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A CRITIQUE OF FRESNEL'S LAW

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Introduction

Fresnel's law gives the speed w of light, at normal incidence, in a medium of refractive index μ, moving with speed v in a vacuum, as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right) \tag{1}$$

Equation (1) is obtained from Einstein's *relativistic law for addition of velocities*, where a ray of light is propagated in a vacuum with speed c in the same direction as the medium also moving in a vacuum with speed v.

An alternative formula is obtained by taking the ratio of relative speed $(c - v)$ between light propagated with speed c in a vacuum and the medium moving in a vacuum with speed v, to the relative velocity $(w - v)$ between light propagated in the medium with speed w and the moving also medium moving with speed v, thus:

$$\frac{c - v}{w - v} = \mu$$

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \tag{2}$$

2. Galileo's velocity addition rule

According to Galileo's *velocity addition rule*, if you run with velocity $\boldsymbol{u}$ in a ship (medium) cruising with velocity **s**, your velocity relative to an observer sitting on the shoreline, is vector sum z, given by:

$$\mathbf{z} = \mathbf{u} + \mathbf{s} \tag{3}$$

Here, the velocities $\boldsymbol{u}$ and $\boldsymbol{s}$ are vectors that can be of any magnitude and in any direction. The velocity relative to an observer moving with velocity $\boldsymbol{v}$, is $\mathbf{z} = \mathbf{u} + \boldsymbol{s} - \mathbf{v}$. This is the Galilean principle of relativity, which is in agreement with observation and natural sense.

3. Einstein's velocity addition rule

According to Einstein's *velocity addition rule,* if you run with speed u relative to a ship (medium) moving with speed s, your speed relative to an observer sitting on the shoreline, is:

$$z = \frac{u+s}{1+\frac{us}{c^2}} \tag{4}$$

where c is the speed of light in a vacuum.

In equation (4), the velocities $\boldsymbol{u}$ and $\boldsymbol{s}$ of magnitudes u and s respectively, must be collinear. In reality, you should be able to run with velocity $\boldsymbol{u}$ in any direction relative to $\boldsymbol{s}$. If $u = c$ (speed of light) or $u = v = c$, the speed z remains as c. Equation (4), more than anything else, had lent support to the principle of constancy of the speed of light, in accordance with special relativity. For speeds much less than c, or if c is infinitely large, Einstein's relativistic formula reduces to the Galilean classical formula.

According to the theory of special relativity, if you were a ray of light running with velocity of light $\boldsymbol{u} = \boldsymbol{c}$, relative to a medium (the ship) moving in a vacuum with velocity $\boldsymbol{s}$, your velocity remains a constant $\boldsymbol{c}$, irrespective of the velocity of the medium (ship). Your velocity (of light) within and with respect to a medium (ship) of refractive index μ is $\boldsymbol{c}/\mu$. Einstein's *velocity addition rule*, with $u = c/\mu$, gives the speed w of light, at normal incidence, in the moving medium (with respect to an observer sitting on the shore) as:

$$w = \frac{\frac{c}{\mu}+s}{1+\frac{cs}{\mu c^2}} = \frac{\frac{c}{\mu}+s}{1+\frac{s}{\mu c}} \approx \left(\frac{c}{\mu}+s\right)\left(1-\frac{s}{\mu c}\right) \tag{5}$$

where s^2/c^2 is negligible if $s \ll c$.

$$w = \frac{c}{\mu} - \frac{s}{\mu^2} + s - \frac{s^2}{\mu c} = \frac{c}{\mu}\left(1-\frac{s^2}{c^2}\right) + s\left(1+\frac{1}{\mu^2}\right) \tag{6}$$

Again, neglecting s^2/c^2 compared with s/c, we obtain:

$$w = \frac{c}{\mu} + s\left(1-\frac{1}{\mu^2}\right) \tag{7}$$

Equation (7), the same as equation (1), is Fresnel's law.

4. Fresnel's Law Versus the Alternative Formula

In equations (1) and (2) the speed v of the moving medium can take any value between 0 and $\pm c$. For $v = 0$, both equations give the speed of light in the medium $w = c/\mu$, as expected. For $\mu = 1$ both equations give $w = c$, also as expected. For $v = c$, equation (2) gives $w = c$, again as expected, but equation (1) gives $w = c(1 + 1/\mu - 1/\mu^2)$, which may be greater than c as $\mu > 1$, and this is untenable. For $v = -c$, equation (2) gives $w = c(2/\mu - 1)$, which is reasonable as $\mu < 2$, but equation (1) gives $w = c(1/\mu^2 + 1/\mu - 1)$, which may be negative if $\mu > 1.618$, and this is absurd.

Equation (2) is an exact expression, its derivation not dependent on any approximation. In contrast, equation (7) is derived on the basis of speed s of the medium being small compared to the speed of light c, so that s^2/c^2 is negligible compared to unity, but there is nothing to prevent s being comparable to c. The conclusion here is that equation (7), Fresnel's law, the relativistic equation for the speed of light in a moving medium, may be wrong and equation (2) is more likely to be correct.