Abstract

The relativistic Doppler shift formula is shown to be based on a contradictory equation, revealing the unviability of the Special Relativity. In addition, the light speed postulate is shown to result in contradictions with the Special Relativity equations, when expressed in terms of the light wave characteristics, with the relativistic Doppler shift formula being applied in the resulting expression. An absurd relation between the wave frequency and wave period would emerge to satisfy the Special Relativity time dilation. An actual time dilation factor emerging from the speed of light principle is revealed.

Keywords: Special Relativity, Relativistic Doppler shift.

Introduction

In his 1905 paper,\(^1\) Einstein derived the Lorentz transformation equations for the space and time coordinates on the basis of the relativity principle and the constancy of the speed of light. Transformation equations for the electric and magnetic forces were then deduced from the Maxwell-Hertz and the former equations. The obtained electrodynamics transformations applied on the wave equations for light led to the general relativistic Doppler shift formula.

It follows that the coherence of the Doppler shift formula is vital for the Special Relativity veracity. In this communication, the Doppler shift is shown to merely result from a contradictory equation requiring that any relative motion velocity must be zero. Hence, the Special Relativity transformation equations leading to the relativistic Doppler formula must be unviable.

Relativistic Doppler Shift

In §3 of the cited paper,\(^1\) the time transformation equation converting event time between two inertial frames in relative motion of velocity \(v\), having the coordinate systems \(\hat{K}(x,y,z,t)\) and \(k(\xi,\eta,\zeta,\tau)\) associated with what’s considered as “stationary” and “moving” frame, respectively, is obtained as
\[ \tau = \beta \left( t - \frac{vx}{c^2} \right), \]  

(1)

where

\[ \beta = \left( \frac{1}{\sqrt{1 - v^2/c^2}} \right)^{-1}, \text{ and } c = \text{speed of light in empty space}. \]

In §7 of the same paper, a light (electrodynamics waves) source, with given wave characteristics, is considered in the stationary system at a sufficiently far distance from the origin. The characteristics of these waves were to be determined when observed from the moving frame. We quote the following passage:

*From the equation for \( \omega' \) it follows that if an observer is moving with velocity \( v \) relatively to an infinitely distant source of light of frequency \( \nu \), in such a way that the connecting line “source-observer” makes the angle \( \phi \) with the velocity of the observer referred to a system of co-ordinates which is at rest relatively to the source of light, the frequency \( \nu' \) of the light perceived by the observer is given by the equation

\[ \nu' = \nu \left( 1 - \frac{\cos \phi \cdot v}{c} \right) \frac{v}{\sqrt{1 - v^2/c^2}}. \]  

(2)

This is Doppler’s principle for any velocities whatever. When \( \phi = 0 \) the equation assumes the perspicuous form

\[ \frac{\nu'}{\nu} = \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}. \]  

(3)

Equation (3) can be written in the form

\[ \nu' = \beta \nu \left( 1 - \frac{v}{c} \right). \]  

(4)

**Relativistic Doppler Shift Formula Contradiction**

Let’s consider the case for \( \phi = 0 \). According to the classical velocity addition, the light wave emitted from the stationary system \( K \) would be traveling with respect to the moving observer at a velocity of \( c + v \). Whereas, according to the Special Relativity’s constancy of the speed of light principle, the wave velocity would be \( c \) relative to the moving observer’s system \( k \). Let’s suppose that this principle shall lead to the following contradiction

\[ c + v = c \]  

(5)
Let the wave period be given by \( t \) in the source frame, and the period measured in the traveling observer’s frame by \( \tau \). According to the basic wave characteristics, we have \( \tau \nu' = t \nu = 1 \) (since \( \lambda \nu = c; \ \lambda' \nu' = c; \ t \nu = 1 - \text{ditto} \tau \nu' = 1 \)). Therefore, Eq. (5) can be rewritten as

\[
(c + v)\tau \nu' = c t \nu;
\]

\[
\tau = t \frac{c}{c + v} \frac{\nu}{\nu'}.
\]

Using Eq. (3) in the above equation, we get

\[
\tau = t \frac{1}{\sqrt{1 + v/c}} \frac{\sqrt{1 + \nu/c}}{\sqrt{1 - \nu/c}};
\]

\[
\tau = t \frac{1}{\sqrt{1 - v^2/c^2}};
\]

\[
\tau = \beta t;
\]

which is in line with Eq. (1) for \( x = 0 \) (source is at origin of \( K \)), thus satisfying the time dilation prediction of the Special Relativity.

It follows that the contradictory Eq. (5) leads to the Special Relativity time dilation Eq.(7), through the application of the relativistic Doppler shift formula given by Eq. (3).

Conversely, Eq. (6) is a legitimate Special Relativity equation, since it leads to its time dilation equation. However, this same equation yields the contradictory Eq. (5).

Based on the above, the Special Relativity is deemed to be unviable.

**Light Speed Principle Contradiction**

On the other hand, the constancy of the speed of light principle can be expressed as

\[
c = \lambda \nu = \lambda' \nu',
\]

where \( \lambda \) and \( \lambda' \) are the wavelengths in the source and observer’s frame, \( K \) and \( k \), respectively.

Considering the wave period perceived as \( t \) and \( \tau \) in \( K \) and \( k \), respectively, Eq. (8) leads to

\[
t \nu = \tau \nu', \text{ or } \tau = t \frac{\nu}{\nu'};
\]

yielding, (by Eq. (3))
\[ \tau = t \frac{\sqrt{1 + \frac{v}{c}}}{\sqrt{1 - \frac{v}{c}}} , \quad (9) \]

or

\[ \tau = t \frac{\sqrt{1 + \frac{v}{c}} \sqrt{1 - \frac{v}{c}}}{1 - \frac{v}{c}} ; \]

\[ \tau = t \frac{1}{\beta(1 - \frac{v}{c})} , \text{ or } \tau = t\beta(1 + \frac{v}{c}) , \quad (10) \]

which is in contradiction with the Special Relativity time dilation Eq. (7).

However, in terms of the wavelength, using \( \tau = \lambda'/c , \quad t = \lambda/c \), Eq. (10) results in

\[ \lambda' = \lambda \beta(1 + \frac{v}{c}) , \quad (11) \]

in line with the Special Relativity wavelength equation (obtained from Eq. (4) using \( \nu = c/\lambda , \quad \nu' = c/\lambda' \)). Hence, Eq. (10) contradicts the Special Relativity in terms of the time dilation, but agrees with it in terms of the wavelength transformation!

It should be noted that in order to obtain the Special Relativity time dilation formula (Eq. (7)) from Eq. (11), we must have \( \lambda' = (c + v)t \), and \( \lambda = ct \), the former being in contradiction with the speed of light principle. Hence, the time dilation Eq. (7) cannot be viable in terms of the speed of light postulate.

Using \( \lambda' = ct \) and \( \lambda = ct \), in line with the speed of light principle, Eq. (11) leads to Eq. (10). Hence, Eq. (10) gives the actual time dilation factor, \( \beta(1 + \frac{v}{c}) \), resulting from the speed of light principle, in conformance with the findings of earlier, related studies.

Furthermore, if the relativistic Doppler shift Eq. (4) was to be written in terms of the wave period, another contradiction would emerge. In fact, since as shown earlier \( \tau \nu' = t \nu = 1 \), Eq. (4) results in

\[ \frac{1}{\tau} = \beta \frac{1}{t} \left(1 - \frac{v}{c}\right) ; \]

\[ \tau = \beta t \left(1 + \frac{v}{c}\right) , \quad (12) \]

returning Eq. (10), in contradiction with the time dilation Eq. (7).

In order to obtain the time dilation equation from Eq. (4), we must have the absurdity

\[ \nu' = \frac{1}{(1 + \frac{v}{c})\tau} ; \]
\[ \frac{c}{\lambda'} = \frac{c}{(c + v)\tau} ; \]
\[ \lambda' = (c + v)\tau, \]

which is in contradiction with the speed of light principle.

**Conclusion**

When the relativistic Doppler shift equation, derived from the Special Relativity transformation equations, is used in a proposed contradictory equation, it leads to the Special Relativity prediction of the time dilation, which demonstrates the unviability of the Special Relativity. In addition, the speed of light principle, expressed in terms of the wave characteristics, is shown to result in contradictions with the Special Relativity equations, when coupled with the relativistic Doppler shift formula.

**References**