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Einstein's Energy-Mass Formula

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In his work "Ist die Trägheit eines Körpers von seinem Energienhalt abhängig" (1905), Einstein derived a relationship between the radiated energy and the change of mass of a body caused thereby, which results from the Einstein relativity principle. A body was regarded from two reference systems and the resulting energy difference of the delivered radiation energy was compared, for which purpose a special case of a "thought experiment" was developed. Special case means here that only symmetrical radiation emission was regarded, so that this experiment cannot be valid generally. As a formula for the relativistic change of radiation energy, Einstein uses the relationship, which results from the averaged Doppler effect (aberration) for amoving radiation point-source emitting in all directions if the consideration is limited to small speeds as shown elsewhere:

$$L' = L\frac{1-\frac{v}{c}\cos\varphi}{\sqrt{1-\frac{v^2}{c^2}}}.$$

Einstein regards the emission of the radiation as the sum of two portions emitted in opposite directions, whereby in the moved system of reference, different Doppler shifts result, which after Einstein simple method, adds because in both cases same average speed of light is used:

$$c' = c\sqrt{1-\frac{v^2}{c^2}}.$$

Thus Einstein receives for this special case the radiation energy of:

$$\Delta L = L\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - L \approx \frac{L}{c^2}\frac{v^2}{2}.$$

If the radiation is emitted only in one direction, one receives the energy-mass formula, which had been already derived by Poincaré in the year 1900 considering only the non-relativistic principle of the reaction for "ordinary matter and in addition that there is an ether" if $\varphi = 0$ [2]:

$$\Delta L = L\frac{1-\frac{v}{c}\cos\varphi}{\sqrt{1-\frac{v^2}{c^2}}} - L \approx \frac{L}{c^2}\frac{v^2}{2} - L\frac{v}{c}\cos\varphi.$$

References

[1] A. Einstein: ”Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig“. Annalen der Physik **18** (1905) 639-641.

[2] H. Poincaré, “La Théorie de Lorentz et le Principe de Réaction”, Archives nèerlandaises des Sciences exactes et naturelles, série 2, volume **5**, pp 252-278 (1900).