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Magnetic Charge

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ABSTRACT: This article aims to clarify some misinterpretation about the magnetic field and magnetic induction that originated an inconsistent magnetic flux equation. It will be demonstrated that what we actually call magnetic flux is really the magnetic charge and, as consequence, demonstrate the correct equations for magnetic flux and induced magnetic scalar potential and show the extended equation for the induced electric potential.

KEYWORDS: magnetic flux, magnetic charge, magnetic current, magnetic potential.

Contents

1 Symbology

In this text we will use the following symbols with its abbreviated unit of measurements:

N = Newton, m = meter, s = second, V = Volt, C = Coulomb, A = Ampere, Wb = Weber.

E = Electric field intensity [N C^{-1}] [V m^{-1}];
D = Surface density of electric charge [C m^{-2}];
V_E = Electric potential [V] [Wb s^{-1}];
Φ_E = Electric flux [N m^2 C^{-1}] [V m];
q_E = Electric charge [C];
I_E = Electric current [C s^{-1}] [A];
J_E = Surface density of electric current [A m^{-2}] [C s^{-1} m^{-2}];
ε_0 = Vacuum electric permittivity [C^2 N^{-1} m^{-2}] [C V^{-1} m^{-1}];
H = Magnetic field intensity [N Wb^{-1}] [A m^{-1}];
B = Surface density of magnetic charge [Wb m^{-2}];
V_M = Magnetic potential [A] [C s^{-1}];
Φ_M = Magnetic flux [N m^2 Wb^{-1}] [A m];
q_M = Magnetic charge [Wb];

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I_M = Magnetic current [Wb s^{-1}] [V];
J_M = Surface density of magnetic current [V m^{-2}] [Wb s^{-1} m^{-2}];
μ_0 = Vacuum magnetic permeability [Wb^2 N^{-1} m^{-2}] [Wb A^{-1} m^{-1}];
l = Length [m];
S = Area [m^2];
t = Time [s].

2 Introduction

Before the internationalization of measurements, there was a profusion of unities in electromagnetic quantities. Thanks to that, some electromagnetic theory books still call **B** magnetic field and **H** stays like an auxiliary field, without giving importance to the units of measurement. Today, with the establishment of the International System of Units – SI, we know that **B** do not have a field unit of measurement, but **H** has it; the unit of measurement of **B** is similar to **D**, that has a surface density of electric charge unit of measurement.

Knowing this, we have to change the misconception that **B** is a magnetic field and put **H** in its place. Then **B** has a surface density of [Wb] unit of measurement, which is a surface density of magnetic charge. With this in mind and the SI system, it may be established the following:

1. The electric field **E** has unit of measurement [V m^{-1}]: it is measured by an electric potential, with unit of measurement Volt [V] over a distance, with unit of measurement meter [m].
2. The electric induction **D** has unit of measurement [C m^{-2}]: it is measured by an electric charge, with unit of measurement Coulomb [C], over an area, with unit of measurement $meter^2$ [m^2]. The correct description of **D** is surface density of electric charge.
3. The magnetic field **H** has unit of measurement [A m^{-1}]: it is measured by a magnetic potential, with unit of measurement Ampere [A], over a distance, with unit of measurement meter [m].
4. The magnetic induction **B** has unit of measurement [Wb m^{-2}]: it is measured by a magnetic charge, with unit of measurement Weber [Wb], over an area, with unit of measurement $meter^2$ [m^2]. The correct description of **B** is surface density of magnetic charge.

In the vacuum of space, there are simple relations between electric/magnetic fields and electric/magnetic induction:

$$\vec{D}=\varepsilon_0 \vec{E} \qquad \vec{B}=\mu_0 \vec{H}$$

The third and fourth items actually seems no sense because we do not recognize magnetic potential like a scalar neither magnetic charge. But we will see below that this is the correct form and what this implies.

3 Electric Flux and Electric Charge

The Gauss's law establishes the electric flux Φ_E in space that pass through a closed surface S that surrounds an electric charge:[1]

$$\Phi_E=\oint_S \vec{E}\cdot d\vec{S}=\frac{q_E}{\varepsilon_0} \qquad q_E=\varepsilon_0\Phi_E=\varepsilon_0\oint_S \vec{E}\cdot d\vec{S}=\oint_S \vec{D}\cdot d\vec{S}$$

When the integral is over an open area, the electric flux indicates the quantity of lines of electric force field that pass through the area. This is, a limited area that has a surface density of electric charge has a total quantity of charge that may be equated by:

$$\Phi_E = \int_S \vec{E} \cdot d\vec{S} \qquad q_E = \varepsilon_0 \Phi_E = \varepsilon_0 \int_S \vec{E} \cdot d\vec{S} = \int_S \vec{D} \cdot d\vec{S}$$

This indicates that the electric induction **D** is really a surface density of electric charge, confirmed by its unit of measurement [C m^{-2}].

4 Magnetic Flux and Magnetic Charge

Actually, the Gauss' law for the magnetic flux φ establishes that:

$$\varphi = \oint_S \vec{B} \cdot d\vec{S} = 0 \qquad \varphi = \int_S \vec{B} \cdot d\vec{S} = \mu_0 \int_S \vec{H} \cdot d\vec{S}$$

Here we have used the inconsistent magnetic flux φ that has unit of measurement [Wb] and has nothing in common with his electric counterpart Φ_E that has unit of measurement [N m^2 C^{-1}] [V m]. When the integral is over an open area, the magnetic flux indicates the quantity of lines of force field that pass through the area. There is no relation with magnetic charge.

But by analogy with the electric flux, we have to equate the real magnetic flux Φ_M with the magnetic field **H** and not with the magnetic induction **B**. The unit of measurement of magnetic field **H** is Ampere by meter, like its electric field counterpart **E**, that has unit of measurement Volt by meter. Than, the correct equation for magnetic flux is:

$$\Phi_M = \int_S \vec{H} \cdot d\vec{S}$$

This way, the unit of measurement of magnetic flux Φ_M is Ampere * meter, that is analog to its electric flux counterpart Φ_E with unit of measurement Volt * meter. With this correct analogy we equate the magnetic charge with the actual inconsistent magnetic flux equation, that has Weber as unit of measurement:

$$q_M = \int_S \vec{B} \cdot d\vec{S} = \mu_0 \int_S \vec{H} \cdot d\vec{S} = \mu_0 \Phi_M$$

We may see that this is the correct equation because the magnetic induction **B** is really a surface density of magnetic charge with unit of measurement [Wb m^{-2}]; its integral over an area gives the quantity of magnetic charge [Wb] superficially distributed on that area.

Actually, our instruments (gaussimeters and teslameters) do not measure magnetic field **H** nor magnetic flux Φ_M, but surface density of magnetic charge **B** by the Hall effect. Knowing **B** we may get **H** with the magnetic permeability μ of the material with $\vec{B} = \mu H$ equation.

5 Magnetic Current

The electric current is the motion of electric charges with unit of measurement [C s^{-1}], so the magnetic current is the motion of magnetic charges and its unit of measurement is [Wb s^{-1}]. It defines the quantity of magnetic charges that passes by unit of time like a conduction current for magnetostatic fields on magnetic conductors. But we may define a magnetic current by the magnetic flux variation by unit of time like a displacement current for magnetodynamic fields on magnetic isolators:

$$I_M = \int_S \vec{J}_M \cdot d\vec{S} = \frac{d q_M}{d t} \qquad I_M = \frac{d}{d t} \int_S \vec{B} \cdot d\vec{S} = \mu_0 \frac{d}{d t} \int_S \vec{H} \cdot d\vec{S} = \mu_0 \frac{d \Phi_M}{d t}$$

So far we have demonstrated that what is actually called magnetic flux is really the magnetic charge, then Faraday's induction law, that establishes the electric motive force (electric potential) V_E induced by a time variation of the inconsistent magnetic flux φ, must be equated by the time variation of both the real magnetic flux and the magnetic charge because $\varphi = q_M = \mu_0 \Phi_M$.

$$V_E = -\frac{d\varphi}{dt} = -I_M - \mu_0 \frac{d\Phi_M}{dt}$$

In which situations does it occur one or the other?

1. Magnetic current or charge variation:
   It occurs when we pass a magnetized bar through a bobbin because the face of the bar may be considered like a surface distribution of magnetic charge. It occurs inside a transformer nucleus: the variation in polarization angle of the magnetic dipoles of the ferromagnetic material, when submitted to the external magnetic field created by the primary bobbin, is like a magnetic current.
2. Magnetic flux variation:
   Occurs in the iron gap of an electromagnetic transformer: when the flux inside the nucleus varies, the surface charge of the ferromagnetic nucleus faces changes and this causes a magnetic flux variation in the isolating air gap. This is like an electric capacitor submitted to a variable electric current: the faces on the electric isolating material changes its surface density of electrical charge and causes an electric flux variation.

We may establish that the integral of the electric field around a closed line that encircle an area passed through by a magnetic current is equal to the magnetic current enclosed by the line. Then, the Faraday's law may be extended to include two forms of magnetic current: conduction and displacement. The complete equations in integral and differential forms are:

$$\oint_L \vec{E} \cdot d\vec{l} = I_M + \mu_0 \frac{d\Phi_M}{dt} \qquad \nabla \times \vec{E} = \vec{J}_M + \mu_0 \frac{\partial \vec{H}}{\partial t} = \vec{J}_M + \frac{\partial \vec{B}}{\partial t} \qquad V_E = -\oint_L \vec{E} \cdot d\vec{l} \qquad \vec{E} = -\nabla V_E$$

These equations tell us that the unit of measurement of the magnetic current [Wb s^{-1}] is the same as the electric potential [V]. This is analogous to its electric current counterpart I_E, with unit of measurement [C s^{-1}], that is the same as the magnetic potential [A].

In this extended form, Faraday's electric induction law is similar to Ampère-Maxwell's magnetic induction law and it show us that there is magnetic current because there is magnetic charge. This does not tell us that magnetic monopole exists:

- A magnetic field is produced by polarizing in the same direction the dipoles in a magnetic material; each face has a magnetic pole defined by a surface distribution of magnetic charges.
- A magnetic current may be understood like a variation in the orientation angles of the magnetic dipoles in a magnetic material and the passing of a magnetized material inside an electric bobbin.

This equation is used in calculating the secondary circuits of electromagnetic transformers. In the nucleus of the transformer there is magnetic current that induces electric potential in each cooper turn of the secondary circuit, that are like multiples closed lines.

6 Magnetic Potential

Ampere's induction law establishes that the integral of the magnetic field around a closed line that encircle an area passed through by an electric current is equal to the electric current enclosed by the line. This is like a conduction current for electrostatic fields on electric conductors. [1]

$$\oint_L \vec{B}\cdot d\vec{l}=\mu_0 I_E \qquad \oint_L \vec{H}\cdot d\vec{l}=\int_S \vec{J}_E\cdot d\vec{S}=I_E=\frac{dq_E}{dt}$$

Maxwell extended this equation with the electric flux variation by unit of time like a displacement current for electrodynamic fields on electric isolators. The analogy with the electric potential lead us to define the magnetic motive force (magnetic potential) V_M induced by an electric current (conduction and displacement). The complete equation in integral and differential forms are:

$$\oint_L \vec{H}\cdot d\vec{l}=I_E+\varepsilon_0\frac{d\Phi_E}{dt} \qquad \nabla\times\vec{H}=\vec{J}_E+\varepsilon_0\frac{\partial\vec{E}}{\partial t}=\vec{J}_E+\frac{\partial\vec{D}}{\partial t} \qquad V_M=-\oint_L \vec{H}\cdot d\vec{l} \qquad \vec{H}=-\nabla V_M$$

This is the magnetic scalar potential; it is not like the magnetic vector potential but is the magnetic counterpart of the electric scalar potential. This equation is used in calculating the primary circuits of electromagnetic transformers. In the multiple cooper turns of the primary circulates an electric current that induces a magnetic potential in the nucleus of the transformer. This potential creates a magnetic current that circulates on the ferromagnetic nucleus and induces an electric potential on the multiple cooper turns of the secondary circuit.

When the integral of the magnetic field is over an open line, we obtain the magnetic potential between the initial and end points. This is used to calculate the magnetic potential induced by an electric current that flows in a bobbin with n turns of electric current i_E. The equation used for this is $Hl=ni_E=I_E$:

$$Hl=\int_a^b \vec{H}\cdot d\vec{l}=V_M=ni_E=I_E$$

We may confirm with this relation that the magnetic potential unit of measurement is the same as the electric current.

7 Magnetic Field and Magnetic Force

The magnetic field produced by an electromagnet actually is calculated simply knowing the total electric current that passes by an electric bobbin $ni_E=I_E$ and the path of the magnetic field, because I_E induces the magnetic potential V_M.

$$\vec{H}=\frac{ni_E}{l}=\frac{I_E}{l}=-\nabla V_M$$

One application of the equations developed here is the calculus of the attraction force between two magnetized bars or electromagnets. The actual equation $F=\mu_0 H^2 S$ consider two fields; but we may transform it to seems like its electric counterpart $\vec{F}=q_E\vec{E}$, that calculates a force between an electric charge and an electric field:

$$\vec{F}=\mu_0\int_S \vec{H}\cdot d\vec{S}*\vec{H}=\int_S \vec{B}\cdot d\vec{S}*\vec{H}=q_M\vec{H}$$

We may treat two magnetic fields like one of them being a surface distribution of magnetic charge and calculate the force between one field and the total magnetic charge in the same way we calculate a force between a surface distribution of electric charge and an electric field.

8 Conclusion

The magnetic field is really **H** [A m^{-1}] whose unit of measurement is magnetic potential V_M [A] by distance [m]. This is similar to electric field **E** [V m^{-1}] whose unit of measurement is electric potential V_E [V] by distance [m].

The unit of measurement of magnetic potential V_M [A] is the same as the electric current I_E [A]. The unit of measurement of electric potential V_E [V] is the same as the magnetic current I_M [V].

What we actually call magnetic flux φ is really the magnetic charge q_M. The Maxwell equations, with the Faraday' induction law, use $d\varphi/dt$ as time variation of the magnetic flux. This is inconsistent because φ is defined like a magnetic charge; the correct form would be the time variation of the real magnetic flux $\mu_0 d\Phi_M/dt$.

With these corrections, the equations for electric and magnetic quantities become similar. Because we did not find out the magnetic monopole, the field equations below are obtained by surface charge distributions.

Greatness	Electric		Magnetic	
	Equation	**Unit**	**Equation**	**Unit**
Field from a surface charge	$E(r)=\frac{1}{4\pi\varepsilon_0}\int_S \frac{\vec{D}\cdot d\vec{S}}{r^2}$	[N C^{-1}] [V m^{-1}]	$H(r)=\frac{1}{4\pi\mu_0}\int_S \frac{\vec{B}\cdot d\vec{S}}{r^2}$	[N Wb^{-1}] [A m^{-1}]
Field from a scalar potential	$\vec{E}=-\nabla V_E$	[N C^{-1}] [V m^{-1}]	$\vec{H}=-\nabla V_M$	[N Wb^{-1}] [A m^{-1}]
Surface charge density	$\vec{D}=\varepsilon_0\vec{E}$	[C m^{-2}]	$\vec{B}=\mu_0\vec{H}$	[Wb m^{-2}]
Flux	$\Phi_E=\int_S \vec{E}\cdot d\vec{S}$	[N m^2 C^{-1}] [V m]	$\Phi_M=\int_S \vec{H}\cdot d\vec{S}$	[N m^2 Wb^{-1}] [A m]
Charge	$q_E=\int_S \vec{D}\cdot d\vec{S}=\varepsilon_0\Phi_E$	[C]	$q_M=\int_S \vec{B}\cdot d\vec{S}=\mu_0\Phi_M$	[Wb]
Current	$I_E=\frac{dq_E}{dt}=\varepsilon_0\frac{d\Phi_E}{dt}$	[C s^{-1}] [A]	$I_M=\frac{dq_M}{dt}=\mu_0\frac{d\Phi_M}{dt}$	[Wb s^{-1}] [V]
Scalar potential	$V_E=-\int_L \vec{E}\cdot d\vec{l}$	[V] [Wb s^{-1}]	$V_M=-\int_L \vec{H}\cdot d\vec{l}$	[A] [C s^{-1}]

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