

Impact of 'Division by Zero' in Einstein's Static Universe and Newton's Equations in Classical Mechanics.

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Abstract

Aristotle (384-322BC) and other ancient philosophers believed that force is always required for movement of body but no equation was formulated for such perception in antiquity. Galileo (1664-1727) argued against the idea and maintained that under specified hypothetical conditions body can move with uniform velocity without any force (if once set in motion). Newton (1642-1727) formulated equation $F = ma$, for Galileo's hypothesis. In definition of inertial mass ($m = F/a$), the denominator acceleration becomes zero under some conditions. The similar was situation (division by zero) with Einstein's model of Static Universe involving Cosmological Constant, which was then purposely withdrawn by Einstein. Exactly similar is the situation in case of inertial mass, the acceleration (a) becomes zero when velocity is uniform. The division by zero in Einstein's equations lead to acceptance of doctrine of Expanding Universe, similarly division by zero Second Law of Motion ($m = F/a$) lead to equation of force which supports the perception of force and motion in pre-Galileo's or Aristotle's days.

1.0 Einstein's equation of Static Universe and Newton's equation of inertial mass (Second Law of Motion) involve division by zero.

Einstein (1879-1955) in 1917 in his research paper proposed a model of Static Universe introduced Cosmological Constant. Alexander Friedmann a Russian cosmologist after five years found that under certain condition Einstein's equation involve division by zero, which is not permissible. George Gamow (1904-1968) Russian-born American nuclear physicist and cosmologist remarked that "it is well known to students of high school algebra" that division by zero is not valid; and Einstein admitted it as the biggest blunder of his life [1].

Contrary to prevalent views continuing over nearly 2,000 years, Galileo argued that in case all the resistive forces (atmospheric, frictional and gravitational etc.) are precisely eliminated (true under hypothetical conditions only) then body once set in motion will maintain its state of perpetual uniform motion (if body once set in motion). It must be noted that nothing was known about Gravitation (theoretically this force extends up to infinity) in Galileo's time otherwise his hypothesis may have been different. Galileo also developed concept of acceleration and formulated various kinematical equations for uniformly accelerated motion, thus acceleration was the main term in his interpretation. These equations were later obtained by method of calculus (discovered independently and simultaneously by Leibnitz and Newton) also, using concept of constant acceleration.

Taking Galileo's hypothesis as a basis Newton formulated Second law of motion or Axiom II as quoted in Book I of *the Principia Mathematica Philosophiae Naturalis* in Latin (first translation in English by done Andrew Motte, 1729, two years after death of Newton) as

The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

Thus Newton provided [the mathematical basis for Galileo's perception of uniformly accelerated motion \[2, 3\]](#). The Second Law is called a basic law of motion as the First Law of motion can be obtained from it; and it defines inertial mass (m) as ratio of net force (F) and acceleration (a) i.e.

$$m = F/a. = Ft / (v-u) \quad (1)$$

where u is initial, v is final velocity and t is time. Then it was established that inertial and gravitational masses are equivalent. The physical quantity acceleration was defined by Galileo before Newton (as the rate of change of velocity); the denominator of Eq. (1) is often written as in disguised form i.e. as 'a' rather than in terms of difference in velocities ($v-u$). Thus acceleration becomes zero if body moves with uniform velocity or is at rest. Thus the situation is precisely similar in Einstein's Static Universe and Newton's Second law of motion, regarding occurrence of division *by zero*, in equations.

The second law is the real law of motion [3] as first law can be obtained from it. Purposely we quote Resnick and Halliday [3] as if $\mathbf{F} = 0$, then $\mathbf{a} = 0$.

In other words if net force on body is zero, then acceleration of body is also zero. Therefore in absence of impressed or resultant force ($\mathbf{F}=0$) the body will move with constant velocity ($\mathbf{a}=0$) or remains at rest, which is first law of motion. Now what is the value of inertial mass under this condition ($F = 0, a = 0$) i.e. when second law of motion reduces to the first law. Obviously undefined

$$M = F / a = 0/0.$$

It is completely meaningless, in this case not only denominator but numerator also becomes zero. It implies that under this condition (i.e. when Newton's second law of motion reduces to first law), the Eq. (1) i.e. $m = F/a$ is not applicable as it gives value of mass (has definite dimensions and units of mass) as undefined. In this case $F = ma$ implies $0 = 0$, which is without dimensions, units and have zero magnitude; hence gives no physical information which is inherent characteristic or prerequisite of equation [3, 4].

2.0 Effects of division by zero in classical mechanics

Thus there is a precise similarity between Einstein's equations regarding Static Universe and Newton's laws of motion (calculation of inertial mass), as far as *division by zero* is concerned. Realizing the limitations Einstein withdrew his arguments in 1931 and theory of expanding universe was accepted, keeping aside the theory of static universe. Likewise in view of this limitation of $F = ma$ another equation of force has been proposed by author [5] in as

$$F = A m(u+v)S/t \quad (2)$$

where, F is net or resultant or impressed force which causes displacement S is distance traveled. And additional term 'A', which is used to remove the sign of proportionality, has nature like Hubble's constant or like coefficient of thermal conductivity or coefficient of viscosity etc). Their magnitudes are determined experimentally e.g. Hubble's constant {50 to 80 kilometres per second-Mega parsec (Mpc)} or coefficient of viscosity (1.05×10^{-3} poise to 19.2×10^{-6} poise) or co-efficient of thermal conductivity ($0.02 \text{Wm}^{-1}\text{K}^{-1}$ to $400 \text{Wm}^{-1}\text{K}^{-1}$).

The interpretation of Eq. (2) is revival of concepts or perceptions of force propagated by Aristotle and others for system in which resistive forces are present i.e. practical system. It consistent with existent concepts in antiquity, Eq. (2) implies that force is always required for movement of body, in this case resistive force (regarded as null by Galileo for hypothetical system) is the main factor. This doctrine was taught for over two thousand years or more, as it too found immediate experimental support, and have even now in countless cases. Galileo put forth that in a medium devoid of resistive forces (frictional, atmospheric and gravitation), body once set in motion will keep on moving with uniform velocity. Thus impact of concept of "division by zero" has not only resulted in adopting the theory of Expanding Universe but in this case it also revives of pre-Galilean or Aristotelian perception of motion of bodies on the basis of facts and logic.

The most practical aspect of this interpretation is that we should formulate a mathematical basis for practical or most abundant cases; then interpretation must be provided for hypothetical system in limiting cases. For example, we formulate an equation of force for systems most abundantly available in daily life i.e. when gravitational, frictional and atmospheric forces etc are present. Then the same equation (mathematical basis) should be used to explain the hypothetical cases (when gravitational, frictional and atmospheric forces etc are present). However Galileo and Newton has established different formulations, they formulated mathematical basis for hypothetical system (devoid of gravitational, frictional and atmospheric forces etc)

Reference.

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