The Origin of Generalised Mass-Energy Equation $\Delta E = Ac^2 \Delta M$; and its applications in General physics and Cosmology.

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Einstein's 27 Sep 1905 paper available at http://www.fourmilab.ch/etexts/einstein/E_mc2/www/

Abstract

Einstein's (Sep. 1905) derivation theorizes that when light energy (L) is emanated by luminous body then its mass diminishes as $\Delta m = L/c^2$ and this equation is speculative origin (without proof) of $\Delta E = c^{2\Delta} m$. The same derivation predicts that mass of luminous body inherently INCREASES ($\Delta m = -0.03490 \, L/cv +L/c^2$) when it emits light energy in some cases, mass of body also remains same ($\Delta m = 0$). Alternate equation $\Delta E = Ac^{2\Delta} M$ has been suggested, which implies that energy emitted on annihilation of mass (or vice versa) can be equal, less and more than predicted by $\Delta E = c^2 \Delta m$. The total kinetic energy of fission fragments of U^{235} or Pu^{239} is found experimentally 20-60 MeV less than Q-value predicted by Δmc^2, it is explainable with $\Delta E = Ac^2 \Delta M$ with value of A less than one. $\Delta E = c^2 \Delta m$ is yet unconfirmed in chemical reactions. Energy emitted by Gamma Ray Bursts (most energetic event after Big Bang) in duration 0.1-100s, is 10^{45}J which can not be explained by $\Delta E = \Delta mc^2$, similar is the case of Quasars. It can be explained with high value of A i.e. 2.57×10^{18}. The mass of particle Ds (2317) discovered at SLAC, have mass lower than current estimates; it can be explained with value of A more than one. $\Delta E = Ac^{2\Delta} M$, explains that mass of universe 10^{55}kg was created from dwindling amount of energy (10^{-444}J or less) and A is 2.568×10^{-471} J or less; and in the end may reduce to small energy. It gives explanation for big bang, annihilation of antimatter in hadron epoch, black holes and for dark matter etc. For origin of inherent gravitational energy it implies that it is another form of mass like other energies, hence gravitation and mass are inseparable.

1.0 Einstein's Light Energy- Mass equivalence $\Delta m = L/c^2$ (Sep. 1905 paper)

The law of conservation of mass or energy existed in literature since 18th century (or may be even before informally) the French chemist Antoine Lavoisier (1743-1794) was the first to formulate such a law in chemical reactions. The very first idea of mass-energy inter conversion was given by Fritz Hasenohrl [1] that the kinetic energy of cavity increases when it is filled with the radiation, in such a way that the *mass of system appears to increase* before Einstein's pioneering work [2]. Then Einstein [2] calculated relativistic form of Kinetic Energy [$KE_{rel} = (m_r - m_o)c^2$] in June 1905. From this equation at later stage Einstein[3] derived result $E_o = m_o c^2$, where E_o is Rest Mass Energy, m_0 is rest mass and c is velocity of

light. Einstein also quoted the same method of derivation of Rest Mass Energy in his other works [4], whereas in some other cases he completely ignored it [5]. Many other celebrated authors quote the derivation in the exactly similar and simplified way [6,7]. Einstein [8] derived or speculated relationship between mass annihilated (Δm) and energy created (ΔE) i.e. $\Delta E = \Delta mc^2$, in his paper widely known as September 1905. For first time the salient mathematical limitations and contradictions of this derivation have been pointed out and alternate equation $\Delta E = Ac^2 \Delta M$ has been proposed by author. This method [8] is critically discussed below for understanding then its possible inconsistencies and alternate equation ($\Delta E = Ac^2 \Delta M$) are pointed out for first time.

(i) Einstein [8] perceived that let there be a luminous body at rest in co-ordinate system (x, y, z) whose energy relative to this system is E_o. The system (ξ, η, ζ) is in uniform parallel translation w.r.t. system (x, y, z); and origin of which moves along x-axis with velocity v. Let energy of body be H_o relative to the system (ξ, η, ζ). Let a system of plane light waves have energy ℓ relative to system (x, y, z), the ray direction makes angle φ with x-axis of the system. The quantity of light measured in system [ξ, η, ζ] has the energy [2].

$\ell^* = \ell\{1 – v/c \cos \varphi \} /\sqrt{[1 – v^2 /c^2]}$

$\ell^* = \ell \beta \{1 – v/c \cos \varphi\}$ (1)

where $\beta = 1 /\sqrt{(1 – v^2 /c^2)}$

The Eq.(1) was proposed by Einstein [2] in Section (8), as an analogous assumption without specific derivation and critical analysis.
(ii) Let this body emits plane waves of light of energy 0.5L (measured relative to x, y, z) in a direction forming an angle φ with x-axis. And at the same time an equal amount of light energy (0.5L) is emitted in opposite direction ($\varphi+180^o$).
(iii) The status of body before and after emission of light energy. According to the Einstein's original remarks ……..
Meanwhile the body remains at rest with respect to system (x,y,z).
So luminous body is not displaced from its position after emission of light energy.
If E_1 and H_1 denote energy of body after emission of light, measured relative to system (x, y, z) and system (ξ, η, ζ) respectively. Using Eq. (1) we can write (equating initial and final energies in two systems)

Energy of body in system (x,y,z)

$E_o = E_1 + 0.5L +0.5L = E_1 + L$ (2)
Or Energy of body w.r.t system (x,y,z) before emission = Energy of body w.r.t system (x,y,z) after emission + energy emitted (L).

$H_o = H_1 + 0.5 \beta L\{(1 – v/c \cos\varphi) + (1+ v/c \cos \varphi) \}$ (3)
Energy of body in system (ξ, η, ζ)

$H_o = H_1 + \beta L$ (4)

Or Energy of body w.r.t system (ξ, η, ζ) before emission = Energy of body w.r.t system (ξ,η,ζ) after emission + energy emitted (βL).

Subtracting Eq. (2) from Eq.(4)

$(H_o - E_o) - (H_1 - E_1) = L [\beta - 1]$ (5)

Or { Energy of body in moving system (ξ, η, ζ) – Energy of body in system (x,y,z)} $_{before\ emission}$ –
{Energy of body in moving system(ξ,η,ζ)–Energy of body in system (x,y,z)}$_{after\ emission}$}
$= L [\beta - 1]$ (5)

Einstein neither used nor mentioned in calculation or description the relativistic variation of mass which is given by
$m_r = \beta m_o = m_o / (\sqrt{(1 - v^2/c^2)})$ (6)

where is relativistic mass (m_r) and m_o is rest mass of the body excluding the possibility that velocity v is in relativistic region. This equation existed before Einstein and was initially justified by Kauffmann [9] and more comprehensively by Bucherer [10]
(iii) Further Einstein [8] assumed the following relations (and tried to justify them at later stage).

$H_o - E_o = K_o + C$ (7)

$H_1 - E_1 = K_1 + C$ (8)

where K is kinetic energy of body, C is additive constant which depends upon the choice of the arbitrary additive constants in the energies H and E. Thus Eq. (5) becomes

$K_o - K = L \{1/\sqrt{(1 - v^2/c^2)} - 1\}$ (9)
From here Einstein deduced using Binomial Theorem without giving any justification the relation between light energy (L) emitted and decrease in mass (Δm) as
$\Delta m = L/c^2$ (10)
or M_a (mass of body after emission) = M_b (mass of body before emission) – L/c^2
Thus mass of body decreases when light energy is emitted. The body may also emit one, two, three or many waves of different magnitudes of light energy (0.499L and 0.501L or L etc.) at different angles (different values of φ) and the velocity v may be non-uniform also. The derivation, should be free of limitations and inconsistencies and should lead to same result $\Delta m = L/c^2$ even under diverse conditions as law of conservation of mass and energy holds good in all cases. These aspects are neither addressed by Einstein nor other scientists at all, and an attempt is made to do so.

1.1 Detailed discussion on some aspects of Einstein's derivation.

Einstein's derivation of $\Delta m = L/c^2$ and its generalisation is in form of brief and compact discussion or note [8] , which contains no sections or sub-sections and equations are also unnumbered. Some significant deductions and conclusions are given by Einstein in straight way without explanation.

The law of conservation of mass-energy is general law, having far reaching importance and consequences, thus the detailed and critical analysis of the derivation in all respects is absolutely necessary. Sharma [11-13] has initiated a logical discussion , which is required to be elaborated taking all factors in account. .

1.11 Einstein used classical conditions of velocity:

If Einstein's remarks in original paper [8] after the stage he derived Eq.(9) are quoted then above aspect (i.e. velocity is in classical region) is crystal clear. Purposely Einstein's original text is being quoted below in two parts in italics.

(i) The kinetic energy of body diminishes as result of the emission of light

(As KE of body is $m_o v^2/2$, it implies decrease in mass of body as it is moving with constant velocity, v)

(ii) *and amount of diminution is independent of properties of body*

The amount of diminution in KE is (diminution in mass)$v^2/2$, Einstein regarded velocity v as constant in the derivation. Thus magnitude of diminution in mass is dependent on original mass, if the mass does not change the diminution in KE remains the same. The mass remains unchanged if velocity is in classical region. Thus if velocity of body is in classical region, then amount of variation of mass of body is independent of velocity of body (property of body). But it is not so if velocity v, is in relativistic region then mass increases as given by Eq.(6), which is neither taken in account nor mentioned at all. Hence Einstein has assumed velocity is in classical region. So mass, diminution in mass and hence kinetic energy are independent of velocity of body (property of body) which is constant in classical region. Further to support this fact Einstein has used Binomial Theorem (v<<c) by quoting [8]

Neglecting magnitudes of fourth and higher orders we may place.

In Eq.(9) the magnitudes of fourth and higher orders (v^4/c^4, v^6/c^6 …..v^n/c^n) occur on application of Binomial Theorem (v<<c), these are neglected. And only magnitude of second order (v^2/c^2) are retained in calculations. In view of it Einstein solved Eq.(9) as

$K_o - K = Lv^2/2c^2$ (11)

If Einstein had considered velocity in relativistic region then equation

$m_r = \beta m_o = m_o / (\sqrt{(1 - v^2/c^2)})$ (6)

would have been taken in account. Thus mass of body would have been increased; but here only decrease in mass has been considered. In the mathematical and conceptual treatment Einstein did not at all mention about this (increase in mass). If velocity of body is not in relativistic region, then it is classical region. The above remarks and exclusion of Eq.(6) specifically exclude this possibility and confirm that the velocity is in classical region. We have,

KE of body before emission –KE of body after emission = $K_o - K = L \{1/\sqrt{(1 - v^2/c^2)} - 1\}$ (9)

Without giving any further mathematical explanation Einstein [8] conclude that

'If a body emits energy L in form of radiations, its mass decreases by L/c^2.'

Following is the obvious mathematical step but not mentioned by Einstein in brief note or paper [8], as he has given final result in straight way as in Eq.(10) i.e. $\Delta m = L/c^2$. Using Binomial Theorem Eq.(9) is written as Eq.(11), which can be further written as

$M_b v^2/2 - M_a v^2/2 = Lv^2/2c^2$ (11)

Mass of body before emission (M_b) – Mass of body after emission (M_a) = L/c^2

Or $\Delta m = L/c^2$ (10)

M_a (mass of body after emission) = M_b (mass of body before emission) – L/c^2

Here L is energy emitted by luminous body i.e. equal to difference between magnitude of final energy

(L_{final}) and initial energy ($L_{initial}$) thus Eq.(10) in more transparent way can be written as

$M_b - M_a = (L_{final} - L_{initial})/c^2$ (10)

If $L_{initial}$ is regarded as zero, then light energy emitted (ΔL) equals L_{final}, L (say).

Thus it implies that when body emits light energy then its mass decreases i.e the mass is annihilated into the light energy. Thus the law of conservation of light energy emitted and mass annihilated is established. Then Einstein speculated that the conservation law not only exists between light energy emitted and mass annihilated but also between all energies and masses, without scientific justification or logical embellishment as a postulate. This aspect is discussed separately.

1.12 Eq.(10) can also be obtained if single wave of light energy L, is emitted.

This case has neither been discussed by Einstein nor others. Consider a body is placed in the system (x,y,z) and emits a single wave of light energy L, which is perpendicular to ray direction ($\varphi = 90^{\circ}$). And it is observed in system (ξ, η, ζ) moving with uniform relative velocity v, exactly in the similar way as in Einstein's derivation. In that case Eq.(3) can be written as

$H_o = H_1 + \beta L\{(1 - v/c \cos 90^{\circ})\}$

$H_o = H_1 + \beta L$ (4)

Also, we have

$E_o = E_1 + L$ (2)

Now proceeding as in Eq.(5) to Eq.(10) we get

$\Delta m = L/c^2$ (10)

M_a (mass of body after emission) $= - L/c^2 + M_b$ (mass of body before emission)

which is the same result as obtained by Einstein.

1.13 Einstein's condition: Body remains at rest before and after emission of light energy
or
Momentum of body before emission is equal to momentum after emission.

Einstein has not even mentioned term conservation of momentum in his derivation at all. However according to the Einstein's original remarks (after describing emission of light energy by luminous body) ……..

Meanwhile the body remains at rest with respect to system (x, y, z).

Einstein's this condition is satisfied in numerous cases when luminous body emits energy and remains at rest. The body remains at rest before and after emitting the light energy L as recoil is vanishing small or zero, hence initial momentum of luminous body is equal to the final momentum. The extent of recoil also depends upon the resistive forces (frictional, gravitational, atmospheric etc.) present in the system. The body may remain at rest after emitting one, two or more waves simultaneously having energy different than 0.5L. In such cases initial and final momenta of the luminous body are equal as the net change after emitting energy is negligibly small. Thus body remains at rest as requisite of Einstein's constraint. Further in description of light emitting body its mass is regarded as in classical region. For example, Einstein has perceived that body (not particle or wave) remains at rest before and after emitting energy. If a wave or particle emits light energy then it will not remain at rest as per Einstein's condition. Thus mass may be of the order of 10 gm or less (such that body remains at rest after emitting light energy as mentioned by Einstein) or more (body may be heavier than 100kg unreservedly). The law of conservation of energy holds good for all bodies, particle or waves; thus the derivation must be in general and applicable to all cases.

The conservation of momentum, which is not at all mentioned by Einstein, holds good when body emits light energy. For example, if a light wave is emitted in visible region ($f = 4 \times 10^{14}$ Hz) has energy (hf) 2.6×10^{-19}J, a wave with this energy is unable to cause any observable recoil (however it tends to do so) in a body of mass in classical region as used or intended by Einstein. As already mentioned the recoil also depends upon resistive forces present in the system . The law of conservation of momentum is obeyed in this case, as in previous case which has been considered by Einstein [8] to derive Eq.(10). The body remains at rest before and after emitting energy which is Einstein's main condition in above derivation. Analogously, consider a man fires a bullet from his gun and as bullet moves forward man recoils observably, and law of conservation of momentum is obeyed (forward momentum of bullet equals backward momentum of man). However exact velocity of recoil also depends upon resistive forces (frictional, atmospheric and gravitational forces) present in the system. Then the man fires a shot from the toy gun, and this shot is unable to cause any observable recoil (however system tends to recoil). The initial momentum remains equal to final momentum. The law of conservation of momentum (in one wave or two waves or more) equally holds good in these cases; the energy of waves is unable to recoil body observably. In this case the initial momentum and final momentum (after emitting light waves) of luminous body are also equal as change in momentum is vanishing small.

The non-displacement of body from original position during and after the emission of light energy, which may be regarded as the simplest or special case. However the law of conservation of mass-energy (which is ultimate result) holds good in all cases when velocity of body may be zero, uniform or non-uniform (motion may be accelerated or non-accelerated) w.r.t. system (x,y,z) and system (ξ, η, ζ). As already mentioned the body also **remains at rest** if light energy emitted by luminous body in one, two or more waves at different angles having different magnitudes of energy. Thus Einstein has discussed the simplest case.

1.2 Einstein did not compare relativistic kinetic energy of slowly accelerated electron i.e. $K_f - K_i = W = m_o c^2 \{1/ \sqrt{(1 - V^2/c^2)} - 1\} = m_r c^2 - m_o c^2$ and kinetic energy from light energy mass equation $K_o - K = L \{1/\sqrt{(1 - v^2 /c^2)} - 1\}$

According to Work-Energy Theorem, work done is equal to change in kinetic energy.
Work done (W) = Final Kinetic Energy (K_f) – Initial Kinetic Energy (K_i)

If body is moving with uniform velocity (a=0), then initial KE is equal to the final KE, thus W is zero; also in this case acceleration is zero thus force (F=ma) and work done (W=FS=0). Then Einstein gave a result in straight way in the paper he introduced the Special Theory of Relativity [2] as
$K_f - K_i = W = m_o c^2 \{1/\sqrt{(1 - V^2/c^2)} - 1\} = m_r c^2 - m_o c^2$ (12)
where V is variable velocity (slowly accelerated motion) and other terms are defined earlier. Further in Eq.(9) L is ΔL i.e. difference in initial and final energy.
Einstein's original remarks (after indirectly justifying that velocity v is in classical region)
Moreover, the difference $K_0 - K$, like the kinetic energy of the electron depends upon the velocity.

These may be regarded as just passing remarks as right hand sides of both the equations depend upon velocity (v, constant and V, variable). In his original thesis [8] Einstein correctly did not equate or mathematically compare both the equations i.e. Eq.(9) and Eq.(12) due to following reasons.
(i) The equations from different derivations can not be compared or equated unless they have same mathematical and conceptual nature or basis and they discuss the identical scientific aspects. The equations from different derivations cannot be mathematically equated just for the reason they have same dimensions. The energy have different forms e.g. heat energy, light energy, sound energy, electrical energy, kinetic energy etc, but have same dimensions. The dimensions of torque and work or energy are the same i.e. ML^2T^{-2}, for this reason both cannot be equated. Mathematically, Torque = $rF\sin\Theta$, a vector quantity and Work = $FS\sin\Theta$, a scalar quantity; thus in one case Θ is angle between F and r (position

vector) and in second case Θ is angle between F and S (displacement). Hence when angle Θ is 0° then Torque is zero but work done is maximum; and when Θ = 90°, Torque is maximum and work done is zero Thus even if two physical quantities have same dimensions, they may represent two different aspects , hence cannot be equated .

(ii) Different ways to measure change in the Kinetic energy in Eq.(9) and Eq.(12)
(a) In Eq.(9),
Change in Kinetic Energy =
Initial kinetic energy (before emission of light energy) – Final kinetic energy (after emission of light energy)
(b) In Eq.(12), according to Work Energy Theorem,
Change in Kinetic Energy =
Final Kinetic energy (after increase in mass) – Initial Kinetic Energy (original mass)
Thus expression for change in KE both the equations i.e. Eq.(9) and Eq.(12) is different.

(iii) Decrease or increase in mass:
(a) In Eq.(9) due to emission of light energy, the final mass of the body DECREASES , the decrease in mass is converted into energy. Thus final mass of the body is less than the original mass.
(b) In Eq.(12) when external force acts on the body and velocity is in relativistic region, the mass of body INCREASES and is known as relativistic mass

$m_r = m_o /\sqrt{(1–V^2/c^2)}$ (6)

The energy which is externally supplied to accelerate the body is converted into mass. Thus Hence in Eq. (12) the mass of the body INCREASES and in Eq.(9) mass of body DECREASES.

(iv) Constant and variable velocity.
(a) In Eq.(9) the velocity v with which system (ξ, η, ζ) moves with respect to system (x,y,z) is constant (thus motion is non-accelerated).
(b) Whereas in Eq.(12), as quoted by Einstein [2] himself represents motion of slowly accelerated electron, thus velocity V is variable.
Hence in Eq.(12) velocity is variable i.e. motion is ACCELERATED, and in Eq.(9) velocity is constant , motion is UN-ACCELERATED.

(v) Velocity in classical region and relativistic region.
(a) In Eq.(9) and subsequent equations, Einstein has done mathematical treatment under classical conditions. For example, Einstein has mentioned that KE of body decreases (hence mass decreases as velocity v is constant) and relativistic increase in mass with velocity is not taken in account at all, thus velocity is in classical region. Had velocity been in relativistic region, then increase in mass would have been taken in account as in Eq.(6). Thus velocity is in classical region.

(b) In Eq.(12), the relativistic KE of slowly accelerated electron has been calculated using Eq.(6)
$m_r = m_o /\sqrt{(1–V^2 /c^2)}$ (6)

which clearly implies velocity V is in relativistic region. In case v =0 or v is negligible compared to c, then $m_r = m_o$ If the velocity is in classical region then Eq.(12), is simply $1/2 m_o V^2$, which implies no change in mass. Hence in Eq.(12) velocity V, is in relativistic region (V~c) whereas in Eq.(9) the velocity v is in classical region (v<<c)

(vi) In Eq.(9) energy is emitted to surroundings and in Eq.(12) energy taken from surroundings
(a) In Eq.(9) and subsequent equations it is implied that mass of body DECREASES and energy is emitted to surroundings as light energy.
(b) In Eq.(12) when mass is slowly accelerated to the relativistic region (V becomes comparable to c), then mass of body INCREASES. The increase mass is due to energy taken from the surroundings.
Hence in Eq.(12) energy is ABSORBED by body from the surroundings, and in Eq.(9) energy is EMITTED by body to surroundings.
Thus Eq.(9) and Eq.(12) are based upon the entirely diverse and conflicting concepts, and hence can not

be mathematically compared simply due to reasons that dimensions are the same.

1.21 Justification of transformation from Eq.(5,7-8) to Eq.(9)

Einstein [8] correctly did not draw any direct conclusion from these equations i.e. Eq.(9) and Eq.(12) simultaneously or equated these two equations . Einstein simply on the basis of the fact that right hand sides of Eq.(9) and Eq.(12) depend upon velocity (nature of both equations is entirely different) , tried to justify two aspects. Firstly, the transformation of Eq. (5,7-8) to Eq.(9) is logical, which means justification of introduction of arbitrary additive constant C. Secondly, assumption that arbitrary additive constant C does not change during emission of light.

1.3 Infinitesimally small variations in parameters (φ and L) cause drastic changes in characteristics and concepts; these are not discussed by Einstein.

1.31 One Wave is emitted

If angle formed by single wave is 89 ^{o} or 91 ^{o} not 90 ^{o}.

(i) When $\varphi = 89^o$:

Consider a body is placed in the system (x,y,z) and emit a single wave of light energy L, making angle 89 ^{o} with direction of propagation , and it is observed in system(ξ, η, ζ) which is moving with uniform relative velocity v. Then

$H_o = H_1 + \beta L(1 - v/c \cos 89^o)$

$H_o = H_1 + \beta L (1 - 0.017452406\ v/c)$

Now proceeding as in Eq.(5) to Eq.(10) we get

$\Delta m = -0.03490 L/cv + L/c^2$ (13)

M_a (mass of body after emission) $= 0.03490 L/cv - L/c^2 + M_b$ (mass of body before emission)

which implies mass of body increases when light energy is emitted. In RHS term $(-0.03490\ L\ /cv + L/c^2)$ is always negative as velocity v is in classical region. The % age difference between Eq.(10) and Eq.(13) is $3.49 c/v$ or 1.047×10^8 if the v is 10m/s (in classical region).

(ii) When $\varphi = 91^o$:

Similarly, if the angle made is 91 ^{o} {$\cos 91^o = -0.0174524$} with direction of propagation then

$\Delta m = +0.03490 L/cv + L/c^2$ (14)

$M_a = -0.03490 L/cv - L/c^2 + M_b$

which implies that mass of body decreases when light energy is emitted. But this decrease in mass is much more than Eq.(10). The magnitude of percentage difference between Eq.(10) and Eq.(14) is $3.49 c/v$ or 1.047×10^8 if the v is 10m/s (in classical region).

If the angles are considered in the derivation 90 ^{o}, 89 ^{o} and 91 ^{o}, then results are all together different. It implies that angle at which light energy is emitted, is significant factor in Einstein's derivation. The law of conservation of mass-energy should not depend upon angle like this.

1.32 Two waves are emitted.

(i) When magnitude of energy emitted is slightly different than 0.5L.
In case it is assumed that this body emits plane waves of light of energy 0.4999L along x-axis i.e $\varphi = 0^o$.

The other wave of light energy 0.5001L is emitted in exactly opposite direction i.e. forming angle 180 ^{o}
[Einstein has assumed light energy 0.5L in both the cases] then equation equivalent to Eq.(3) becomes

$H_o = H_1 + 0.501 \beta L (1 - v/c \cos 0^o) + 0.499 \beta L (1 - v/c \cos 180^o)$

After proceeding in similar way as above we get

$K_o - K = -0.002 \ v/c + Lv^2/2c^2$ (15)

Or Δm = Mass of body before emission (M_b)–Mass of body after emission (M_a)

$= -0.004L/cv + L/c^2$ (16)

or $M_a = 0.004L/cv - L/c^2 + M_b$

which implies that when Light Energy, L (0.499L along x-axis, 0.501L in opposite direction) is emitted, then mass of luminous body increases ($v \ll c$); so $0.004L/cv$ is always more than L/c^2 . It is contrary to Einstein's deduction i.e. Eq.(10). The %age difference between Eq.(10) and Eq.(16) is $0.4c/v$ or $1.2 \ x10^7$ if velocity is 10m/s i.e. in classical region.

(ii) When magnitude of angle at which one wave is emitted is 181° rather than 180° (as considered by Einstein) and other at 0° .

$H_o = H_1 + 0.5 \ \beta \ L \ (1 - v/c \ cos0^o) + 0.5 \ \beta \ L \ (1- v/c \ cos181^o)$

$H_o = H_1 + \beta \ L \ (1 - 0.00007615v/c)$

$K_o - K = -0.00007615Lv/c + Lv^2/c^2$ (17)

$\Delta m = -0.0001523 \ L/cv + L/c^2$ (18)

or $M_a = 0.0001523 \ L/cv - L/c^2 + M_b$

which implies mass of body increases when light energy is emitted, as in classical region (10m/s) is more than L /c^2 . If angle $\varphi = 181^o$, thus difference in 1° of angle caused drastic changes in results. Earlier Einstein has considered if angle φ is 180° then mass of body decreases on emission. If the angles are considered in the derivation 90°, 89° and 91°, then results are all together different. It implies that angle is significant factor in Einstein's derivation. The law of conservation of mass-energy should not depend upon angle like this.

2.0 Some feasible cases neglected by Einstein and contradictions of the derivation.

The only condition put by Einstein on luminous body is that the body must **remain at rest** before and after emission of light energy. This condition is satisfied in numerous cases and hence Einstein's derivation is applicable. Einstein has discussed the simplest or typically special case when two light waves (of equal energy, 0.5L) are emitted in opposite directions; which is just one from numerous possible cases. But the following genuine possibilities (which are absolutely essential) have neither been discussed by Einstein and nor by others.

(i) Many waves or single wave.

The sources of light may emit many light waves simultaneously or just a single wave; then angle φ and magnitude of light energy (L) may be different. Einstein has discussed the simplest and special case when two waves are emitted. Realistically it is a particular case and body may emit a single or many waves depending upon its characteristics and **remain at rest**. The bodies can be fabricated to have wide range of parameters to check all aspects of the derivation.

(ii) Angle at which light waves are emitted.

Einstein considered just two angles φ & $\varphi + 180^o$ at which light waves are emitted. The body remains at rest if it emits light energy at numerous angles. This is Einstein's only condition regarding this i.e. body remains at rest before and after emission. In general luminous body can emit light waves at various angles. Even slight variations in angle(s) cause very significant change in concepts as discussed in Eqs (10, 14, 16, 18). But this aspect has not been taken in account by Einstein.

(iii) Magnitude of light waves.

In the derivation Einstein has considered that total light energy emitted by luminous body is L, and

emitted in two waves each having energy 0.5L, moving in opposite directions. It can be 0.499L and 0.501L or different. Further luminous body can emit large number of waves, then light energy L will be distributed among the waves equally or unequally and body **remains at rest.** If energy emitted is different from 0.5L then results are different from $\Delta m = L/c^2$ as in Eqs.(21,24,26)

(vi) Velocity v may be non-uniform.
Einstein considered systems which are in uniform relative motion, v w.r.t. each other. The velocity of system may be uniform or non-uniform may be in classical or relativistic region or at rest the law of conservation of mass and energy holds good under all circumstances .Thus the derivation should have been applicable for all values of velocities.

(v) Application of Binomial Theorem. Further this derivation is stage sensitive as $\Delta m = L/c^2$ is only obtained if Binomial Theorem is applied at particular stage i.e. Eq.(9). The same is also applicable to Eq. (1) at much earlier stage, if applied here then we get contradictory results i.e. $\Delta m (M_b - M_a) = 0$ instead of $\Delta m = L/c^2$, as shown in Eq.(29) Thus in this derivation it is application of Binomial Theorem which makes or mars the law of conservation of energy, hence the derivation is inconsistent. However in derivation of relativistic form of kinetic energy [$KE_{rel} = (m_r - m_o)c^2$], Binomial Theorem may be applied at any stage, but the result is same i.e classical form of kinetic energy ($m_o v^2/2$).

(vi) Nature and limitations of Eq.(1) The central equation in derivation is Eq.(1) which gives variation of light energy with velocity, this equation has been quoted (but without mathematical details) by Einstein in paper in which he gave Special Theory of Relativity [2] in Section 8. The nature of this equation is different from equation which existed before Einstein's work e.g. relativistic variation of mass $[m_r = m_0/(1-v^2/c^2)^{1/2}]$, length contraction [$L = L_o (1-v^2/c^2)^{1/2}$] and time dilation [$T = T_o (1-v^2/c^2)^{1/2})$]. This equation has also serious mathematical limitations. All the above aspects are described below along with impacts and consequences.

2.1 Violation of law of conservation of mass and energy i.e. mass and energy are created out of nothing simultaneously.

This case can be discussed considering the cases when luminous body emits one or more (even or odd) waves.

Four waves are emitted.
Consider the case when four waves at angles φ & φ+180 ^{o}, Θ & Θ +180 ^{o}; having magnitude of light energy 0.255 L and 0.245L. The angles at which four waves are emitted are 0 ^{o} & 180 ^{o} and 75 ^{o} & 255 ^{o} (75+180 ^{o}), in view of this Eq.(3) can be written as

$H_0 = H_1 + 0.255Lβ[1–v/c \cos0^o] + 0.245Lβ[1—v/c \cos180^o] + 0.255Lβ[1—v/c \cos75^o] +0.245Lβ[1–v/c \cos255^o]$ (19)
or $H_o = H_1 + Lβ [1— 0.012588 v/c]$ (19)
Using Eq.(2) and Eq.(19), we get as in previous cases,

$K_o – K = L [− 0.012588 v/c + v^2/2c^2]$ (20)

$\Delta m (M_b - M_a) = − 0.025176 L/cv + L/c^2$ (21)

Or M_a (mass after emission) $= 0.025176 L/cv − L/c^2 + M_b$ (mass before emission) (22)

The above equation is derived with application of Binomial Theorem (v<<c); under this condition Δm from Eq. (21) is always negative. The %age difference between Eq.(10) and Eq.(21) is 3.7764×10^7 (if v

is 10m/s). Hence from Eq.(22) following two conclusions are absolutely clear,
(a) Body is emitting light energy, L (say, very stocky amount of energy) continuously.
(b) Simultaneously as light energy L is emitted, the mass of body (matter) also increasing.

Conclusions

From this interpretation following two conclusions are clear
(i) The increase in mass of body when body emits light energy L.
Einstein's 1905 derivation also predicts that luminous body emits energy and simultaneously mass of body increases. Both the creation of energy and mass is at cost of nothing or cipher. It is clear contradiction of law of conservation of mass and energy.
(ii) Contradiction with relativistic variation of mass.
According to relativistic variation of mass i.e. Eq.(6) i.e $m_r = \beta\, m_o$, mass increases when velocity of body is comparable with speed of light, this aspect was also used by Einstein from existing literature in the paper he introduced the Theory of Relativity [2]. This result is being contradicted by Einstein himself. This derivation implies that mass of body can also increase if it moves with classical velocity (v<<c) and surprisingly emits light energy (which Einstein established is form of mass) . Thus this derivation also contradicts the relativistic variation of mass with velocity.

Five waves are emitted.

If five waves are emitted such that each wave makes angles 0° , 90° , 145°, 300° and 340° with the x-axis and each wave carries energy equal to 0.2L. Then equation equivalent to Eq.(3) be written as

$H_0 = H_1 + 0.2L\beta[1-v/c\ \cos0^o] + 0.2L\beta[1—v/c\ \cos90^o] + 0.2L\beta[1—v/c\ \cos145^o] + 0.2L\beta[1-v/c\ \cos300^o]$

$+ 0.2L\beta[1—v/c\ \cos340^o]$ (23)
$H_o = H_1 + 0.2L\beta\,[\,5 - 0.620543\ v/c\,]$ (23)
Using Eq.(2) and Eq.(23), we get as in previous cases.
or $K_o - K = L\,[\beta\,(1- 0.1241081\ v/c) -1]$

$\Delta m = -\,0.2482\ L/cv + L/c^2$ (24)
$M_a = 0.2482\ L/cv - L/c^2 + M_b$
mass of body increases which is contradictory, as discussed earlier in case of Eq.(13) and others.

Six waves are emitted

Consider the case when six waves are emitted by the luminous body making angles 0°, 180°, 75°, 255°, 60° and 69° ; the magnitudes of light energy emitted along these directions are 0.25L, 0.24L, 0,25L, 0.24L, 0.01L and 0.01L respectively. In view of this Eq.(3) can be written as

$H_o = H_1 + 0.25\ L\beta\,(\,1-v/c\ \cos0^o) + 0.24\ L\beta\,(\,1- v/c\ \cos180^o) + 0.25\ L\beta\,(\,1-v/c\ \cos75^o) + 0.24\ L\beta\,(\,1-$

$v/c\ \cos225^o\,) + 0.01\ L\beta\,(\,1-v/c\ \cos60^o) + 0.01\ L\beta\,(\,1-v/c\ \cos69^o)$ (25)
Substituting various values and rearranging terms,
$H_o = H_1 + L\beta\,(\,1- 0.02117v/c)$
In terms of change in mass as calculated in Eq.(24) and other equations
$\Delta m = -\,0.04234\ L/cv + L/c^2$
$M_a = 0.04234\ L/cv - L/c^2 + M_b$ (26)
Hence the similar contradictory result, as discussed in case of Eq.(24) or Eq.(13) and others
Like wise results can be discussed if luminous body emits many waves.

2.2 Violation of law of conservation of mass-energy i.e. energy is created out of nothing, in Einstein's derivation

(i) As already mentioned v is the velocity with which co-ordinate system (ξ, η, ζ) moves w.r.t system (x, y, z). If velocity v is regarded as zero (v = 0) i.e. system (ξ, η, ζ) is at rest and body emits light energy, L as before. Under this condition Eq.(1) becomes

$\ell^* = \ell$ (27)

Re-writing Eq.(3) in view of above we get

$H_o = H_1 + 0.5L + 0.5L = H_1 + L$ (28)

From Eq. (28) and Eq. (2),

$(H_o - E_o) - (H_1 - E_1) = 0$

Now proceeding in the similar way as earlier, we get

or $K_o - K = 0$

$\Delta m (M_b - M_a) = 0$
Or Mass of body before emission (M_b) = Mass of body after emission (M_a) (29)

Thus when system (ξ, η, ζ) is at rest and luminous body emits energy (say, stocky amount), then its mass remains constant i.e. energy is being emitted out of nothing, the law of conservation of mass and energy is clearly contradicted in Einstein's derivation. In all nuclear reactions, chemical reactions etc., mass is converted into energy, energy is obtained at the cost of mass. But here derivation predicts energy L (say stocky amount of energy) is created without loss of mass at all as in Eq.(29) mass remains the same, which is inconsistent results in all respects.
Just for glacial scrutiny the same condition i.e. v = 0 is applied to Eq.(9) then
0=0

In this case there is no non-zero term for interpretation, that how light energy L is emitted. Thus body is emitting light energy light L which is created out of nothing or cipher, as mass of body remains the same.

2.3 Contradiction of $\Delta m = L/c^2$ itself i.e. energy emitted is more than $L = \Delta m c^2$

(i) Similarly consider that source emits light energy L in single wave such that it makes angle 180° with x-axis,

$H_o = H_1 + \beta L (1+v/c)$ (30)

Now proceeding in the similar way as earlier, we get

$K_o - K = \beta L [1+v/c] - L$

or $\Delta m = 2L /cv + L/c^2$ (31)

If the value of v is 10m/s, then Eq. (31) becomes

$\Delta m (M_b - M_a) = 2L / 3 \times 10^9 + L / 9 \times 10^{16}$ (32)

$M_a = -2L / 3 \times 10^9 - L / 9 \times 10^{16} + M_b$

Also in this case Eq. (10) becomes,

$\Delta m (M_b - M_a) = L / 9 \times 10^{16}$

or $M_a = - L / 9 \times 10^{16} + M_b$ (33)

The magnitude of Eq.(32) is much higher than that of Eq.(10). Now the percentage difference between Eq.(32), and Einstein's original equation i.e. Eq.(10) is 200c/v or 6×10^9 (if v = 10m/s). Thus results given by this derivation are self contradictory. Many such examples can be quoted in this regard.

3.0 If Binomial Theorem is applied to Eq. (9) the law of conservation of matter and energy is originated and if the Binomial Theorem is applied to Eq. (1) then the law is contradicted.

(i) If Binomial Theorem is applied at Eq. (9) as in case of Einstein's derivation, then as already mentioned the result is

$\Delta m = L/c^2$ (10)

(ii) When Binomial Theorem is applied at Eq. (1) i.e. v<<c

$. \ell^* = \beta \ell (1 - v/c \cos\varphi) = \ell$ (27)

Thus, the subsequent equations can be written as

$E_o = E_1 + L$ (2)

$H_o = H_1 + L$ (4)

$(H_o - E_o) - (H_1 - E_1) = 0$

$K_o - K = 0$ or $\Delta m = 0$

Mass of body before emission (M_b) = Mass of body after emission (M_a) (29)

The Eq. (29) implies that body emits light energy, then its mass remains the same, it is contrary to Eq. (10) given by Einstein which implies that when light energy is emitted then mass of body decreases. Thus this derivation has a limitation as the stage at which Binomial Theorem is applied makes or mars law of conservation of mass and energy. However, the law of conservation of mass-energy holds good under all conditions and must not depend upon the stage at which Binomial Theorem (a mathematical tool) is applied. In addition the results from this derivation are affected by other factors as well.

3.1 The velocity constraints on Einstein's Light Energy Mass Equivalence ($\Delta m = \Delta L/c^2$)

From above discussion it is evident that Einstein's this derivation has over dependence on velocity. It only holds good if velocity is in classical region, gives contradictory results if body is at rest (v =0) and

is not applicable if v is in relativistic region or $v \rightarrow c$.

(i) When velocity is in classical region ($v \ll c$) then Binomial Theorem is applicable to Eq.(9)

$$K_o - K = L \{1/\sqrt{(1 - v^2/c^2)} - 1\} \text{ (9)}$$

Or $\Delta m = L/c^2$ (10)

or M_a (mass of body after emission) = M_b (mass of body before emission) $- L/c^2$

Thus mass of body decreases when light energy is emitted.

(ii) When body is at rest, $v = 0$

or $K_o - K = 0$

$\Delta m (M_b - M_a) = 0$

Or Mass of body before emission (M_b) = Mass of body after emission (M_a) (29)

Thus when system (ξ, η, ζ) is at rest and luminous body emits energy (say, stocky amount), then its mass remains constant i.e. energy is being emitted out of nothing, the law of conservation of mass and energy is clearly contradicted in Einstein's derivation of $\Delta m = \Delta L/c^2$.

(iii) When velocity is in relativistic region or $v=c$

Under this condition there is relativistic variation of mass of body and Eq.(6) is applicable. This factor is not at all taken in account, in the mathematical and conceptual treatment. Hence Einstein's Light Energy Mass equation is not applicable in this case.

Thus there are constraints due to velocity, angle and magnitude of light energy on the derivation. In fact law of conservation of mass and energy is a basic law, and should not have constraints like this.

3.2 Comparison of Einstein's Light Energy mass equation and Relativistic form of KE

Using relativistic variation of mass

$$m_r = \beta m_o = m_o / (1 - v^2/c^2)^{1/2} \text{ (6)}$$

Einstein obtained relativistic form of KE

$dK = Fdx$ or $(KE)_{rel} = (m_r - m_o)c^2$ (12)

(i) If Binomial Theorem is applied to Eq.(12) then

$(KE)_{rel}$ when $v \ll c$ = $(KE)_{classical}$ = $m_o v^2/2$ (34)

which is usual form of KE as obtained by other methods. In equation $KE = m_o v^2/2$, m_o is rest mass. It cannot be interpreted as that m_o decreases then KE increases.

(ii) If Binomial Theorem is applied at Eq.(6) then

$m = m_o$.

or $dK = Fdx = dp/dt\, dx$ or $K = m_o v^2/2$ (34)

Thus we apply the Binomial Theorem at Eq.(6) or Eq.(12), we get the same equation for kinetic energy, $m_o v^2/2$ i.e. Eq.(34).

Analogously if Binomial Theorem is applied at Eq.(1) and Eq.(9), then different values of change in mass i.e. $\Delta m = 0$ or $\Delta m = L/c^2$ are obtained (in one case mass remains the same and in the second mass decreases). In fact result should have been the same as in Eq.(34); irrespective of the fact whether Binomial Theorem is applied at Eq.(1) or Eq.(9). It clearly highlights the limitations of Einstein's derivation, which is unnoticed by scientific community.

4.0 Eq. (10) is based upon inconsistent equation

(i) Inconsistency of dimensional homogeneity when v→c, by Eq.(1)
The central equation in this derivation is Eq (1) which has been quoted (but without mathematical details) by Einstein himself in Section (8) in the same paper he introduced the Special Theory of Relativity [2]. If $\varphi = 0^o$, then Eq. (1) becomes,

$$\ell* = \ell \, (1 - v/c) \, / \, \sqrt{(1 - v^2/c^2)} \quad (35)$$

This equation is applicable when velocity v is uniform, if the velocity is non-uniform in the interval; its application requires estimations of sub-intervals when velocity is uniform for its applicability. If the system [ξ, η, ζ] moves with velocity equal to that of light i.e. v = c which realistically means that velocity v tends to c i.e. v → c. (some Quasars or other heavenly bodies may attain such high velocities) .Thus,

$$\ell^* = 0/0$$

which is undefined or $\ell*$ tends to 0/0 which has the same meaning.
The dimensions of LHS are M L^2 T^{-2} [energy] and that of RHS undefined. It is the inherent requirement that an equation must obey the principle of dimensional homogeneity [14-15] but it is not so in case of Eq.(1) under this particular condition. Hence under this condition the central equation which leads to $\Delta m = L/c^2$ is not applicable. Such central equation should be free from the limitations.
(ii) Non-compliance of identity $a^2 - b^2 = (a+b)(a-b)$ by Eq.(1).
Further contradictory results are also self-evident if Eq. (35) is solved and same condition (v = c or v → c) is applied i.e. $\{1 - v^2/c^2\} = (1-v/c)(1+v/c)$ is simple algebraic result.

$$\ell* = \ell \, \sqrt{(1 - v/c)} \, / \sqrt{(1 + v/c)} \quad (35a)$$

Now again if the velocity v tends to c i.e. v → c above equation becomes,

$$\ell^* = 0 \quad (36)$$

Under this condition the Eq.(35), in unsolved form is $\ell^* = 0/0$

Thus the same equation (in solved and unsolved forms) under similar conditions (v → c) gives different results ($\ell^* = 0/0$ and $\ell^* = 0$), which is purely arbitrary and illogical. Thus results from Eq.(1) are contradictory to basic identity of algebra; and in addition the result is not consistent with dimensional homogeneity.
The basic principle of conservation of mass and energy should not be based upon an equation which is full of limitations e.g it disobeys dimensional homogeneity and basic algebraic identities. Thus Eq.(1) is relativistic in nature but its numerator varies with v , even under classical conditions. The variation in magnitude of numerator may be a factor for inconsistent results.
If angle $\varphi = 180^o$, then $\cos 180^o = -1$, thus under condition when velocity tends to c (v → c) or becomes equal to c then,
$$\ell^* = \infty$$

Thus entirely different results are obtained, i.e. $\ell^* = \infty$ simply if angle of wave is exactly reversed ($\varphi = 180^o$) compared to above case (when . $\varphi = 0$, then $\ell^* = 0/0$ or $\ell = 0$)
Similarly Einstein [16] developed theory of Static Universe in 1917, which has limitation Einstein divided with a term which becomes zero under certain cases. This limitation was pointed out by that

cosmologist. Friedmann and later on Einstein accepted the same as the biggest blunder of life, quoted by George Gamow [17]. The similar prospective situation is here also.

Thus in totality, critically analysing all aspects, this derivation is obviously inconsistent and theoretically unchecked as well. As Einstein [8] did not address this aspect at all along with many others which he should have, hence Einstein's unfinished task is completed here.

5. 0 Einstein's arbitrary generalisation of $\Delta m = L/c^2$ for all energies i.e. $\Delta m = \Delta E/c^2$

Further Einstein generalized the equation from Eq. (10) in the following way just in two sentences [8], *"Here it is obviously inessential that energy taken from the body turns into radiant energy, so we are led to more general conclusion: The mass of a body is a measure of its energy content: if the energy changes by L, the mass changes in the same sense by $L/9x10^{20}$ if the energy is measured in ergs and mass in grams."*

The meaning of first sentence dubious in regard to the discussion. In second sentence Einstein [8] speculated general equation from Eq.(10) just in analogous way as (energy emitted = mass annihilated c^2) i.e.

$$\Delta m = \Delta E/c^2 \text{ or } (M_b - M_a) = (E_{final} - E_{initial})/c^2 \text{ (37)}$$

where ΔE is energy produced on annihilation of mass.
Now it is regarded as true for all types of energies *(mass of a body is measure of its energy content).* Does it mean that when sound energy is produced from source, then mass of body decreases ? When mass is annihilated then huge amount of sound or noise may be produced which can be termed as sound bomb or noise bomb analogous to nuclear bomb. The most likelihood of the phenomena may be speculated at instant of Big Bang or in supernova explosions etc. At small scale sound energy is produced when bomb or cracker explodes. In the discussion no equation for sound energy or other forms of energy (in frames having relative motion or at rest) equivalent to Eq. (1) and Eq.(10) has been quoted, the above speculation needs its insightful theoretical justification .
In case conversion of mass to sound energy is feasible then speed of sound may be more than 332m/s under some characteristic cases. The same arguments can be checked for speed of light if mass annihilated spontaneously in bulk in some emblematic reaction and mainly light energy is produced. Such reactions may be more feasible in heavenly bodies. In the existing literature there are prospective proposals [18-19] that speed of light can be less or more than c. The same is true for all other forms of energy, as law of conservation of mass-energy holds in all circumstances. In its generalisation from light energy-mass equations all aspects to be accounted for here. Does all form of energies obey Eq.(1) or have similar dependence as in Eq.(1)?. But as already discussed in section (4.0) has inherent limitations. About its experimental confirmation Einstein [8] quoted that ……..*"It is not excluded that it will prove possible to test this theory using bodies whose energy content is variable to a high degree (e.g., radium salts) "* Thus at that time Einstein has in his mind the loss of weight resulting from radioactive transformations[20]. At that time heat produced on loss of mass in body in burning (chemical change) was also the most obvious example. Planck [21-22] was the first to remark that mass-energy equation bears on binding energy for a mole of water, in 1907. In 1910, Einstein himself remarked [23] *"for the moment there is no hope whatsoever for the experimental verification of mass–energy equivalence".* But similar estimates for nuclear binding energy (conversion of mass to energy) were made nearly after a quarter of century when atomic and nuclear models were developed and nuclear characteristics were understood.

For this period Einstein's mathematical derivation was not critically analysed, and after its experimental

confirmation in nuclear reaction, such necessity for theoretical analysis was not felt, as is done now. In nuclear phenomena this equation $\Delta E = \Delta mc^2$ is regarded as standard or reference. For first time, in view of true scientific spirits Einstein's derivation is critically analysed.

5.1 Einstein derived five equations relating to various types of masses { rest mass(m_o) , relativistic mass (m_r) , mass annihilated (Δm) and mass created (Δm) } and energies {rest mass energy (E_o) relativistic energy (E_{rel}) energy annihilated or energy created, (ΔE)}

The inherent characteristics of the equations derived by Einstein are discussed below , only closely related equation existed at time of Einstein was that for kinetic energy i.e. $K = m_o v^2/2$.

(i) $KE_{rel} = (m_r - m_o)c^2$ (12)

This equation is equivalent to $K = m_o v^2/2$, in relativistic mechanics.

(ii) $E_o = m_o c^2$ (Energy when body is at rest, v =0, dx = 0 or dK = Fdx =0) (38)

E_o is derived from relativistic form of kinetic energy, which is further derived from dK = Fdx

(iii) $E_{rel} = KE_{rel} + m_o c^2 = m_r c^2$ (energy when velocity of body is v~c) (39)

E_{rel} is derived from Work Energy Theorem i.e.

Work done = change in kinetic energy = KE_{final} -$KE_{initial}$

and relativistic variation of mass as given by Eq.(6) i.e. $m_r = \beta m_o$

(vi) $\Delta L = \Delta mc^2$ (conversion of light energy to mass and vice-versa) (10)

ΔL is derived from energy considerations of emission of light energy in two systems having relative motion under classical conditions using Binomial Theorem.

(v) $\Delta E = \Delta mc^2$ (conversion of any form of energy to mass and vice-versa) (37)

$\Delta E = \Delta mc^2$ has been speculated by Einstein from $\Delta L = \Delta mc^2$

Now the five equations were derived by Einstein are $L = \Delta m c^2$, $\Delta E = \Delta m c^2$, $KE_{rel} = (m_r - m_o)c^2$ $E_{rel} = m_r c^2$ and $E_o = m_o c^2$ appear to be similar as have same units and dimensions but conceptually are entirely different like four directions . Even the cradle of $E_o = m_o c^2$ is questionable, in view of scientific logic.

The similar equations of energy which are useful in understanding conceptual differences of Eqs.(10,12, 37-39) are kinetic energy (KE) and potential energy (PE),

$KE = m_o v^2/2$ (34)

$PE = m_o gh$ (40)

where g is acceleration due to gravity and h is height at which body is placed.

5.2 Origin and interpretation of Rest Mass Energy

It can be carefully noted that equations $E_o = m_o c^2$, $E_{rel} = m_r c^2$, $KE_{rel} = (m_r - m_o)c^2$, $K E = m_o v^2/2$, PE $= m_o gh$ are originated from the same equation i.e.

dW= dK = FdxcosΘ (41)

If the force (F) causes displacement (dx) in its own direction then, $\Theta = 0°$ or $cos 0° = 1$, thus

dK= dW = Fdx (42)

Einstein gave a direct result [2] relativistic form of kinetic energy for slowly accelerated electron in

Section (10) and then generalized it for all ponderable masses. In the same paper he introduced Special Theory of Relativity. If velocity is in relativistic region(v~c) then Eq.(42) becomes

$dK = dW = Fdx = \frac{d}{dt}\{m_r v\}\, dx = m_r v dv + v^2\, dm$

Differentiating Eq.(6) i.e $m_r = \beta m_o$ we get

$m_r v dv + v^2\, dm = c^2\, dm$

Or $dK = dW = c^2\, dm$ $KE_{rel} = W_{rel} = c^2 (m_r - m_o)$ (12)

{ Kinetic Energy attained by body due to influence of external force when body moves in direction of force with velocity v, which is comparable to c, speed of light }

$= c^2 (m_r - m_o) =$ (Increase in mass of body due to relativistic velocity)c^2 (12)

or

{Work done by body due to influence of external force when body moves in direction of

force with velocity, v which is comparable to c, speed of light}

$= c^2 (m_r - m_o) =$ (Increase in mass of body due to relativistic velocity) c^2

From Eq.(12) Newtonian form of kinetic energy can be obtained.

If m_r and m_o are the same then relativistic form of kinetic energy or work done is zero.

If velocity v is in classical region (v <<c) then Eq.(12) reduces to classical form of KE i.e

$K = (m_o /\sqrt{} (1- v^2/c^2\ 1) = m_o (1+ v^2/2c^2 + 3v^4/8c^4 + \ldots\ldots.. - 1) = m_o v^2/2$ (34)

If this condition is applied to original Eq. (42) then Newtonian form of kinetic energy is obtained

$K = Fdx = = \frac{d}{dt}\{m_o v\}\, dx = m_o v^2/2$ (34)

5.3 Deduction of Rest Mass Energy by Einstein from Eq.(12)

In 1907 Einstein [3] deduced Rest Mass Energy after terming $m_r c^2$ as total energy or relativistic energy (E_{rel}) as m_r is relativistic mass.

$KE_{rel} = (m_r - m_o)c^2$ (12)

$m_r c^2 = KE_{rel} + m_o c^2$ (12a)

Einstein re-christened Eq.(12a) as relativistic energy E_{rel} or total energy

$E_{rel} = m_r c^2 = KE_{rel} + m_o c^2$ (39)

When body is at rest v =0, dx =0 or dK = Fdx =0) then Einstein wrote

E_{rel} (when v =0) $= m_o c^2 = 0 + m_o c^2$

Then Einstein expressed E_{rel} (when v =0) as Rest Mass Energy (E_o). Thus,

E_{rel} (when v =0 dx =0 or dK = Fdx =0) or Rest Mass Energy, $E_o = m_o c^2 = 0 + m_o c^2$ (38)

This inference needs to be critically discussed when body is at rest (v=0 , dx =0) as very first equation

(dK = Fdx =0) is zero and intermediate equations do not exist. Hence Einstein's the rest mass energy is a mathematically and conceptually non-existent physical quantity, as obtained from invalid mathematical operation.

5.4 Invalidity or inconsistencies in Einstein's deduction i.e. in Eq.(39)

If an equation is re-arranged or re-christened (simply transposing without multiplication, division, addition and subtraction of any term), then results from original equation and new transformed equation must be the same. Also final equation cannot be interpreted under the condition when very first equation is zero and preceding equations don't exist. But his simple logic is not satisfied or obeyed by Einstein's interpretation of rest mass energy. We have,

$KE_{rel} = (m_r - m_o)c^2$ (12)

$KE_{rel} + m_o c^2 = m_r c^2$ (12a)

Total energy or Relativistic Energy $= m_r c^2 = KE_{rel} + m_o c^2$ (39)

Now the Eq.(12), Eq.(12a) and Eq.(39) are exactly the same. Thus these equations under similar conditions e.g. when body is at rest (v =0, dx=0, dK = Fdx = 0) must lead to similar results. Hence substituting v=0, in all three equations.
(a) Firstly, in Eq.(12),

$0 = (m_o - m_o)c^2 = 0$

Thus under this condition v =0, Eq.(12) does not give any physical information. The reason is that under this condition first equation i.e. dK = Fdx =0, hence subsequent equations don't exist and final equation can not be interpreted.
(b) Secondly, in Eq.(12a)

$0 + m_o c^2 = m_o c^2$

or 1 =1 or 1-1 =0

Thus again in this case Eq.(12a) does not give any physical information, hence cannot be interpreted regarding E_o and m_o . The reason is exactly as above.
(c) Thirdly, in Eq.(39),

Relativistic Energy (v =0) $= m_o c^2 = 0 + m_o c^2$ (43)

The equation can be interpreted as when body is at rest then,
Relativistic energy (dependent on velocity) is zero as in this case kinetic energy is zero. Thus,

$0 = m_o c^2 = 0 + m_o c^2$

$0 = m_o c^2 = m_o c^2$

m_o and c^2 both are non-zero and their product cannot be zero. Hence, as in previous cases under this condition (v=0, Fdx = 0) the equation cannot be interpreted. It is further supported by Eq.(42) as dK = Fdx or dK = F.0 = 0

Thus if very first equation and subsequent equations do not exist thus further interpretation is not only invalid but impossible.

5.5 Einstein's arbitrary explanation

Without taking the fact in account that when body is at rest then dK = Fdx =0, Einstein tried to interpret the equation in such a way that result may have resemblance with Eq.(10) or Eq.(37) i.e.

$\Delta L = \Delta mc^2$ or $\Delta E = \Delta mc^2$

Relativistic Energy { v =0, dx= 0 or dK = Fdx =0 } = Rest Mass Energy, E_o

Thus Eq.(39) becomes

Relativistic Energy { v =0, Fdx =0 } $E_o = 0 + m_o c^2 = m_o c^2$

or $E_o = m_o c^2 = m_o c^2$

Einstein interpreted the equation as relation between rest mass, m_o and rest mass energy E_o. In the process following facts were ignored

(i) When body is at rest then very first equation vanishes i.e.

dK =dW = Fdx = 0

and all other equations are non-existent. Thus to draw any conclusion from the imaginary equations is purely HYPOTHETICAL and has no logical and mathematical basis.

(ii) The Eq.(12) and Eq.(12a) are originating forms of Eq.(39). Under similar conditions the results from Eq.(12) and Eq.(12a) are the same but from Eq.(39) , which is just other form of Eq.(12) and Eq.(12a) are different . Hence Eq.(39) may be regarded as illogically re-christened.

5.6 Δm in equations $\Delta m = \Delta L /c^2$ or $\Delta m = \Delta E /c^2$ is different from m_o in $E_o = m_o c^2$, $KE_{rel} = (m_r - m_o)c^2$, $KE = m_o v^2/2$ or $U = m_o gh$ or $p = m_o v$

This conclusion is justified by following illustrations

(i) Origin and meaning of Δm:

Out of above equations only Eqs(10,37) represent mass annihilated to energy or energy materialized to mass. Here Δm is mass actually annihilated to energy or mass created on annihilation of energy.

For hydrogen isotope deuterium [7] ,

The expected mass is 2.0165 amu i.e.

Mass of $_1H^1$ atom + Mass of neutron = 1.0078 amu + 1.0087 amu = 2.0165 amu (44)

Experimentally measured mass of deuterium = 2.0141amu (45)

Mass defect = Expected mass – measured mass = 0.0024 amu (46)

Energy equivalent to mass defect = (0.0024amu) x 931MeV /amu = 2.2 MeV (47)

Here Δm is mass defect 0.0024amu, i.e. mass which is actually converted to energy. Hence Δm is not equal to actual mass of deuterium i.e. 2.0165 amu. Thus here Δm (0.0024amu) is given in Eq.(10) or Eq. (37) i.e. $\Delta m = \Delta L /c^2$ or $\Delta m = \Delta E/c^2$ rather than m_o in equation $m_o = E_o /c^2$

Einstein has considered in his derivation that mass of body decreases when light energy is emitted and given by

$M_b v^2 /2 – M_a v^2 /2 = Lv^2 /2c^2$ (11)

Or Mass of body before emission (M_b) – Mass of body after emission (M_a) = L/c^2

Analogously above equation for deuterium binding energy can be written as

Mass of of free nucleons – Mass of bound nucleons = Binding Energy (B.E.)/c^2

$\Delta m (M_b - M_a) = \Delta L/c^2 = (BE)/c^2$ (10)

Then from Eq.(10) general form mass energy equation i.e $\Delta E = \Delta mc^2$, is speculated

(ii) The meaning of m_o is same as in all equations

The rest mass m_o is related with relativistic mass m_r as

$m_r = m_o / \sqrt{(1 - v^2/c^2)}$ (6)

This equation existed before Einstein's Special Theory of Relativity and was used by him in the same [2]. Kauffman [9] verified it in 1901 and more comprehensively by Bucherer [10] in 1908. According to this if body is at rest or moving with slow velocity, then

Relativistic mass = Rest mass i.e $m_r = m_o$

The term rest mass occurs in equations, $E_o = m_o c^2$, $KE_{rel} = (m_r - m_o)c^2$, $m_o v^2/2$ and $m_o gh$. Now equation $E_o = m_o c^2$ cannot be interpreted as energy E_o produced on annihilation of rest mass m_o, or when energy E_o is materialised then rest mass m_o is produced, as these are not defined for the purpose. If so then it is equally feasible to interpret that when m_o is annihilated then KE and PE both must increase and must be given by equations $m_o v^2/2$ and $m_o gh$. In these terms c is not involved. The same argument is valid for $KE_{rel} = (m_r - m_o)c^2$. Can similar explanation be speculated for momentum $p = m_o v$ (defined under classical conditions)? Each equation is derived under characteristic conditions and is applicable under those conditions only. Hence annihilation of mass to energy or materialization of energy to mass is only explained on the basis of Eq.(10) only or its speculative generalisation i.e. $\Delta E = Ac^2 \Delta M$.

6.0 A New or Generalised Form of Mass Energy Equivalence as $\Delta E = Ac^2 \Delta M$

In view of serious mathematical limitations of Einstein's derivation, the Mass-Energy equation is derived in independent ways. Here we start with a new method which is non-relativistic in nature {as Eq.(6) is not involved}, as mass can be annihilated to energy when velocities of reactants are in classical region. This aspect is justified as in fission of $_{92}U^{235}$, secondary neutrons produced originally have energy 1-2 MeV (velocity in relativistic region), it is reduced to 0.025 MeV (2185 m/s) with help of moderator (such as heavy water); otherwise nuclear fission does not take place. This velocity is classical in region and even orbital velocity of the earth 3×10^4 m/s is in classical region Some anomalies are observed in nuclear fission fragments of U^{235} or Pu^{239}, as total kinetic energy is observed experimentally [24-26] is 20-60MeV less, which is explained on the basis of wave mechanics equation, $H = mv^2$, obviously given by de Broglie from his wave mechanics. Also alternatively liquid-drop model of Bohr and Wheeler is also extended [27-28] to explain the deficiency.

Interestingly derivation of Einstein's Eq. (10) started from relativistic equation i.e Eq.(1), but finally derived results classically by applying Binomial Theorem as cited above. In the whole derivation relativistic variation of mass i.e. $m_r = \beta m_o$ as given by Eq.(6) is not taken in account. Further scientists are continuously suggesting the variations (increase or decrease) in magnitudes of c [18-19] if materialised then this derivation of $\Delta E = \Delta mc^2$ will become invalid or energy emitted will be less or more than predicted by Einstein's equation. Then existing phenomena cannot be explained with help of Einstein's $\Delta E = \Delta mc^2$.

In 18th century Lavoisier stated that the law of conservation of matter in chemical reactions. The first idea of mass energy Interco version was given by Fritz Hasenohrl. [1] in 1904. However there are a few scientifically unconfirmed counterclaims about it ($\Delta E = \Delta mc^2$). In 1905 Einstein mathematically derived interconvert ability equation between mass and energy as $\Delta L = \Delta mc^2$, according to this conversion factor between mass and energy is precisely and rigidly is c^2. Whereas according to $\Delta E = Ac^2 \Delta M$, the conversion factor between mass and energy may or may not be c^2.

Here the derivation involves calculation of infinitesimally small amount of energy dE when small amount of mass dm is converted (in any process) into energy (energy may be in any form i.e. light energy, sound energy, energy in form of invisible radiations etc); then

$dE \propto dm$

In the existing literature conversion factor c^2 between mass and energy has been experimentally confirmed. Thus in above proportionality, it can be taken in account as,

$dE \propto c^2 dm$ or $dE = Ac^2 dm$ (48)

where A is used to remove the sign of proportionality it has nature like Hubble's constant {50 to 80 kilometres per second-Mega parsec (Mpc)} or like coefficient of viscosity (1.05×10^{-3} poise to 19.2×10^{-6} poise) or coefficient of thermal conductivity ($0.02 Wm^{-1}K^{-1}$ to $400\ Wm^{-1}K^{-1}$ etc) or decay constant λ in radioactivity, also force constant or spring constant, k is similar other example. It may be termed as conversion–coefficient, as it highlights extent of conversion of mass to energy or vice –versa and depends upon the characteristics and intrinsic conditions of a particular process.

Further, conversion–coefficient A is consistent with Newton's Second law of motion or Axiom II as quoted in Book I of *the Principia* in Latin (first translation in English by Andrew Motte, 1729 subsequent disseminations has directly used acceleration).

"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed".

Mathematically,

$F \propto a$ or $F = m_o a$ (49)

where m_o (mass) is constant of proportionality which varies from one situation to other, similar is nature of A (conversion co-efficient) or Hubble's constant etc. Likewise in law of gravitation G is constant.
Let in some conversion process mass decreases from M_i to M_f and energy increases from E_i to E_f .Initially when no mass is converted into energy, $E_i = 0$. Thus integrating Eq. (48) we get,

$E_f - E_i = Ac^2 (M_f - M_i)$ (50)

$\Delta E = Ac^2 \Delta M$ (51)

Energy evolved $= Ac^2$ (decrease in mass)

(i) If the initial and final masses remain the same, then $M_i = M_f$, then ΔE, the Energy evolved is zero. (ii) If the characteristic conditions of the process permit and whole mass is converted into energy i.e. $M_f = 0$, then Eq.(50) becomes

$\Delta E = - Ac^2 M_i$ (52)

Here energy evolved is negative implies that energy is created at the cost of mass; and reaction is exothermic in nature, also energy is scalar quantity, hence only magnitude is associated with it and not direction. In case of annihilation of electron-positron pair all the mass is converted into energy i.e. $M_f = 0$,

so the energy emitted is consistent with Einstein's Mass- Energy Equivalence i.e. $\Delta E = Ac^2 \Delta M$. Thus in this case value of A is unity, hence in this case Eq.(50) becomes,

$$\Delta E = - c^{2\Delta} M = - c^2 M_i \ (53)$$

(iii) If ΔE is positive, means the additional mass has been supplied to the system of M_f is more than M_i but in Eq.(50) annihilation of mass into energy is being discussed, so this case is irrelevant.

Further Eq.(50) implies that if initial and final energies are equal ($E_f = E_i$), then no mass is annihilated. The created mass is maximum if all the energy is materialised into mass i.e. E_f =0 (e.g. materialisation of gamma ray photon)

Thus mass-energy equivalence may be stated as

"The mass can be converted into energy or vice-versa under some characteristic conditions of the process, but conversion factor may or may not always be c^2 (9×10^{16} m^2/s^2) or c^{-2} "

6.1 The Eq.(51) can be obtained by method of Dimensions.

Let the energy emitted (ΔE) on annihilation of mass, depends upon annihilated mass (ΔM) as dimensions; a, depends upon speed of light c, as dimensions b and depends upon time t as dimensions c. Thus as in other cases in existing physics, ΔE can be expressed as
$$\Delta E \propto (\Delta M)^a c^b t^c$$
$$\Delta E = A (\Delta M)^a c^b t^c \ (54)$$
where A is constant of proportionality and is called Conversion Co-efficient. According to principle of dimensional homogeneity, the dimensions of both the sides must be the same. Hence
$$ML^2T^{-2} = A M^a (LT^{-1})^b T^c = A M^a L^b T^{-b+c}$$
or a=1, b=2 and $-2 = -2 +c$ or c =0
or $\Delta E = A \Delta Mc^2 t^0 = A c^2 \Delta M \ (51)$

Hence the same result is obtained by the method of dimensions also.

6.2 The variation in magnitude of 'A' is consistent with existing Physics.

Now obvious question is how the co-efficient A should vary i.e. on what factors does it depend or get influenced? The answer for question of dependence or variation of co-efficient of proportionality A is precisely same as answer for all other proportionality constants or co-efficients in existing physics. All such constants or co-efficients of proportionality in existing physics depend upon the intrinsic characteristics conditions and parameters which influence the results directly or indirectly; hence the same is precisely true for A. The co-efficient A is dimensionless due to reason that it is introduced in existing equation of energy, and dimensions of energy has to be ML^2T^{-2} same in both sides, in F=kma , k is also dimensionless

For example, Hubble's constant (50 to 80 kilometres per second-Mega parsec (Mpc)), Co-efficient of thermal conductivity (0.02Wm^{-1}K^{-1} to 400 Wm^{-1}K^{-1}), Co-efficient of viscosity (1.05$\times$ 10^{-3} poise to 19.2$\times$ 10^{-6} poise), decay constant etc, all are determined experimentally. All these vary from one situation to other. The value of G in various experiments is appearing to be higher than the current accepted value

Gundlach [29]

For example, $_{92}U^{235}$ undergoes nuclear fission but not $_{26}Fe^{57}$. Further for nuclear fission to take place, in chain reaction fast moving neutrons (energy 1-2 MeV, velocity in relativistic region) are produced, these are slowed down and called thermal neutrons (0.025 MeV, velocity 2185 m/s) with help of moderator. The total kinetic energy of fission fragments of U^{235} or Pu^{239} is found experimentally 20-60 MeV less than Q-value (200MeV) predicted by Δmc^2, Whereas an intrinsic condition for nuclear fusion is that temperature must be of the order of 10^6K , the feasible conditions to cause fission of $_{26}Fe^{56}$ have not been achieved yet. Such characteristics conditions are taken in account by constant of proportionality this aspect is established in existing physics, and in generalised form of mass-energy equation the same is taken in account by A. Energy may be in different forms e.g. heat energy, light energy, sound energy electrical energy, chemical energy, atomic energy etc. under different conditions. Or in some particular process, the energy may coexist in different forms under different conditions e.g. in explosion of nuclear bomb, heat, sound, light and energies in forms of invisible radiations are released simultaneously. Thus along with characteristics and intrinsic conditions of the processes, the value of A varies from unity. The constant of proportionality may arise by method of conceptual derivation or by method of dimensions always determined experimentally. In physics the same entity may behave in different ways under different conditions. For example a single wave of radiations behaves like both wave and particle; also atomic particle electron behaves like both wave and particle depending upon characteristic conditions. Thus status of conversion co-efficient A and its magnitude is consistent with existing physics.

What determines A : The value of A is determined by magnitudes of mass annihilated to energy or energy materialized to mass. i.e. $A = \Delta E/c^{2\Delta} M$. Following two intrigues are important in understanding of determination of A

(i) The mass of which particular element will be annihilated? If mass is annihilated then what is magnitude of mass annihilated (ΔM) out total mass M? These both aspects depend upon inherent characteristics of elements.

(ii) Which factor determines (for the same annihilated mass) whether value of A will be less, more or equal to one?

The value of A depends upon inherent characteristics of elements. For determination of magnitude and trends of variations of A all above reactions are required to be theoretically and experimentally extensively conducted. The general trend of variation of A and its magnitude is expected to be different in each type of reactions, due to different inherent conditions prevailing in nuclear reactions, chemical reactions and heavenly processes etc. Thus due to diverse conditions of the processes, the mathematical formula for magnitude and variation of A in $\Delta E = Ac^2 \Delta M$ can only be developed after extensive experimentations over wide range of parameters. It is supported as below.

Determinations of Hubble's constant H and A are equivalent. This determination of A is consistent with determination of Hubble's constant H which is given by H = V/D. Now substituting value of V (velocity of recession) and D (distance), Hubble's constant can be measured. The determination of A is also identical, as it is determined after measurement of ΔE and ΔM. The range of variation of Hubble's constant experimentally measured is 50 to 80 kilometres per second-Mega parsec (Mpc), likewise A will have its own characteristic range (Hubble constant H may be better Hubble Co-efficient due its variation in magnitude).

Resistance R in Ohm's Law. Ohm's Law establishes relationship between electromotive force E, current I and resistance R i.e E $^\alpha$ I

or $E = R I = [\rho l/a]I = [m_e l/ne^2\tau a]I$ or $E = [\rho_o exp(E_g/kT) l/a] I$ (55)

where m_e is mass of electron, n is number of electron per unit volume, τ is relaxation time, l is length a is area of the conductor. Thus the resistance R which is constant of proportionality in Ohm's law is further determined as in eq.(4), as there are direct experimental observations. Presently this type of determination

as in eq.(4) of A is not possible due to lack of experimental data as described above

Further equation $\Delta E = Ac^2 \Delta M$ involves all types of reaction thus separate experiments for determination of A variation and trend of A in each case required.

7.0 Conversion of energy from one form to other.

In electric bulb electrical energy changes to light energy, in radio electrical energy is converted into sound energy , in cell chemical energy is changed to electrical energy, in photocell light energy changes to electrical energy there are many such examples of inter conversion of one form of energy to other. How much mass is associated with these inter-conversions of energy from one form to other? If these are pure examples of inter-conversion of energy from one form of energy to other. The energy in one form may be regarded as proportional to other e.g.

Light energy = k Electrical energy. (56)

where k is a co-efficient which determines the characteristics of conversion of light energy to electrical energy. The Eq. (56) is similar to relation between heat and work (W=JH, J = 4.2×10^7 erg cal^{-1}). Likewise similar equations can be written for other forms of energy and can be verified.

8.0 Unstudied aspects of of E =mc^2 and applications of $\Delta E = Ac^{2\Delta} M$

The experimental confirmation of $\Delta E = \Delta mc^2$ came in 1920s after two decades of its speculation, realistically at the dawn of experimental nuclear physics era. Then it was regarded as standard of measurements of energy such as MeV, and whole the observations were based upon it. Then qualitative nuclear uncontrolled fission established it further, no necessity was felt to theoretically analyse it. It rigidly establishes the conversion factor between mass and energy as c^2. Even then there are some compelling evidences that E =mc^2 is not complete but these evidences were either overlooked or no serious attempt was made to pursue in that direction, possibly due to lack of viable alternative such as Δ **E = Ac$^{2\Delta}$ M**

8.1 $\Delta E = c^2 \Delta M$ is unconfirmed in chemical reactions.

Further $\Delta E = \Delta mc^2$ has not been confirmed in the most abundant chemical reactions; reason cited is that currently experimental precisions are not so accurate to measure the mass annihilated and energy emitted **[7,31-32]**. Purposely a reaction from the existing literature is cited as quoted by Halliday **[32]**, suppose that 1.0 mol of (diatomic) oxygen interacts with 2.0 mol of (diatomic) hydrogen to produce 2.0 mol of water vapour, according to reaction,

$2H_2 + O_2 = 2H_2O + Q$ (4.85×10^5 J)
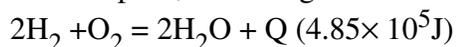
The energy released in the reaction is 4.85×10^5 J. According to $\Delta E = \Delta mc^2$ this energy is equivalent to mass 5.39×10^{-12} kg (or 5.92×10^{18} electronic masses). Thus the mass of reactants decreases and converted into energy and mass is fraction of electronic masses. It means that hydrogen and oxygen (hence electron, protons and neutrons) become lighter by mass 5.39×10^{-12} kg.

Or does it mean that masses of the order of fraction of electron are possible (for transference, annihilation or creation), as charges of the order of fraction of electronic charge are considered in Quark concept? If so then this deduction is consistent with concept of proposal that electrons consist of sub-particle. So all possibilities has to be discussed. But at the moment lack of experimental precision keeps such in speculative predicament only. But in this regard key factor (to answer every query) is precise measurement of annihilated mass, which has not been done yet.

Apparently there is immense conceptual difference between annihilation of electron positron pair, in nuclear reaction, and in combustion (e.g. burning of paper) as described above. So these experiments are required to be specifically conducted. If emitted energy is found less than Δmc^2, then value of A is less than one, which is an open prospect so far .

8.2 Total Kinetic Energy of fission fragments is 20-60MeV less than predicted by $\Delta E = c^2 \Delta m$.

In laboratory it is confirmed [**24-26**] that using thermal neutrons the total kinetic energy (TKE) of fission fragments that result from of U^{235} and Pu^{239} is 20-60MeV less than Q-value of reaction predicted by Einstein's famous $\Delta E = \Delta mc^2$ (200MeV for U^{235}). It is typically assumed to explain it that energy [Q-Value –TKE of fragments] is lost in unobserved effects.
Attempts has been made to explain the Q-value of reaction on the basis of de Broglie's wave-mechanical equation [8] i.e.
H= mv^2 (**57**)
where H is energy , m mass and v is velocity (v<c). Also attempts [**27-28**] have been made to explain the total kinetic energy of fission fragments by extending the successful liquid-drop model of Bohr and Wheeler. These existing attempts use the fact that TKE is the Coulomb potential energy equation and in a specific case (TKE should have minimum value), the magnitudes of TKE as given by extended model coincide with wave mechanical equation H=Δmv^2. The theme of discussion is that

extension in Bohr-Wheeler model yields the same value of TKE [**24**] as H=Δmv^2; further the equation $\Delta E = Ac^2 \Delta M$, is consistent with H=Δmv^2 with value of A less than unity. Let the TKE of fission fragments is 175MeV (as experimentally it is observed less), instead of expected 200MeV, then according to $\Delta E = Ac^2 \Delta M$ value of A is 0.875 i.e.
A = $\Delta E / c^2 \Delta M$ = 175/200 = 0.877 (**58**)

Thus energy of fission fragments of U^{235} and Pu^{239} is given by
$\Delta E = 0.877c^2 \Delta M$ (**59**)
Thus energy obtained corresponding to annihilation of mass ΔM is less, hence lesser the total kinetic energy of nuclear fragments. This problem is due to reason that according to $\Delta E = \Delta mc^2$ conversion factor between ΔE and Δm for all cases is harshly c^2 , but it is not always true as indicated by $\Delta E = Ac^2 \Delta M$.

8.3 Discovery of particle having mass less than predicted mass

Recent work at SLAC confirmed discovery of a new particle dubbed as Ds (2317) having mass 2,317 mega-electron volts. But this mass is far less than current estimates, is a mathematical puzzle [30]. This discrepancy can be explained with help of equation $\Delta E = Ac^2 \Delta M$ with value of A more then one.

8.4 Applications of $\Delta E = Ac^2 \Delta M$ in cosmological and related phenomena

For all such phenomena which involve inter-conversion of mass and energy, $\Delta E = Ac^2 \Delta M$ provides reasonably good explanation with value of A less or more than one. For determination of A, the value of ΔM i.e. mass annihilated in case of heavenly body is required; which can not be directly measured like many other parameters. Thus, initially for simplicity or calibration (standard or reference can be chosen) the magnitude of value of ΔM is regarded as 4.322×10^9kg i.e. mass annihilated in case of sun

(luminosity of the sun is 3.89×10^{26} Js^{-1}), thus

$\Delta M = \Delta E/c^2 = 3.89\times 10^{26}$ Js^{-1}/ 9×10^{16} m^{2}s^{-2} = 4.322×10^9kg (60)

If for some cases the value of ΔM is experimentally measured then its actual value (ΔM) can be used instead of Eq.(60). Many more such phenomena may be revealed as innovative precision in investigative measurements is increasing continuously

8.41 Creation of mass of universe (10^{55} kg) before Big Bang

The Big Bang Theory assumes that initially (t=0) whole mass 10^{55}kg of universe was infinitely compact and in singular state enclosing a space even smaller than an atomic particle instantaneously exploded in gigantic detonation and ever since the universe is expanding, Hawking [33]. How the whole mass of universe was formed and condensed to infinitely compact point? How this compressional energy was created and whole mass became hot? How explosion was triggered causing expansion, reduction in temperature and density drastically? Which source provided energy for these events? All these intrigues are neither answered by detractors nor adherents of Big Bang Theory, and are open for plausible elucidation.

Further additional energy (which may be infinitely large i.e. unimaginably high to be appraised) is required to change mass 10^{55}kg *into a point of exceedingly high density*, and *raise the temperature*, trigger an explosion and to impart *kinetic energy* to it , even now accelerating outward. This additional energy may be far more than energy required for creation of mass 10^{55}kg. .

Creation of Mass 10^{55}kg on the basis of Δ E=Δ mc^2. A gamma ray photon of energy at least 1.02 MeV (1.623×10^{-13} J) when passes near the nucleus gives rise to electron and positron pair of mass i.e. 18.2×10^{-31} kg, it is consistent with ΔE=Δ mc^2. The mass of universe is estimated to be nearly 10^{55} kg, thus as above it must have been materialized from energy (ΔE=Δ mc^2) i.e. 9×10^{71}J. Now it has to be assumed that energy 9×10^{71}J and spectacular amount of additional energy as mentioned above is created from nothing or naught or cipher automatically and spontaneously. The law of conservation of energy does not permit creation of mass out of nothing at all (further on such highest scale), hence the law was not obeyed at that stage, according to Δ E=Δ mc^2. How the energy of the order of 9×10^{71}J was produced along with additional energy? Thus answer of a query is another query.

Creation of Mass 10^{55}kg on the basis of Δ E = Ac^2 Δ M. Thus the equation Δ E = Ac^2 Δ M predicts that in this primordial bang (exceptionally-2 super special event), diminishingly small pulse of energy, say 10^{-4444} J (or less) equivalent to 2.4×10^{-4443} calorie (or less), can manifest itself in mass 10^{55}kg if the value of A is regarded 2.568×10^{-4471}. The energy 10^{-4444} J or less is regarded as to exist inherently in the universe, when process of formation of space started i.e. even when there was no material particle. The primordial value of conversion coefficient A$_{uni}$ (due to lack of value of Δ M$_{uni}$, eq.(60) is used) can be estimated as

A$_{uni}$ = 10^{-4444}/$9\times 10^{16}\times$ 4.32×10^9 = 2.568×10^{-4471} (61)

Thus Δ E = Ac$^{2\Delta}$ M, is the first equation which at least theoretically predicts that universe (10^{55}kg) has been created from minuscule or immeasurably small amount of energy (10^{-4444}J or less, which may be easily available compared to 9×10^{71} J).Thus the generalized equation explains the origin of mass of universe with ease and simplicity.

8.42 Big Bang, how and why?

The most peculiar phenomena like big bang require peculiar perceptions for the understanding. Here intrigues are how whole mass of universe (already formed) condensed to a dense point with exceptionally

high temperature and how explosion was triggered? These aspects can be easily explained like formation of primeval mass on the basis of $\Delta E = Ac^2\Delta m$ and generalization of existing established facts. The manifestation of energy from one form to other is well established (H = W/J i.e. heat energy is converted into mechanical energy, J= 4.2×10^7 erg cal^{-1}). The analogous relation between energy emitted on annihilation of mass and gravitational energy (measure of gravitational force or pull) as

Energy emitted in annihilation of mass ($Ac^2\Delta m$) = k Gravitational energy (U_g)

or Gravitational energy (U_g) = Energy emitted in annihilation of mass ($Ac^2\Delta m$) / k **(62)**

(i) Annihilation of mass to gravitational energy: It is experimentally observed in uncontrolled nuclear fission light energy, heat energy, sound energy and energy in form of invisible radiations is emitted. It implies that energy changes from one form to other i.e. heat energy to sound energy or light energy etc. The mass may firstly change to energy (including heat energy) then the energy may further transform to gravitational energy like heat energy changes to work. Thus it can be speculated that the earliest mass is converted into energy according to $\Delta E = Ac^2\Delta m$ with exceptionally high value of A, thus corresponding to annihilation of small mass huge amount of energy is emitted. This energy got transformed into gravitational energy as given by Eq.(**62**), smaller the value of k , higher would be congregation of gravitational energy. Hence when whole mass of universe is condensed to a point then value of A in $\Delta E = Ac^2\Delta m$ was exceptionally large and value of k exceptionally less, thus gravitational energy and temperature of universe was unimaginably high. The gravitational energy is universally prevalent and is inherent property of bodies, and here it can be understood how it came into being with ease and simplicity with help of $\Delta E = Ac^2\Delta m$ and Eq.(**62**). Thus gravitational energy is another form of mass, as it is obtained on annihilation of mass like heat, light, sound energy etc.(these energies can be physically felt). Also higher the mass of body higher the gravitational force, gravitational potential energy and gravitational potential.

The body with higher or maximum gravitational energy or force attracted other constituents towards its centre. When lighter bodies stuck with heavy body (under extreme conditions of temperature and gravitation) then gravitational force of the system increases. Thus high temperature and high gravitational pull caused universe to contract to a single point, further some other effects might have facilitated the same. As the process of annihilation of mass continued rise in temperature and increase in gravitational energy continued. Thus bodies were quickly attracted being extremely hot they compressed to small size easily consequently radius of the universe decreased. This process of increase in gravitational energy continued with annihilation of mass, causing compression of mass to optimum limit. When the gravitational energy further increase on annihilation of mass and tried to compress the mass of universe beyond optimum limit, then it changed into repulsive or anti-gravitational force which caused distruptive big bang.

During the bang considerable amount of mass of universe is annihilated to energy with value of A exceptionally greater than one. This energy obtained after annihilation of mass, transformed into kinetic energy of constituents, hence they are continuously receding even now.

(ii) **C**omparison of anti-gravitation force with intermolecular forc**e**. When size of universe is decreased beyond the optimum limit, then attractive force suddenly changed into anti-gravitational force. This repulsive force, also known as dark energy caused explosion and recession between various constituents of universe. This transformation of gravitational to anti-gravitation energy is also consistent or identical with well established intermolecular potential energy (intermolecular force). At larger distance compared to r_o (equilibrium distance), the potential energy is negative i.e. attractive force between various molecules (like gravitation) and when distance between molecules becomes less than r_o then potential energy is positive i.e. force between molecules becomes repulsive (like anti-gravitational force). The nuclear force is also repulsive.

(iii) Mass and gravitation are inherently related. In nucleus the binding energy is also due to annihilation

of a part of mass of nucleons, higher the mass annihilated higher is the binding energy according to $\Delta E = \Delta mc^2$. Analogously it can be concluded or speculated that the gravitational energy has also similar nature, higher the mass annihilated higher is the gravitational energy according to $\Delta E = Ac^{2\Delta} M$. Mathematically, gravitational force or energy of body is proportional to mass of body and it has emerged (other form of mass of body) when mass is annihilated. Hence they are so inherently associated with each other. In general higher the bulk mass of body, higher the mass annihilated higher would have been the gravitational energy as in Eq.(**62**) .The gravitational energy created is proportional to mass annihilated. It may be compared with magnetizing an iron bar by rubbing magnet on it. Gradually the degree of magnetization increases likewise had been the case of gravitational energy. All the heavenly bodies have been inherited characteristics of gravitation when all bodies when the mass formed and whole mass of universe condensed at a point. There may be typical heavenly bodies in universe which may have gravitation just to keep its mass together and not able to attract other external bodies on its surface.
(iii) Every perception of origin of universe also requires its assessment about the possible end. In view of current observations that all heavenly bodies are accelerating outward. It implies the universe is in initial stages as accelerating due to impact of energy that has been obtained during big bang. At final stages this acceleration will slow down and may become zero. At final stages the whole mass of universe may further convert into energy according to $\Delta E = Ac^2 \Delta m$ (with value of A less than one). Thus whole mass of universe may convert into small energy. Then mass of universe will again be created according to $\Delta E = Ac^2 \Delta m$ (value of A less than one) from small energy left from previous cycle. This whole process may take infinitely large time for completion and these cycles of mass energy inter-conversion may continue indefinitely.

8.43 Annihilation of antimatter in hadron epoch

(i) **U**niverse began with the explosion of a "primeval atom".The uncertainty principle of quantum mechanics prevents our speculating on times shorter than 10^{-43} seconds after the big bang. After time 10^{-35} seconds in the Universe roughly equal amounts of matter and antimatter were created. Now mass of universe is regarded as 10^{55} kg. But so far, no antimatter domains have been detected in space within 20 megaparsecs of the Earth [34]. The antimatter has been annihilated at 10^{-6}s, in the end of hadron epoch (10^{-35} - 10^{-4} s, temperature, 10^{27} - 10^{12} K) and temperature of the universe in next lepton epoch (10^{-4} - 10^{2} s,) reduced to temperature 10^{12} - 10^{9} K.

It leaves a ticklish situation which has been overlooked, when huge amount of antimatter is instantly annihilated, then huge amount of energy according to $\Delta E = c^2 \Delta m$ would have been created further increasing the temperature. But almost simultaneously temperature of universe decreased. Thus the generalized equation $\Delta E = Ac^2 \Delta m$ can be used to explain the same, with value of A exceptionally less than one . Thus at the end of hadron epoch huge amount of antimatter is annihilated and very small amount of energy is emitted. Thus $\Delta E = Ac^2 \Delta m$ (with value of A less than one) explains both the aspects i.e. why antimatter is not observable now and temperature of universe decreased usually.

8.44 Gamma Ray Bursts

Gamma ray bursts (GRBs) are intense and short (approximately 0.1-100 seconds long) bursts of gamma-ray radiation that occur all over the sky approximately once per day and originate at very distant galaxies (several billion light years away). GRBs are the most energetic events after the Big Bang in the universe and energy emitted is approximately 10^{45} J with the most extreme bursts releasing up to 10^{47} J. This energy cannot be explained with $\Delta E = c^{2\Delta} m$ (precisely confirmed in nuclear reactions). This is also the amount of energy released by 1000 stars like the Sun over their entire lifetime! It implies that for

annihilation of dwindling mass in short time unimaginably high amount of energy is emitted, which can be explained with help of $\Delta E = Ac^{2\Delta} M$ with exceptionally high value of A. If for simplicity the value of ΔM can be taken standard as in Eq.(60) as actual estimate of ΔM for GRBs is not available, then $A_{grb} =$

$$\Delta E / c^2 \Delta M = 10^{45}/ 9\times 10^{16\times} (4.32\times 10^9) = 2.57\times 10^{18} \text{ (63)}$$
or $\Delta E = 2.57\times 10^{18} c^2 \Delta M$ (64)

Hence all conversions of mass to energy in nature, is not always according to $\Delta E = c^{2\Delta} m.$, where c is the conversion factor like universal constant. In the GRBs intense and short bursts of gamma-ray radiation are emitted; which implies for small mass (simply gamma rays), in small region, in small time huge amount of energy is liberated. It is direct confirmation for $\Delta E = Ac^{2\Delta} M$ with very high value of A i.e. for annihilation of small mass (burst of Gamma Ray), in short time enormous amount of energy is emitted (in this case 2.31×10^{32} J for annihilation of 10^{-3} kg) which is 2.57×10^{18} times more than $\Delta E = c^{2\Delta} m.$ However the actual value of A_{grb} will be more when exact values of Δm corresponding to energy emitted will be experimentally determined, instead of standard value as given by eq.(63).

8.45 Quasars

The observations taken with the 2.5-meter Isaac Newton Telescope at La Palma in the Canary Islands reveals that the quasar is 4 million-billion (15.56×10^{41} Js^{-1}) to 5 million-billion times brighter than the Sun or this energy is thousand times more than emitted by the brightest galaxy. The most peculiar characteristics of Quasar is reported by Arav et al. [35] that this prodigious amount of energy is generated in a small region approximately **one light year** across. By comparison the diameter of the Milky Way is about **100, 000 light years.** It implies corresponding to a small region (a measure of mass and its hence annihilation) mammoth amount of energy is emitted in case of Quasars. $\Delta E = Ac^{2\Delta} M$ is useful in explaining such aspects. Now

$$A_{qu} = \Delta E / c^2 \Delta M = 15.56\times 10^{41} Js^{-1}/9\times 10^{16\times} (4.32\times 10^9) = 4\times 10^{16} \text{ (65)}$$

For simplicity the value of ΔM is regarded same in Eq.(65), which is standard for the sake of calculations as exact estimate of ΔM is not available**.**
With this value of the generalized mass-energy inter convertibility equation becomes,
$\Delta E_{qu} = 4\times 10^{16} c^2 \Delta M$ (66)
Thus corresponding to small mass (size) energy emitted is more thus comparatively smaller quasars or in general smaller bright objects are feasible. So in small region even when small amount of mass is annihilated, huge amount of energy is emitted. The lower limit of Quasars mass is not yet determined, Vestergaard [36]. It is further justified from the fact that the Quasars possibly or inexorably ending as super massive black holes, presently the maximum mass is of the order of 2×10^{40} kg, Vestergaard [37].Thus inspite of emitting huge amount of energy in own life time, significant amount of matter is remnant in Quasar and which are expected to behave like super massive black hole, this aspect is easily explained on the basis of $\Delta E = Ac^{2\Delta} M$, with high value of conversion co-efficient, A. Normally a black hole have density of the order of 10^{18}kg/m^3, and even light cannot escape from it, may be regarded as formed after numerous cycles.
It can be concluded that to attain such state Quasars must under go series of large number of exceptionally intense compressions utilizing energy produced in itself. But energy used for this purpose (internal changes) is not taken in account in current measurements of luminous energy, implying that total energy (including measurable and immeasurable) is far higher than current estimates i.e. A_{qu} may be more than 4×10^{16} (it is only for luminous energy). This large amount of energy emitted by Quasar and

other heavenly bodies is consistent with $\Delta E = Ac^{2\Delta} M$ with higher values of A. Similarly energy emitted by supernova and other bodies can be explained. Thus according to this equation $\Delta E = Ac^{2\Delta} M$ more energetic and abundant such explosions in universe are feasible and universe is more long lived compared to predictions of $\Delta E = \Delta mc^2$ as for smaller mass huge amount of energy is emitted.

8.46 Dark Matter

There are compelling evidences that predominant mass of the Universe may be in the form of non-luminous "dark matter". The candidates for dark matter are numerous, and include low mass dwarf stars, neutron stars, hydrogen , black holes, massive neutrinos, magnetic monopoles, particles predicted by supersymmetry (the as yet undetected gravitinos and photinos) or again undetected axions, (particles with extremely small masses). The above explanation (for annihilation of antimatter) of $\Delta E = Ac^2\Delta m$ favors that dark matter in form of the lightest neutrinos, axioms etc. Thus bulk of mass of antecedents according to $\Delta E = Ac^2\Delta m$ (value of A less than unity) is annihilated and small energy is emitted, thus they reduce to the lightest descendants e.g. neutrinos, axioms etc. which is consistent with existing perceptions . The reason and effect of dark of dark energy is already explained.

8.47 Black Holes

On the basis of $\Delta E = Ac^2\Delta m$ the reason for formation of black hole is that due to annihilation of mass the temperature rises and energy thus produced is converted into gravitational energy. In some huge heavenly body e.g. star with mass 5 solar masses is pre-requisite, as considerable amount of mass is required to be annihilated then energy created (rise in temperature) and will be transformed to gravitational energy as in Eq.(62).
Thus gravitational energy of star became high and high contracting its size and even not allows the light to escape. This process of annihilation of mass to energy then transformation of created energy to gravitational energy continues causing compression (increase in density) to optimum limit. Thus eventually the black holes, whose gravitational energy try contract them beyond optimum limit they may end in form of baby or black bang in universe. According to this perception even smaller heavenly bodies may become black hole, if value of A is sufficiently high , and value of k is small in eq.(62). Thus for small mass annihilation, huge amount of gravitational energy is gained by body which even does not allow light to escape. The lightest black holes reside in the youngest galaxies

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