# "g-factor and the Helical Solenoid Electron Model"

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*Abstract*

*A new model of the electron with Helical Solenoid geometry is presented. This new model is an extension of the Parson's Ring Electron Model and the Hestenes' Zitter Electron Model. In this new electron model, the g-factor appears as a simple consequence of the geometry of the electron. The calculation of the g-factor is performed in a simple manner and we obtain the value of 1.0011607. This value of the g-factor is more accurate that the value provided by the Schwinger's factor.*

## Introduction

In 1915, Alfred Lauck Parson proposed a new ring-shaped model for the electron. Some important physicists of his time, as Arthur Compton, conducted studies supporting the Parson's model. All these studies were compiled in 1918 by H. Stanley Allen in "The Case for a Ring Electron". According to the "Ring Electron Model", an electric charge circulates at the speed of light by a ring of radius equal to the compton wavelength of the electron, this circular motion of the electric charge is the cause of the magnetic moment of the electron. Furthermore, in 1953 Kerson Huang proposed a semiclassical interpretation of the Dirac equation, whereby zitterbewegung is the mechanism that produce the angular momentum of the electron (spin), and this angular momentum produces the magnetic moment of electron. Some renowned researchers, as Asim Barut or David Hestenes, have worked on this "Zitterbewegung Electron Model". Both models have many similarities and offer a semiclassical alternative to the current electron model of Quantum Mechanics.

In the **"Helical Electron Model" [1]**, we presented a new model of the electron based on the "Ring Electron" and "Zitterbewegung Electron" models. In this work a refinement of the "Helicoidal Electron Model" is presented as "Helicoidal Solenoid Electron Model".

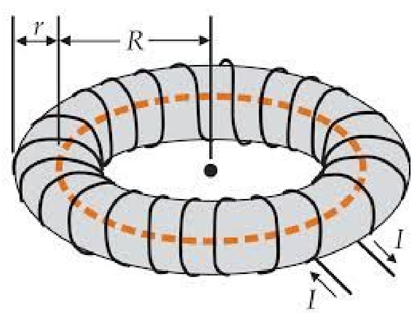
To date, no model based on Parson's Ring Electron has been able to explain the existence of the anomalous magnetic moment of the electron (g-factor). The simplest hypothesis to explain the existence of this anomalous magnetic moment is to assume a substructure of the electron beyond the main helical structure. This new model allows to obtain the anomalous moment of the electron as a direct consequence of its own geometry.

## Toroidal Solenoid

Winston Bostick [2], a disciple of Arthur Compton, discovered in 1956 the existence of "plasmoids". A plasmoid is a toroidal coherent structure of plasma and magnetic fields. Plasmoids are structures so stable that can behave as individual objects and interact between them. From Parson's Ring Electron Model, Bostick proposed a new structure for the electron, similar to the structure of the plasmoids. The proposed model for the electron was a Toroidal Solenoid where an electric unit of charge circulates at the speed of light. Bostick's electron model has been extended by other researchers like Charles W. Lucas [3], allowing a semiclassical alternative theory to Standard Model of Particle Physics.

In a Toroidal Solenoid, the magnetic flux generated is confined within the toroid. This feature is consistent with the idea that the mass of a particle matches the electromagnetic energy contained therein. Devices based on store electromagnetic energy in a Toroidal Solenoid Superconductor without loss of energy are called SMES (superconducting magnetic energy storage). According to this model, an electron is a microscopic version of a SMES.

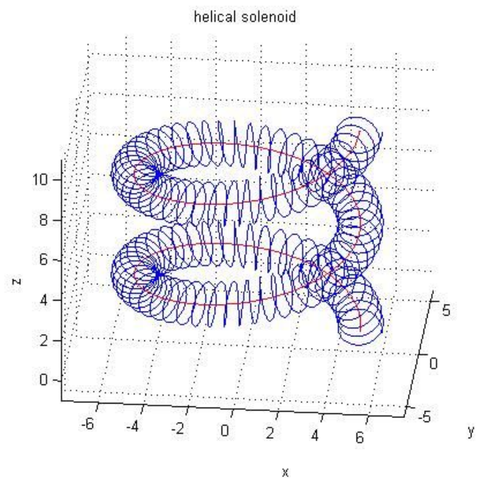
The Toroidal Solenoid geometry is well known in the electronics' field and it is used to design inductors and antennas. Toroidal Solenoid provides two additional degrees of freedom relative to the ring geometry. Other than the radius (R) of the torus, two new parameters appear: The thickness of the torus (r) and the number of turns around the torus (N), being N an integer.




The Electron Model of Bostick is just the electron substructure we were looking for for the Helical Electron Model. In the present model, we assume that the electric charge is a point particle and that the Toroidal Solenoid simply represents the trajectory of that point electric charge.

We define the velocity of the electron (v) as the velocity of the center of mass (CM) of the electron. By symmetry, the CM is the center of the ring. Therefore, we consider that the electron is at rest if the center of the ring is static, since in that case there is only a rotation movement of the electric charge without any real translation movement.

Both the Ring and the Toroidal Solenoid geometries represent a static electron. For a moving electron, with a constant velocity "v", the Ring geometry become a Circular Helix geometry, while the Toroidal Solenoid become a geometry known as "Helical Solenoid", as shown in the following graph:




For a static electron (v=0), the "Helical Solenoid Model" is equivalent to the Bostick's "Toroidal Solenoid Model." On the other hand, if we neglect the thickness of the toroid (r=0), the "Helical Solenoid Model" is reduced to the "Helical Model" described in [1]. If we join both approximations (v=0 and r=0) we obtain the "Ring Electron" model.

| **Geometry** | **v = 0** | **v > 0** |
|---|---|---|
| **r = 0** | Ring | Helix |
| **r > 0** | Toroidal Solenoid | Helical Solenoid |

The universe generally behaves in a fractal way, so the most natural solution is to assume that the substructure of the electron is of the same type as the main structure. That is, an helix on an helix.

**Rotational Velocity**

We can parameterize the trajectory of the electron with the equation of the helical solenoid:

$$\begin{aligned} x(t) &= (R + rcosNwt)coswt \\ y(t) &= (R + rcosNwt)sinwt \\ z(t) &= rsinNwt + vt \end{aligned}$$

The calculation of the tangential velocity is described in **ANNEX A** of this work and the result is:

$$|r'(t)|^2 = (R + rcosNwt)^2 w^2 + (rNw)^2 + v^2 + 2vrNwcosNwt$$

As a main premise of the Electron Helical Model, we postulated that the electron always moves with a tangential velocity exactly equal to the speed of light. Keeping the same criteria but applied for the case of the Helical Solenoid, we obtain:

$$|r'(t)| = c$$

$$|r'(t)|^2 = (R + rcosNwt)^2 w^2 + (rNw)^2 + v^2 + 2vrNwcosNwt = c^2$$

This equation has been obtained directly from the Helical Solenoid geometry without any approximation. Therefore, we can consider this equation as the **Fundamental Equation of Electron** in this model

For the "Ring Electron" model (v=0, r=0), where w is the angular velocity, equal to the speed of rotation around the torus divided by the radius of the torus, the fundamental equation reduces to:

$$c^2 = (Rw)^2$$

As expected, we get a speed of rotation equal to the speed of light.

$$\begin{aligned} &w = c/R = v_r/R \\ &v_r = c \end{aligned}$$

For the "Helical Electron" model [1], (v> 0, r = 0). The fundamental equation reduces to:

$$c^2 = (Rw)^2 + v^2$$

And we get the speed of rotation as the speed of light between the Lorentz factor.

$$
\begin{aligned}
v_r^2 &= c^2 - v^2 \\
v_r &= \sqrt{c^2 - v^2} = c\sqrt{1 - v^2/c^2} \\
v_r &= c/\gamma
\end{aligned}
$$

For the Toroidal Solenoid model (v = 0, r> 0). The fundamental equation reduces to:

$$(R + rcosNwt)^2 w^2 + (rNw)^2 = c^2$$

For R >> rN, we can obtain the velocity of rotation as:

$$c^2 = (Rw)^2 + (rNw)^2$$

$$c/Rw = \sqrt{1 + (rN/R)^2}$$

$$c/v_r = \sqrt{1 + (rN/R)^2}$$

The second factor depends only on the geometry of electron. We call this value the "helical g-factor". If R>>rN, this factor is slightly greater than 1.

$$gfactor = \sqrt{(1 + (rN/R)^2}$$

As a result we obtain a rotation speed dependent on the g-factor and slightly lower than the speed of light.

$$c/v_r = g$$

$$v_r = c/g$$

Finally, for the Helical Solenoid model (v> 0, r> 0) we use the complete fundamental equation:

$$c^2 = (R + rcosNwt)^2 w^2 + (rNw)^2 + v^2 + 2vrNwcosNwt$$

Rearranging the terms we observe a component of the fundamental equation of the electron that oscillates at very high frequency, with an average value of zero. This implies a direct prediction of this model: the electron g-factor is not fixed but oscillating. The main consequence of this prediction is that there is a maximum level of precision with which the g-factor can be measured, since the value oscillates. This prediction is completely new to this model and is in opposition to the predictions of QED.

$$c^2 = (Rw)^2 + 2Rrw^2 cosNwt + (rwcosNwt)^2 + (rNw)^2 + v^2 + 2vrNwcosNwt$$

$$c^2 = (Rw)^2 + (rNw)^2 + v^2 + (2Rrw^2 + r^2w^2cosNwt + 2vrNw)cosNwt$$

For R >> rN we can neglect this oscillating component and the fundamental equation reduces to:

$$\begin{aligned}
c^2 &= (Rw)^2 + (rNw)^2 + v^2 \\
c^2 &= (Rw)^2(1 + (rN/R)^2) + v^2 \\
c^2 &= (v_r)^2 g^2 + v^2 \\
g^2 v_r^2 &= \sqrt{c^2 - v^2} \\
g v_r &= c\sqrt{1 - v^2/c^2} \\
g v_r &= c/\gamma
\end{aligned}$$

Finally we obtain the speed of rotation as a function of the speed of light, the Lorentz factor and the g-factor.

$$v_r = c/g\gamma$$

**Arc Length**

We have calculated the velocity of rotation, and now we also need to get the length of a turn of the toroidal solenoid. This length is called “arc length”.

To calculate the arc length, we need to perform the integral over one turn:

$$l = \int \sqrt{|r(t)|^2} dt$$

For an electron at rest (v = 0),

$$l = \int \sqrt{(R + rcosNwt)^2 w^2 + (rNw)^2} dt$$

Approximating for R >> r, replacing the g-factor and being "T" the rotation period, we have:

$$l = gRw \int dt = gRwT$$

Substituting w and T by:

$$w = v_r/R \qquad T = 2\pi R/v_r$$

We obtain the value:

$$l = 2\pi g R$$

This means that the arc length of a toroidal solenoid is equivalent to the length of the circumference of a ring of radius R '= gR

$$l = 2\pi R'$$

**Angular Momentum**

In the Helical Model of the Electron [1], we postulated that the angular momentum is always equal to the reduced Planck constant.

$$L = mRv_r = \hbar$$

The radius R is an invariant factor of the electron and coincides with the reduced compton wavelength. For an electron at rest (v = 0) with a toroid negligible thickness (r = 0), the velocity of rotation is equal to the speed of light.

In the Toroidal Solenoid Model of the electron, we must take into account the helical g-factor. If the velocity is reduced by a factor "g", the equivalent radius must be increased by the same factor "g". This coincides with the value of the arc length previously calculated

$$\begin{aligned} \hbar &= mR'v_r \\ \hbar &= m(gR)(c/g) \\ R &= \hbar/mc \end{aligned}$$

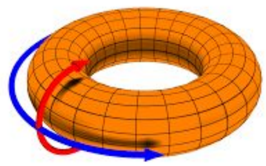
The value of the velocity of rotation is reduced in the same proportion that the equivalent radius, so the angular momentum is constant.

If we extend the model for v> 0, the velocity of rotation is reduced by both the g-factor and the Lorentz factor. The equivalent radius compensates the g-factor while the mass increasing compensates the Lorentz factor, so the angular momentum equal to the constant reduced Planck again.

$$
\begin{aligned}
\hbar &= m'R'v_r \\
\hbar &= (\gamma m)(gR)(c/g\gamma) \\
\hbar &= mRc
\end{aligned}
$$

**Magnetic moment**

The electric current flowing through a Toroidal Solenoid has two components, a toroidal component (red) and a poloidal component (blue).



By symmetry, the magnetic moment due to the poloidal components (red) are canceled, while the toroidal component (blue) remains fixed. No matter how large the number of turns in the toroidal solenoid, there always exist a toroidal component that generates a corresponding axial magnetic moment. This effect is known in the design of toroidal antennas and can be canceled by using various techniques.

This axial magnetic moment is calculated in **ANNEX B** to this work and in [3]. The exact value of the axial magnetic moment is:

$$m = I\pi R^2[1 + \frac{1}{2}(\frac{r}{R})^2]$$

If we compare the Toroidal Solenoid Electron Model (v=0, r>0) with the Ring Electron Model (v=0, r=0), the radius still coincide with the reduced Compton length, while the electric current is slightly lower, since as we calculated earlier, the speed of rotation of the electron is also slightly lower.

$$I\pi R^2 = (ef)\frac{2\pi R^2}{2} = \frac{ewR^2}{2} = \frac{ev_r R}{2} = \frac{e}{2}(\frac{c}{g})\frac{\hbar}{mc} = \frac{e\hbar}{2mg} = \frac{\mu_B}{g}$$

$$m = \frac{\mu_B}{g}(1 + \frac{1}{2}(\frac{r}{R})^2)$$

In the calculation of the angular momentum, the rotation speed decreases in the same proportion as the equivalent radius increases, so the g-factor is compensated. However, in the calculation of magnetic moment, the speed of rotation decreases by a factor g, while the equivalent radius increases by a factor approximately equal to g squared.

The g-factor of the magnetic moment is slightly different of the helical g-factor. However, we can make this approach:

$$\frac{1+\frac{1}{2}(\frac{r}{R})^2}{\sqrt{1+(\frac{rN}{R})^2}} \simeq \sqrt{1+(\frac{rN}{R})^2} = g.factor$$

And we obtain a value of the magnetic moment of the electron approximately equal to one Bohr magneton multiplied by the helical g-factor

$$m \simeq g\mu_B$$

**Quantitative calculation of the g-factor**

The value of R is the reduced compton wavelength of the electron. However, the value of the helical g-factor depends on two other hidden variables, whose values (r and N) are unknown. Assuming R>>rN:

$$gfactor = \sqrt{(1+(rN/R)^2}$$

Using this expansion serie:

$$\sqrt{1+(a)^2} = 1 + 1/2(a)^2 + ...$$

The helical g-factor can be expressed as :

$$\sqrt{1+(\frac{rN}{R})^2} = 1 + \frac{1}{2}(\frac{rN}{R})^2 + ...$$

The QED also calculates the g-factor by an expansion serie where the first term is 1 and the second term is the Schwinger factor:

$$g.factor(QED) = 1 + \frac{\alpha}{2\pi} + ...$$

The result of the two series are very similar. Equaling the second term of the helical g-factor serie to the Schwinger factor, we obtain the relationship between the radius of the torus and the thickness of the torus:

$$\frac{1}{2}(\frac{rN}{R})^2 = \frac{\alpha}{2\pi}$$

$$\frac{rN}{R} = \sqrt{\alpha/\pi}$$

What gives a value of helical g-factor of:

$$g = \sqrt{1 + \alpha/\pi}$$

This gives us a value of the **g-factor =1.0011607.** This result is as simple as the Schwinger factor and it offers a value much closer to experimental value.

**History of QED**

Quantum Electrodynamics (QED) is considered the most accurate theory of the history of physics. All the prestige of this theory is based on the accuracy of calculating the g-factor of the electron. Therefore, it seems absurd to suggest an alternative theory to the QED for calculating the g-factor. However, the history QED is not as successful as its creators want we believe it.

The history of QED is not explained as actually happened. This is the true history:

In 1928 Dirac published his famous equation of the electron. This equation was considered the definitive equation of the electron according to quantum mechanics. This equation predicts a value of the magnetic moment of the electron exactly equal to a "Bohr magneton". For 20 years the equation worked perfectly.

But in 1947, a series of experiments on magnetic resonance made by I. Rabi showed experimental data that did not match to the theoretical data. To explain these discrepancies, G. Breit [5] proposed that the magnetic moment of the electron was not exactly a Bohr magneton, but a slightly higher value. This anomalous magnetic moment of the electron was called g-factor. Kush and Foley [6] obtained an experimental value of g-factor of 1,00119.

Immediately, theoretical physicists tried to find an explanation for this anomaly. In 1948, Julian Schwinger [7] proposed that the cause of this anomaly was not in the Dirac equation but in the interaction of the electron with the quantum fluctuations of the vacuum. He made a calculation of these interactions and he obtained the value known as "Schwinger factor”:

$$gfactor = \alpha/2\pi$$

The resulting value is 1,00116, which match to the experimental value obtained by Kush and Foley

In 1949, Gardner and Purcell [8] obtained a more precise experimental result of 1,001146. With this experimental value, the Schwinger factor was not sufficiently accurate, so the theory had to be expanded.

Then Feynman appeared and he proposed that Schwinger factor was only the first factor of a serie. The calculation of each factor in the serie required the resolution of an exponential number of extremely complex equations called "Feynman's diagrams". The main problem was that the calculation of these diagrams diverges to infinity. With the application of several mathematical tricks named "renormalization", they succeeded in eliminating these "infinite values" and achieve concrete results for diagrams. QED was born.

To confirm this theory, 2 researchers (Karplus and Kroll [9]) were commissioned to do the calculation of the second coefficient of the serie. It took 1 year to calculate the 7 necessary Feynman diagrams. In 1950 they published their work obtaining a value of "2,973", implying a g-factor value of 1,0011454, very close to the experimental value obtained by Gardner and Purcell.

As the calculation had been carried by two teams independently and they had obtained the same result in both cases, it was impossible for any errors in the calculation. Neither it was possible to imagine that the result could be obtained by chance. It was the ultimate test. QED had triumphed. Along the way, they had to give up the logic in physics, and even they had dispensed with the rigorous mathematics [14]. But it did not matter, the theoretical calculations matched the experimental data with unprecedented accuracy in history. There was no more to say.

However, in 1956, Franken and Liebes [10] published a new experimental data showing a very different g-factor value (1,001165), implying a value of the second coefficient equal to "0,7". The difference between the "2,973" and "0,7" was huge and unjustifiable, so that all the probative value of the QED turned against it. Necessarily the QED theory had to be wrong.

Quickly, surprising facts began to appear. First, the confession of Karplus and Kroll that they had not reached the same result independently, but they had reached a consensus outcome and errors in calculation could exist. Then, Petermann [11] detected an arithmetic error in the calculations of Karplus and Kroll (no one had detected it in the 8 years that the article was published). Finally, the correct calculation offered a result of "0,328", almost 10 times less than the previous estimate value of "2,973". This new theoretical value of the g-factor (1,0011596) matched with the new experimental value (1,001165).

“Miraculously” QED had been saved and their creators (Feynman, Schwinger and Tomonaga) were awarded with the Nobel prize in Physics in 1965.

In 1961, Schupp, Pidd and Crane [12] obtained an experimental value of 1,0011609. Finally, in 1963, Wilkinson and Crane [13] published a new experimental value of 1,0011596, exactly the same value that the theoretical value of Peterman. Thereafter, the experimental values have been adjusted to the theoretical values with unprecedented accuracy in the history of physics.

But such accuracy is suspect. For example, in 1999 Lautrup and Zinkernagely [15] raised serious doubts about the confidence in the experimental values of the g-factor, because in his view they were heavily influenced by the expected theoretical values. In examining the explanation of the experimental work it gives the impression that, consciously or unconsciously, measuring devices are calibrated to obtain the theoretical value calculated by QED. This error is known as "Experimenter's bias" and is much more common in the history of modern physics than might be expected, as indicated in [16].

## Conclusions

As we have shown by reviewing the history, we can not rely on the theoretical calculations of the QED nor on the current experimental values. The last reliable experimental value of the g-factor is that performed by Schupp, Pidd and Crane [12] in 1961. Thereafter, all experimental values are suspect of an "Experimenter's bias" with the only aim to validate the theoretical values of QED.

On the other hand, in this work we have shown a semiclassic model of the electron with a geometry of a Helicoidal Solenoid, in which the g-factor appears naturally due to the electron's own geometry. Forcing a quantitative value for the g-factor, we obtain a simple expression and an exact value much closer to the experimental value of 1961 than the Schwinger factor and the Petermann g-factor.

| Author | g-factor | Value | Error |
|---|---|---|---|
| Schupp, Pidd & Crane | The last trusted experimental value (1961) | 1,0011609 | -- |
| Schwinger | $1 + \alpha/2\pi$ | 1,0011614 | 15 ppm |
| Petermann | $1 + \alpha/2\pi - 0,328(\alpha/\pi)^2$ | 1,0011596 | 13 ppm |
| Solenoid Helical Electron | $\sqrt{1 + \alpha/\pi}$ | 1,0011607 | 2 ppm |

## References

[1] Oliver Consa, "Helical Model of the Electron". viXra: 1408.0203 (2014)

[2] Bostick, Winston H., "Mass, Charge and Current: The Essence and Morphology," Physics Essays, Vol 4, No. 1, pp.. 45-49 (1991)

[3] Charles W. Lucas, Jr. "A Classical Theory of Elementary Particles Electromagnetic Part 2, intertwining Charge-Fibers," The Journal of Common Sense Science Foundation of Science, Vol. 8 No. 2 . (2005)

[4] Marinov, Boardman, Fedotov, Zheludev "Metamaterial Toroidal". New Journal of Physics 9 (2007)

[5] G. Breit, Does the Electron Have an Intrinsic Magnetic Moment ?. Phys. Rev. 72, 984 (1947)

[6] P. Kusch, Foley HM Phys. Rev. 74 (1948)

[7] Schwinger, "On-electrodynamics and the quantum magnetic moment of the electron," Physical Review, 73 (1948)

[8] J. H. Gardner and EM Purcell, Phys. Rev. 76 (1949)

[9] Karplus and Kroll, "Fourth-order corrections in quantum electrodynamics and the magnetic moment of the electron," Physical Review, 77 (1950)

[10] P . S. Franken and Liebes, "Magnetic Moment of the Proton in Bohr magnetons" Phys. Rev. 104 (1956)

[11] A. Petermann, "Fourth order magnetic moment of the electron," Helvetica Physica Acta, 30 (1957)

[12] AA Schupp, RW Pidd, and HR Crane. "Measurement of the g factor of Free, High-Energy Electrons". Phys. Rev. 121 (1961)

[13] DT Wilkinson and HR Crane. "Precision Measurement of the g factor of the Free Electron". Phys Rev. 130 (1963)(1):.

[14] Youhei Tsubono, “QED anomalous magnetic moment (2-loop) is fake". http://www7b.biglobe.ne.jp/~kcy05t/trigf.html

[15] B. Lautrup and H. Zinkernagely, “g-2 and the trust in experimental results”  Studies in History and Philosophy of Science Part B 30 (1):85-110 (1999)

[16] Jeng, M. “A selected history of expectation bias in physic”, American Journal of Physics, 74: 578-583 (2006)

**ANNEX A: Tangential velocity**

We can parameterize the electron trajectory with the equation of the helical solenoid, which is as follows:

$$\begin{aligned} x(t) &= (R + rcosNwt)coswt \\ y(t) &= (R + rcosNwt)sinwt \\ z(t) &= rsinNwt + vt \end{aligned}$$

Differentiating the above equations we obtain the equation of the tangential velocity

$$\begin{aligned} x'(t) &= -(R + rcosNwt)wsinwt - rNwsinNwtcoswt \\ y'(t) &= (R + rcosNwt)wcoswt - rNwsinNwtsinwt \\ z'(t) &= rNwcosNwt + v \end{aligned}$$

The module of the tangential velocity:

$$|r'(t)|^2 = x'(t)^2 + y'(t)^2 + z'(t)^2$$

Substituting the values and rearranging terms we obtain:

$$\begin{aligned} |r'(t)|^2 &= (R + rcosNwt)^2 w^2 sin^2 wt + (rNw)^2 sin^2 Nwtcos^2 wt \\ &+ 2(R + rcosNwt)wsinwt(rNw)sinNwtcoswt \\ &+ (R + rcosNwt)^2 w^2 cos^2 wt + (rNw)^2 sin^2 Nwtsin^2 wt \\ &- 2(R + rcosNwt)wcoswt(rNw)sinNwtsinwt \\ &+ v^2 + 2vrNwcosNwt + (rNw)^2 cos^2 Nwt \end{aligned}$$

$$\begin{aligned} |r'(t)|^2 &= (R + rcosNwt)^2 w^2 (sin^2 wt + cos^2 wt) + \\ &+ (rNw)^2 sin^2 Nwt(sin^2 wt + cos^2 wt) \\ &+ v^2 + 2vrNwcosNwt + (rNw)^2 cos^2 Nwt \end{aligned}$$

$$\begin{aligned} |r'(t)|^2 &= (R + rcosNwt)^2 w^2 + (rNw)^2 sin^2 Nwt + (rNw)^2 cos^2 Nwt \\ &+ v^2 + 2vrNwcosNwt \end{aligned}$$

Finally we obtain the equation of the tangential velocity of a Helical Solenoid:

$$|r'(t)|^2 = (R + rcosNwt)^2 w^2 + (rNw)^2 + v^2 + 2vrNwcosNwt$$

**ANNEX B: Magnetic Moment**

According to the definition of magnetic moment

$$m = \frac{1}{2}\int r\text{x}j\,dV = \frac{I}{2}\int_0^{2\pi} r\text{x}\frac{dr}{d\varphi}d\varphi$$

We define “P” as the vector to be integrated :

$$P = r\text{x}\frac{dr}{d\varphi}d\varphi =< P_x, P_y, P_z >$$

By symmetry, the components <x> and <y> of the integral of the vector are zero, and only would the component z, perpendicular to the head circumference vector.

$$\int_0^{2\pi} P_x = \int_0^{2\pi} P_y = 0$$

To calculate the z vector component, we make a change of variable

$$\varphi = wt$$

We derive the three components of the vector

$$\begin{aligned} x(t) &= (R + rcosN\varphi)cos\varphi \\ y(t) &= (R + rcosN\varphi)sin\varphi \\ z(t) &= rsinN\varphi \end{aligned}$$

$$\begin{aligned} x'(t) &= -(R + rcosN\varphi)sin\varphi - rNsinN\varphi cos\varphi \\ y'(t) &= (R + rcosN\varphi)cos\varphi - rNsinN\varphi sin\varphi \\ z'(t) &= rNcosN\varphi \end{aligned}$$

The z component of the vector is:

$$P_z = xy' - x'y$$

$$\begin{aligned} P_z &= (R + rcosN\varphi)cos\varphi[(R + rcosN\varphi)cos\varphi - rNsinN\varphi cos\varphi] \\ &+ [(R + rcosN\varphi)sin\varphi + rNsinN\varphi cos\varphi](R + rcosN\varphi)sin\varphi \end{aligned}$$

$$P_z = (R + rcosN\varphi)[(R + rcosN\varphi)cos\varphi cos\varphi + sin\varphi(R + rcosN\varphi)sin\varphi]$$

$$P_z = (R + rcosN\varphi)^2[cos\varphi cos\varphi + sin\varphi sin\varphi]$$

$$P_z = (R + rcosN\varphi)^2$$

Therefore we have to solve a simplified integral

$$m = \frac{I}{2}\int_0^{2\pi}(R + rcosN\varphi)^2 d\varphi$$

$$m = \frac{I}{2}\int_0^{2\pi}(R^2 + 2RrcosN\varphi + r^2cos^2N\varphi)d\varphi$$

$$m = \frac{I}{2}[R^2\varphi|_0^{2\pi} + \frac{2RrNsen\varphi}{N}|_0^{2\pi} + r^2(\frac{N\varphi}{2N}|_0^{2\pi} + \frac{4sen2N\varphi}{4N}|_0^{2\pi})]$$

$$m = \frac{I}{2}[R^2 2\pi + r^2\pi]$$

And we get the expected result:

$$m = I\pi R^2[1 + \frac{1}{2}(\frac{r}{R})^2]$$

This result coincides with that used by Marinov and Boardman in his article "Toroidal Metamaterial" [4].