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AN INTERPRETATION OF RELATIVISTIC MASS IN CIRCULAR MOTION OF AN ELECTRON

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1 Introduction

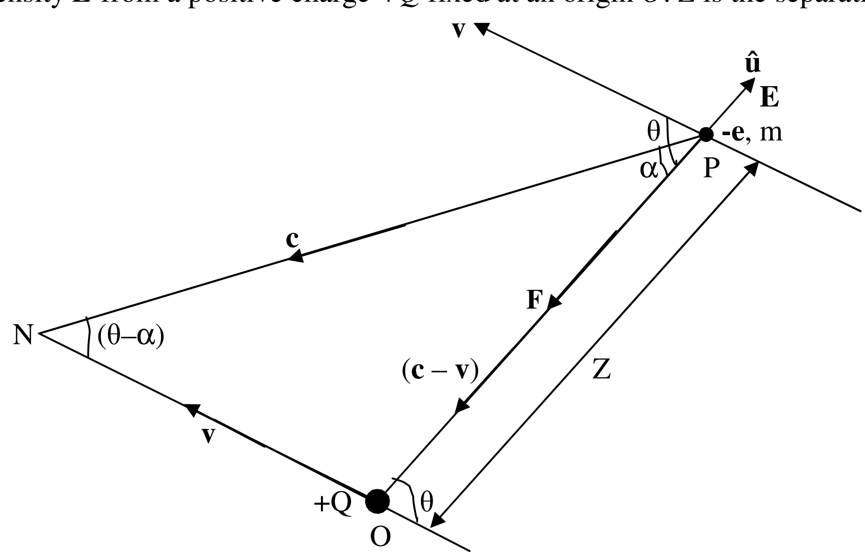
The theory of special relativity gives the mass *m* of a particle moving with speed *v* as:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

where m_o is the rest mass and *c* the speed of light in a vacuum. This formula has a difficulty with mass *m* becoming infinitely large if $v = c$. In this paper, it is shown that equation (1) is correct for circular motion only and that *'m'* is not a physical mass, which has weight, but the ratio of *electrostatic force on a revolving charged particle* to *centripetal acceleration in circular motion.* This ratio may become infinitely large, without any difficulty.

2 Aberration of Electric Field

The Figure below depicts an electron of charge -e and constant mass $m = m_o$ moving at a point *P* with velocity ***v*** at an angle θ to the accelerating force ***F*** due to an electric field of intensity ***E*** from a positive charge +*Q* fixed at an origin *O*. *Z* is the separation of the charges..



An electron of charge –e and mass *m* moving, at *P*, with velocity ***v***, at an angle θ to accelerating force ***F***. The unit vector $\hat{\boldsymbol{u}}$ is in the direction of the electrostatic field ***E*** due to a charge +Q fixed at O.

As a result of motion, the electron "sees" the electric field along **PN**, defined by velocity vector ***c***, displaced by angle of aberration α from the line ***PO*** joining *Q* and *-e*, such that:

$$\sin\alpha = \frac{v}{c}\sin\theta \tag{2}$$

where c is the speed of light. The angle α is due to aberration of electric field, a phenomenon similar to aberration of light discovered in 1728 by English astronomer James Bradley.

3 Electrostatic force of attraction

With reference to the Figure above, the force $\boldsymbol{F}$ of attraction between Q and $-e$ is put, in accordance with Coulomb's law of electrostatics and Newton's second law of motion, as:

$$\mathbf{F} = \frac{Qe}{4\pi\varepsilon_o cZ^2}(\mathbf{c}-\mathbf{v}) = \frac{eE}{c}(\mathbf{c}-\mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{3}$$

where ε_o is the permittivity of a vacuum, Z is the distance between the charges, E is the magnitude of electric field at P and $(d\boldsymbol{v}/dt)$ the acceleration at time t. The vector $(\boldsymbol{c} - \boldsymbol{v})$ is the relative velocity of transmission of the force with respect to the moving electron of charge $-e$.

Equation (3) is expressed as the modulus of a vector with the accelerating force $\boldsymbol{F}$ as:

$$\mathbf{F} = \frac{-eE}{c}\sqrt{c^2 + v^2 - 2cv\cos(\theta-\alpha)}\,\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt} \tag{4}$$

where (θ – α) is the angle between the velocities $\boldsymbol{c}$ and $\boldsymbol{v}$ and $\hat{\mathbf{u}}$ is a unit vector in the direction of the electric field of intensity $\boldsymbol{E}$ and magnitude E, Solving the differential equation (4) in v and t, with equation (2), it is shown that the electron can move in a straight line with θ = 0, to reach a maximum speed equal to c. In motion with θ = π radians the electron is decelerated to a stop and then accelerated in the opposite direction to reach a maximum speed –c.

Where θ = π/2 radians and with $\cos(\theta - \alpha) = \sin\alpha = v/c$, equations (2) and (4) give:

$$\mathbf{F} = -eE\sqrt{1-\frac{v^2}{c^2}}\,\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt} \tag{5}$$

For θ = π/2 radians, we have motion in a circle of radius r, in a radial electric field E, with constant speed v and centripetal acceleration $(d\boldsymbol{v}/dt) = -(v^2/r)\hat{\boldsymbol{u}}$. Equation (5) with $m = m_o$, is:

$$\mathbf{F} = -eE\sqrt{1-\frac{v^2}{c^2}}\,\hat{\mathbf{u}} = -m\frac{v^2}{r}\hat{\mathbf{u}} = -m_o\frac{v^2}{r}\hat{\mathbf{u}}$$

$$eE = \frac{m_o v^2}{r\sqrt{1-\frac{v^2}{c^2}}} = \frac{\varsigma v^2}{r} \tag{6}$$

$$\varsigma = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{eE}{v^2/r} \tag{7}$$

where ζ (zeta) is the relativistic mass.

4 Conclusion

In equation (7), relativistic mass ζ increases with speed in accordance with equation (1). Equations (1) and (7) are identical but obtained from two different points of view. While equation (1) is misconstrued as mass m increasing with speed v, in equation (7) the relativistic mass ζ is interpreted as the ratio of *electrostatic force (eE)* to *centripetal acceleration (v^2/r)* in circular motion. This ratio can become infinitely large, as is the case in rectilinear motion (circular motion of infinite radius) with constant speed c.

Equation (1) is a good example of *Beckmann Correspondence theory* whereby the mathematics is correct but the physics is wrong. Equation (7) is the correct interpretation of relativistic mass ζ that can become infinitely large at the speed of light c. This resolution of relativistic mass, being different from inertial mass, should bring great relief to physicists.