** = q**E**$(1 - v/c)$. For motion against the field with $\theta = \pi$ radians, **F** = q**E**$(1 + v/c)$. For motion perpendicular to the field, with $\theta = \pi/2$ radians, **F** = q**E**$\sqrt{(1 - v^2/c^2)}$. It is deduced that the relativistic mass-velocity formula is correct where a charged particle moves perpendicular to an electric field and that circular revolution of an electron, round a central force, is stable. An experiment is proposed, with charged particles moving perpendicular to crossed electric and magnetic fields, to show that mass is constant but accelerating electric field depends on velocity.

Keywords: Aberration, acceleration, circular motion, electric and magnetic fields, Lorentz force, magnitude, mass, special relativity, speed, vector, velocity.

1. Introduction

In classical and relativistic electrodynamics, the accelerating force on a particle of charge q moving in an electric field of intensity **E**, is a vector q**E** independent of velocity **v** of the charge. As such a particle may be accelerated to an infinitely high speed if the force is applied long enough. But experiments with electrons, the lightest particles known in nature, in high-energy accelerators, showed that they could not be accelerated to a speed beyond that of light. The theory of special relativity explains this limitation in speed by positing that the mass of a particle increases with its speed, relative to an observer, becoming infinitely large at the speed of light. Since an infinitely large mass cannot be accelerated any faster by a finite force, the speed of light should be the ultimate limit.

In this paper, in an alternative electrodynamics, it is shown that the mass of a moving particle, under acceleration, remains constant, at the rest mass, while the electric field or the accelerating force decreases with speed, reducing to zero at the speed of light. With zero electric field and zero accelerating force the speed remains constant at that of light, thus giving the ultimate speed without infinite mass. Constancy of mass with speed is an outcome of the alternative electrodynamics that should be of interest to physicists.

An experiment is devised with charged particles moving at a defined velocity **v** made to pass perpendicularly through crossed electric field of intensity **E** and magnetic flux of intensity **B** at right angles. For a passage through the crossed fields, without deflection, the Lorentz force on the moving charged particles must be zero. In other words, the electrical force should be equal and opposite of the magnetic force. For a straight passage, classical electrodynamics and relativistic electrodynamics give a linear relationship between the magnitude v of velocity and magnitude E of electric field. The alternative electrodynamics gives a non-linear relationship, showing that it is the electric field and the accelerating force which should vary with speed v, not the particle mass.

An experiment, being in accordance with a theory, does not necessarily make the theory correct. But an experiment, contradicting a theory, should invalidate the theory. In this paper an experiment to disprove the theory of special relativity, with respect to the mass-velocity formula, is proposed. The theory of special relativity [1][2] predicts increase in the mass *m* of a particle with its speed *v*, relative to an observer, as:

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_o \tag{1}$$

where m_o is the rest mass, γ the Lorentz factor and *c* the speed of light in a vacuum. Once the mass-velocity formula is invalidated, the theory of special relativity stands disproved.

In an alternative electrodynamics [3][4] the author showed that the mass-velocity formula, equation (1), is mathematically correct only where a charged particle is moving perpendicular to an electric field. This is so in circular revolution of an electron round a positively charged nucleus [5]. As the force due to an electric field is propagated at the speed of light, a moving charged particle is subjected to aberration of electric field.

1.1 Motion of a Charged Particle in an Electric Field

According to classical and relativistic electrodynamics, the force $\mathbf{F}_E$ on a particle of charge *q* moving with velocity **v** at an angle θ to an electric field of intensity **E** and magnitude *E*, is given by Coulomb's law as vector:

$$\mathbf{F}_E = q\mathbf{E} = qE\hat{\mathbf{u}} \tag{2}$$

where $\hat{\mathbf{u}}$ is a unit vector in the direction of the electric field. In equation (2) $\mathbf{F}_E$, in the direction of the electric field, is independent of the direction or magnitude *v* of velocity.

An alternative electrodynamics, in this paper, puts the electrical force $\mathbf{F}_E$ as vector:

$$\mathbf{F}_E = \frac{qE}{c}(\mathbf{c} - \mathbf{v}) \tag{3}$$

where *E* is the magnitude and **E** the intensity of the field, **c t**he velocity of light and **v** the velocity of the particle. In equation (3) the force $\mathbf{F}_E$ is also in the field direction $\hat{\mathbf{u}}$.

The issue here is between equation (2) of classical and relativistic electrodynamics and equation (3) of the alternative electrodynamics advanced in this paper. An experiment is proposed to ascertain which equation is correct.

1.2 Motion of a Charged Particle in a Magnetic Field

Lorentz law gives the force $\mathbf{F}_B$ on a particle of charge *q* moving with velocity **v** at an angle β to a magnetic field of intensity **H** and flux intensity $\mathbf{B} = \mu\mathbf{H}$, as vector product, in the direction of unit vector $\hat{\mathbf{a}}$, as:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = \hat{\mathbf{a}} qvB\sin\beta \tag{4}$$

where *v* and *B* are the respective magnitudes. It is assumed that equation (4) is correct for classical and relativistic electrodynamics as well as the alternative electrodynamics.

In classical and relativistic electrodynamics, the total force **F** on a charged particle moving in electric and magnetic fields is Lorentz force, given by equations (2) and (4) as:

$$\mathbf{F} = qE\hat{\mathbf{u}} + \hat{\mathbf{a}} qvB\sin\beta \tag{5}$$

According to the alternative electrodynamics, the total force is given by equations (3) and (4) as vector:

$$\mathbf{F} = \frac{qE}{c}(\mathbf{c} - \mathbf{v}) + \hat{\mathbf{a}} qvB\sin\beta \tag{6}$$

where the vector (**c** – **v**) is in the $\hat{\mathbf{u}}$ direction of the electric field. Equations (5) and (6) will be used in the proposed experiment with crossed electric and magnetic fields.

1.3 Motion of a Charged Particle Perpendicular to an Electric Field

In the proposed experiment, the electric field of intensity **E**, magnetic flux of intensity **B** and velocity **v** are mutually perpendicular so that in equation (4) $\beta = \pi/2$ radians. The unit vector **â** is equal to -**û** so that equations (5) and (6) respectively become:

$$\mathbf{F} = qE\hat{\mathbf{u}} - \hat{\mathbf{u}}qvB \tag{7}$$

$$\mathbf{F} = \frac{qE}{c}(\mathbf{c} - \mathbf{v}) - \hat{\mathbf{u}}qvB \tag{8}$$

Equation (7), with a linear relationship between the magnitudes E and v and equation (8) with a non-linear relationship, will be used in the proposed experiment to disprove special relativity. Before then, let us consider the stability of circular revolution of an electron.

1.4 Stability of Circular Revolution of an Electron

Revolution of an electron, in a circle of radius r, is with constant speed v and acceleration v^2/r. It revolves in a stable circular orbit, with no change in potential or kinetic energy and no radiation. This is contrary to classical electrodynamics where Lamor formula [6] gives radiation power as proportional to the square of acceleration. If dislodged from the circular orbit the electron oscillates in an elliptic path with radiation of energy at the frequency of revolution, before reverting back into a stable circular orbit. The author [3] showed that radiation power is zero in circular revolution, where a charged particle moves perpendicular to an electric field. Stabilization of Rutherford's nuclear model of the hydrogen atom, outside Bohr's quantum mechanics [7], is one of the achievements of the alternative electrodynamics, based on aberration of electric field.

2. Aberration of Electric Field

The English astronomer, John Bradley, discovered aberration of light in 1725. This was one of the most significant discoveries in science but now relegated to the background in favor of special relativity. In aberration of light, a distant star (considered as stationary) under observation, by a moving astronomer, appears displaced in the forward direction (from the instantaneous line joining the star and the observer) through a small angle α, called the angle of aberration. Aberration of electric field is a phenomenon similar to aberration of light. The force due to an electric field of magnitude E and intensity **E** in the direction of unit vector **û**, is propagated with velocity of light **c** of magnitude c, so that:

$$\mathbf{E} = E\hat{\mathbf{u}} = \frac{E}{c}\mathbf{c} \tag{9}$$

Figure 1 depicts a particle of charge q and mass m at a point P moving with velocity **v** at an angle θ to electric field of intensity **E**, in the direction of unit vector **û**, from a source

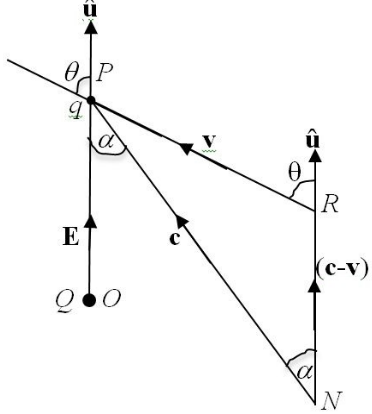

Figure 1. A particle of charge q at P moving with velocity **v** at angle θ to electric field **E**

charge +Q at O. The electric field appears to be propagated along NP with velocity of light **c**, at aberration angle α from the line OP, such that the sine rule in triangle NPR gives:

$$\sin\alpha = \frac{v}{c}\sin\theta \tag{10}$$

Relative velocity between the particle moving with velocity **v** and the electrical force propagated with velocity of light **c**, is vector (**c** – **v**) in the **û** field direction. In the alternative electrodynamics, expressed by equation (3), the electric field $\mathbf{E}_v$, experienced by a charged particle moving with velocity **v** in a field of magnitude E, is given by vector:

$$\mathbf{E}_v = \frac{E}{c}(\mathbf{c} - \mathbf{v}) \tag{11}$$

The *modulus* of (**c** – **v**) or the cosine rule in triangle NPR, with the angle between the vectors **c** and **v** (Figure 1) being $(\theta - \alpha)$, give equation (11) as vector:

$$\mathbf{E}_v = \frac{\mathbf{E}}{c}\sqrt{c^2 + v^2 - 2cv\cos(\theta - \alpha)} \tag{12}$$

For a charged particle moving at speed v under acceleration in the direction of an electric field of intensity **E**, where $\theta = 0$, equations (10) and (12) give $\mathbf{E}_v$ as vector:

$$\mathbf{E}_v = \mathbf{E}(1 - \frac{v}{c}) \tag{13}$$

For a charged particle moving at speed v under deceleration against the direction of electric field of intensity **E**, where $\theta = \pi$ radians, equations (10) and (12) give $\mathbf{E}_v$ as:

$$\mathbf{E}_v = \mathbf{E}(1 + \frac{v}{c}) \tag{14}$$

For a charged particle moving at speed v perpendicular to an electric field of intensity **E**, where $\theta = \pi/2$ radians, equations (10) and (12) give $\cos(\theta - \alpha) = \sin\alpha = v/c$ and:

$$\mathbf{E}_v = \mathbf{E}\sqrt{1 - \frac{v^2}{c^2}} = \frac{\mathbf{E}}{\gamma} \tag{15}$$

3. Circular Motion of an Electron

An interesting case is where an electron of charge $q = -e$ and mass m revolves at constant speed v in a circle of radius r, perpendicular to a radial electric field of magnitude E and intensity $\mathbf{E} = E\hat{\mathbf{u}}$ from a positively charged stationary nucleus, with centripetal acceleration $-(v^2/r)\hat{\mathbf{u}}$. In classical electrodynamics, where mass m is a constant equal to the rest mass m_o, the accelerating force **F**, given by Newton's second law of motion, is:

$$\mathbf{F} = -eE\hat{\mathbf{u}} = -m_o\frac{v^2}{r}\hat{\mathbf{u}} \tag{16}$$

In relativistic electrodynamics, with $m \neq m_o$, accelerating force on the electron is $-eE\hat{\mathbf{u}}$, independent of speed of the electron in the field. In this case equation (1) and Newton's second law of motion give:

$$\mathbf{F} = -eE\hat{\mathbf{u}} = -m\frac{v^2}{r}\hat{\mathbf{u}} = -\frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{v^2}{r}\hat{\mathbf{u}} \tag{17}$$

In the alternative electrodynamics, with $m = m_o$, equation (15) gives accelerating force as:

$$\mathbf{F} = -eE\hat{\mathbf{u}}\sqrt{1 - \frac{v^2}{c^2}} = -m_o\frac{v^2}{r}\hat{\mathbf{u}} \tag{18}$$

Equation (17) from relativistic electrodynamics and equation (18) from the alternative electrodynamics, give:

$$eE = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} = \zeta\frac{v^2}{r} \tag{19}$$

$$\zeta = \frac{eEr}{v^2} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \tag{20}$$

In equation (19) it is as if the accelerating force is constant at eE but the mass varies with speed v in accordance with equation (1). This is a mathematical misinterpretation of a physical phenomenon. The relativistic mass m in equations (1) is not a physical quantity but the ratio of electrostatic force eE to acceleration v^2/r in circular motion as expressed by ζ (zeta) in equation (20). This quantity (zeta), that has the dimension of mass, can become infinitely large, for linear motion, which is motion in an arc of a circle of infinite radius.

It is not the physical mass which increases with speed but the electric field which varies with speed in accordance with equations (13), (14) or (15). An experiment, proving equation (15) as correct, invalidates equation (1) of the theory of special relativity. This paper sets out to prove equation (15) with a proposed experiment.

4. Proposed Experiment

In the experiment, particles each of charge q, moving with velocity **v**, pass through a crossed electric field of intensity **E** and a magnetic flux of intensity **B**, at right angles, as shown in Figure 2.

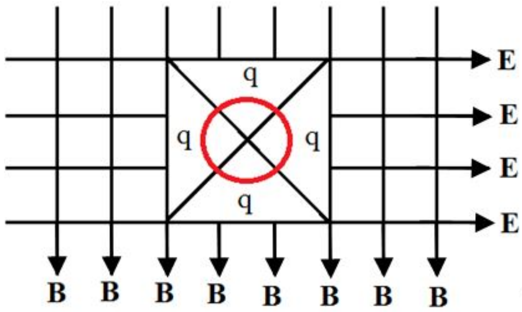

Figure 2. A particle of charge q passing with velocity **v**, perpendicularly into crossed electric field of intensity **E** and magnetic flux of intensity **B**.

A particle of charge q goes through the crossed fields at velocity **v**, without deflection, if it experiences zero electric field and zero force. This makes the Lorentz force, as given by equation (7) or (8), zero. If equation (7) is applicable, in accordance with classical and relativistic electrodynamics, then we have a linear relationship between E and v, thus:

$$E = vB \tag{21}$$

If equation (8) applies, in accordance with the alternative electrodynamics, we have:

$$E\sqrt{1-\frac{v^2}{c^2}} = vB$$

$$\frac{E}{Bc} = \frac{v}{c\sqrt{1-\frac{v^2}{c^2}}} = \frac{v}{c}\gamma \tag{22}$$

Graphs of *v/c* against *E/Bc* are shown in Figure 3. Classical and relativistic electrodynamics give the solid line *AL*, indicating a linear relationship between *E* and *v*, with *E* reaching a maximum value *Bc* at the speed of light *c*. The alternative electrodynamics gives the dotted curve *AK*, for equation (22). It shows a non-linear relationship between *E* and *v*, where, for a given *B* greater than zero, the magnitude *E* becomes infinitely large as the speed of light *c* is approached.

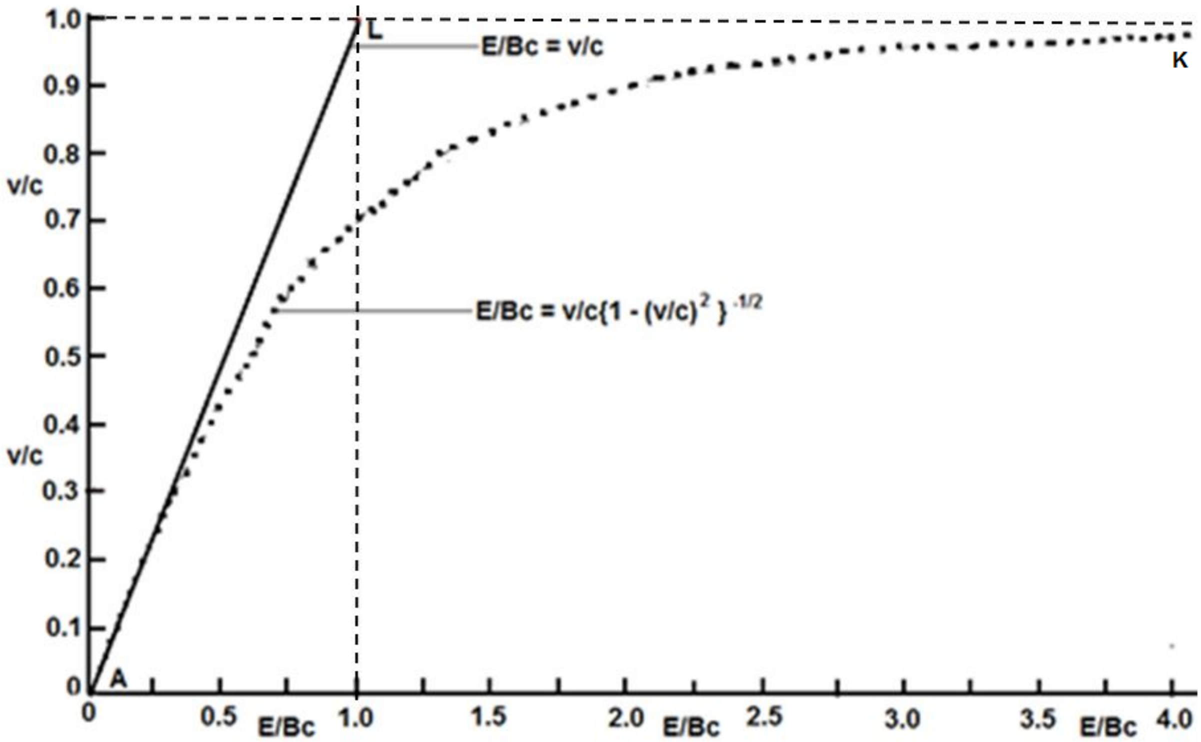

Figure 3. Graphs of speed *v*, in units of speed of light *c*, and electric field of magnitude *E*, in units of *Bc*, for a charged particle passing through a crossed electric field of intensity **E** and a magnetic flux of intensity **B**, at right angles, without deflection; the solid line *AL* according to classical and relativistic electrodynamics and the dotted curve *AK* according to an alternative electrodynamics.

6. Conclusions

The following conclusions may be drawn:

1. Circular revolution of an electron round a positively charged nucleus, perpendicular to the radial electric field, is without radiation and inherently stable.
2. As neither mass nor energy is involved in obtaining equation (15) with the Lorentz factor γ, it is not the mass of a moving charged particle, but the electric field experienced, that depends on velocity of the particle in the electric field.
3. The force exerted by an electric field, on a moving charged particle, is dependent on the magnitude and direction of velocity of the particle in the field.
4. Equation (1) is a mathematical misinterpretation of a physical phenomenon. It is correct for circular revolution, not because mass increases with speed, but as a result of accelerating field depending on speed as in equations (13), (14) and (15).
5. The relativistic mass $m = \gamma m_o$ in equation (1) is not a physical quantity but the ratio of electrostatic force eE to acceleration v^2/r in circular motion. This ratio (eEr/v^2) can be infinitely large, as in rectilinear motion, which is motion in an arc of a circle of infinite radius, without any problem of infinitely large masses.
6. Equating m in equation (1) with physical mass that has weight, is a case of mistaken identity. This paper seeks to correct an unfortunate mistake.

7. Applying equation (1), in rectilinear motion, is wrong.
8. A positively charged particle moving at the speed of light, in the positive direction of an electric field, experiences no force.
9. A positively charged particle moving at the speed of light, in the opposite direction of an electric field, experiences twice the force on a stationary particle.
10. A charged particle moving at the speed of light, perpendicular to an electric field, experiences no force.
11. An experiment giving a non-linear curve *AK*, in Figure 3, disproves the mass-velocity formula (equation 1) of the theory of special relativity.

References

[1] A. Einstein; "On the Electrodynamics of Moving Bodies", *Ann. Phys.*, 17 (1905), 891.
[2] A. Einstein; & H.A. Lorentz, *The Principles of Relativity*, Matheun, London (1923).
[3]https://www.academia.edu/7973538/An_Alternative_Electrodynamics_to_the_Theory_of_Special_Relativity
[4]https://www.academia.edu/16045596/Aberration_of_Electric_Field_and_Radiation_Power_due_to_a_Moving_Electron
[5]https://www.academia.edu/10962823/An_Interpretation_of_Relativistic_Mass_in_Circular_Motion_of_an_Electron
[6] D Griffith; *Introduction to Electrodynamics*, Prentice-Hall, Englewood Cliff, New Jersey (1981), pp. 375 – 378.
[7] N. Bohr; On the Constitution of Atoms and Molecules, *Phil. Mag,*, Series 6, 26 (1913).