The Real Consequence of the Speed of Light Postulate: Failure of the Special Relativity

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In a previous study,[1] the Lorentz transformation was shown to be merely a set of restricted particular equations, the application of which to events having restricted coordinates—essential for the interpretation of time dilation and length contraction—resulted in mathematical contradictions. In this paper, the particular terms, erroneously generalized in the Lorentz transformation, are replaced with their correct expressions, resulting in a transformation conforming to the speed of light postulate, but having detrimental consequences on the Special Relativity predictions. The essential anomaly in the Lorentz time transformation equations leading to their fatal contradictions is identified, and the Special Relativity “established” predictions turn out to be overwhelmingly refuted.

Introduction

The Lorentz transformation equations constitute the backbone of the Special Relativity theory in which their interpretations lead to the peculiar predictions of the space-time distortion characterized by the length contraction and time dilation. The Lorentz transformation was derived on the basis of the constancy of the speed of light postulate[2-3]. The sought transformation, converting between the space and time coordinates of two inertial reference frames, say $K(x, y, z, t)$ and $K'(x', y', z', t')$, in relative motion at speed $v$, was assumed to take the following general form

\begin{align*}
x' &= ax + bt \\
y' &= y \\
z' &= z \\
t' &= kx + mt
\end{align*}

where $a, b, k, m$ are unknown real terms.

Whereas, the constancy of the speed of light postulate was expressed by the assumption that a spherical light wave front, emitted from the coinciding frame origins, would be observed as a light sphere centered at the frame origin, with its radius being expanded at the speed of light $c$, with respect to either frame:

\begin{align*}
x^2 + y^2 + z^2 &= c^2 t^2 \\
x'^2 + y'^2 + z'^2 &= c^2 t'^2
\end{align*}

leading to

\begin{equation}
x^2 - x'^2 = c^2 t^2 - c^2 t'^2
\end{equation}

In the customary derivation of the Lorentz transformation, the latter speed of light constancy equation along with the above proposed space and time transformation equations and given particular conditions would
be solved for the unknown terms, yielding the following Lorentz transformation equations:
\[
\begin{align*}
    x' &= \gamma(x - vt); \\
    y' &= y; \\
    z' &= z; \\
    t' &= \gamma \left( t - \frac{vx}{c^2} \right); \\
    \gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. 
\end{align*}
\]

The above approach is rather complex, which makes inconsistent operations performed in the derivation process easily bypassed. For instance, the above constancy of the speed of light equation was obtained in the original paper on Special Relativity\textsuperscript{[2]} through constructing it from the basic conversion expressions \(x = ct\); \(x' = ct'\) presenting the speed of light invariance in the relative motion direction:
\[
\begin{align*}
    x &= ct; \quad x^2 = c^2 t^2; \quad x^2 - c^2 t^2 = 0 \\
    x' &= ct'; \quad x'^2 = c^2 t'^2; \quad x'^2 - c^2 t'^2 = 0 \\
    x^2 - c^2 t^2 &= x'^2 - c^2 t'^2; \\
    x'^2 - x^2 &= c^2 t^2 - c^2 t'^2. 
\end{align*}
\]

Obviously, the intrinsic property of the basic expressions = \(ct\); \(x' = ct'\), requiring \(x = 0\) when \(t = 0\)—thus leading to \(x' = 0\) and \(t' = 0\)—is lost in the above constructed speed of light equation. To remedy this inconsistency, the above constructed equation should be restricted to non-zero coordinate values.

Furthermore, we can equally use the basic expressions \(x = ct\); \(x' = ct'\) to construct the following equation, by squaring each one and adding the resulting expressions:
\[
x^2 + x'^2 = c^2 t^2 + c^2 t'^2,
\]
which would make the \(y, y', z,\) and \(z'\) coordinates equal to zero in the above light sphere equations.

Consequently, to avoid the encountered inconsistencies in the above conventional derivation approach, a straight forward method is used in this study to derive the actual transformation resulting from the speed of light postulate, and reveal the inherent conflicts in the Lorentz transformation.

The speed of light constancy principle equations, as well as the Lorentz transformation, have been the subject of analytical studies,\textsuperscript{[1, 4-6]} in which mathematical contradictory results, attributed to the Lorentz transformation and the speed of light postulate, have been unveiled.

This study takes a step further to correct the contradictory terms in the Lorentz transformation, resulting in the actual transformation that should follow from the Special Relativity constancy of the speed of light postulate. The obtained transformation effect is in total contradiction with the Special Relativity essential predictions.

The Actual “Light Speed Postulate” Transformation

Consider two inertial reference frames, \(K(x, y, z, t)\) and \(K'(x', y', z', t')\), in relative uniform motion along the overlapped \(x\)- and \(x'\)-axes, at a speed \(v\). The transformation relating the space and time coordinates of the two frames is to be determined. If the time duration was considered to be unchanged from one frame to another, the coordinate conversion equation would then be governed by the Galilean transformation, namely
\[
x' = x - vt
\]
with unchanged \(y\) and \(z\) coordinates (i.e. \(y = y'; z = z'\)).

It would then be inferred that the general transformation should have the following linear form;
\[
x' = \gamma x + \beta t,
\]
where \(\gamma\) and \(\beta\) are real terms to be determined—\(y\) and \(z\) remain invariant.

For both cases described by equations (1) and (2), the origin of \(K'\) is traveling at speed \(v\) with respect to \(K\) origin. Therefore, we can conclude that the coordinate \(x' = 0\) in \(K'\) would be transformed to \(x = vt\) in \(K\), by both equations. Hence, plugging the particular conversion \(x' = 0; x = vt\) in the general transformation equation (2) yields the particular equation \(0 = \gamma vt + \beta t,\) or \(\beta = -\gamma v\) (for \(t \neq 0\),...
leading to a simplified general transformation equation

\[ x' = \gamma(x - vt). \]  

(3)

Furthermore, under the principle of the constancy of the speed of light, another particular conversion related to the \( x \)-coordinate of the tip point of a light ray propagating in the relative motion direction is readily available, and can be expressed as \( x = ct; x' = ct' \), which, when plugged in equation (3), leads to the time transformation equation

\[ ct' = \gamma(ct - vt); \]

\[ t' = \gamma t \left(1 - \frac{v}{c}\right); \]  

(4)

which is a general time equation since it only involves the time variables.

It should be noted that if \( t \) was replaced by \( x/c \) in the term \( vt \) of equation (4), in line with \( x = ct; x' = ct' \), it would lead, with equation (3), to the Lorentz transformation, as demonstrated in an earlier study,\([1]\) which was shown to be inconsistent when applied to events with zero time \( (t \text{ or } t') \) or zero longitudinal spatial coordinate \( (x \text{ or } x') \).

Now, owing to the fact that the reference frame \( K \) is traveling at a speed of \(-v\) with respect to \( K' \), and to the essential symmetrical property of the transformation with respect to the reference frames, the inverse of the general transformation given by equation (3) can be written as

\[ x = \gamma(x' + vt'), \]  

(5)

requiring by symmetry the restriction \( t' \neq 0 \).

Similarly, under the principle of the constancy of the speed of light, plugging the particular conversion of the tip point \( x' \)-coordinate of a light ray propagating in the relative motion direction, expressed as \( x' = ct' \); \( x = ct \), in the general transformation equation (5) leads to the time transformation equation

\[ t = \gamma(ct' + vt'); \]

\[ t = \gamma t' \left(1 + \frac{v}{c}\right). \]  

(6)

Substituting equation (4) into equation (6) leads after simple simplification to

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \]  

(7)

It is worth mentioning that dividing equation (3) by equation (4) leads to the following velocity transformation

\[ u' = \frac{x'}{t'} = \frac{\gamma(x - vt)}{\gamma t \left(1 - \frac{v}{c}\right)}; \]

\[ u' = \frac{u - v}{1 - \frac{v}{c}}. \]

Similarly, dividing equation (5) by equation (6) leads to

\[ u = \frac{u' + v}{1 + \frac{v}{c}}, \]

where \( u \) and \( u' \) are the velocity of an object with respect to \( K \) and \( K' \), respectively, traveling in the relative motion direction.

It is noted that the obtained velocity transformation returns the speed of light \( c \) when either \( u \) or \( u' \) is replaced with \( c \).

**Conflicting Findings**

It follows that, the light speed constancy principle leads to equations (4) and (6) presenting the relationship between the time coordinates \( t \) and \( t' \) in the two reference frames \( K \) and \( K' \). However, these equations are in total disagreement with the respective Lorentz transformation equations given by

\[ t' = \gamma \left(t - \frac{vx}{c^2}\right); \]

\[ t = \gamma \left(t' + \frac{vx'}{c^2}\right). \]

To identify the cause of this discrepancy, let’s rewrite equations (4) and (6) in the following form
Obviously, the term $vt/c$ in equation (10) represents the time it takes a light signal to propagate across the distance $vt$ traveled by $K'$ origin with respect to $K$ at time $t$. Now, if, for instance, two light signals, separated by a time interval $t$, were emitted from the origin of $K'$ (i.e., $x'=0$), the distance $vt$ becomes equal to $x$, and the term $vt/c$ reduces to $x/c$, simplifying equation (10) to

$$t' = \gamma \left(t - \frac{vt}{c}\right); \quad (10)$$

$$t = \gamma \left(t' + \frac{vt'}{c}\right). \quad (11)$$

It should be noted that $x/c$ in the above particular equation (for the particular case of $x'=0$) is the light signal propagation time through the distance $x$ (that replaced the general term $vt$ for the special case of $x'=0$), and not the time $t$ (equals to the travel time of the reference frame $K'$ through the distance $x$ at the speed $v$). Also, setting $x=0$ in the above equation doesn’t merely mean that $t' = \gamma t$, since the above equation is resulting from the time transformation equation (10) for $x = vt$, i.e. for $x' = 0$, and the condition of having both $x$ and $x'$ equal to zero corresponds to $t = t' = 0$, or to no relative motion between the reference frames.

Comparing the above equation with Lorentz transformation equation (8), we notice the contradiction in the term $vx/c^2$, requiring $v = c$.

The same reasoning can be applied to equations (11) and (9) to draw a similar contradiction from equation (9) (i.e. $v = -c$).

Indeed, by a simple analysis of the spatial Lorentz transformation equations, or the effect of the frames relative motion on the perception of the time coordinates of a light signal, the term $vx/c^2$ in the Lorentz transformation equation (8) must actually represent the travel time of a light signal emitted from the origin of $K$ at time $t$ to reach the origin of $K'$ at a distance of $vt$ from that of $K$. Therefore,

$$\frac{vx}{c^2} = \frac{vt}{c},$$

leading to

$$x = ct.$$

Hence, the term $x$ in the Lorentz transformation equation (8) is actually confined to the value of $ct$ (erroneously replaced with $x$), which is contradicted with the fact that $x$ takes the value of $vt$ when $x'=0$, making $v=c$ for $x'=0$; this is indeed the source of the Lorentz transformation contradiction obtained when $x'=0$ (as well as when $x=0$).[1,4,5] A similar contradiction emerging from Lorentz transformation equation (9) can be demonstrated (i.e. erroneously using $x' = ct'$ in the term $vt'/c$ leads to $v = -c$, since for $x = 0; x' = -vt'$).

Furthermore, when $t'=0$, Lorentz transformation (8) leads to $t = vx/c^2$. But, as shown above, $x = ct$ in equation (8), yielding the contradiction $t = vct/c^2$, or $v = c$.

Similarly, Lorentz transformation (9) can lead to a similar contradiction for $t = 0$ (i.e. $v = -c$).

Hence, once again, the Lorentz transformation equations are demonstrated to be overwhelmed with critical contradictions, and therefore unviable.

It follows that the Lorentz time transformation equations (8) and (9) are invalid, and the correct structure of these equations is given by equations (4) and (6), which totally overthrow all of the Special Relativity predictions in terms of the length contraction and time dilation, and their
resulting interpretations. Moreover, the actual transformation emerging from the light speed postulate has no realistic interpretation, ending up with an unrealistic postulate.

Conclusion

The Lorentz time transformation equations are demonstrated to erroneously confine the involved spatial coordinates (in the terms $\nu x/c^2$ and $\nu x'/c^2$) to the specific values of $x = ct$ and $x' = ct'$, which results in conflict with the frame origin coordinates with respect to one another (i.e. $x = vt$ and $x' = -vt'$). In using the correct term in these equations, the resulting transformation is found to be in total disagreement with the Lorentz transformation, leading to the refutation of its predictions.

References


