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Potential Energy and Radiation for an Accelerated Electron in Three Systems of Electrodynamics

Abstract

The potential energy of an electron accelerated by an electric field is given in classical, relativistic and an alternative electrodynamics. In the alternative electrodynamics, mass of a moving electron remains constant and change in its potential energy is not equal to the change in kinetic energy due to energy radiation. At the speed of light the radiation reaction force is equal to the accelerating force and the electron continues to move at that speed as a limit. An electron, moving at the speed of light, is easily decelerated to a stop may then be accelerated backwards to the speed of light.

Keywords: acceleration, electrodynamics, energy, radiation, relativity

1 Introduction

In classical electrodynamics [1, 2], the mass of a particle is independent of its speed and a charged particle, such as an electron, can be accelerated, by an electric field, beyond the speed of light. But observations on accelerated electrons, the lightest particles known in nature, showed that their speeds could not exceed that of light. Relativistic electrodynamics [3, 4] and an alternative electrodynamics [5] deal with the issues that restrain accelerated charged particles from going beyond the speed of light.

Relativistic electrodynamics explains the speed of light being a limit by positing that the mass m of a moving particle increases with its speed v, becoming infinitely large at the speed of light c. The mass-velocity formula, of special relativity, is:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_o \tag{1}$$

where m_o is the rest mass. In this equation (1), the difficulty with infinite masses, at the speed of light is avoided by insisting that the speed v may be as near as possible, but it never really becomes equal to c. Photons, as a stream of “particles”, supposed to move at the speed of light, are given zero rest mass.

An alternative electrodynamics proposes that the speed of light c is an ultimate limit because the accelerating force exerted by an electric field, on a moving electron, decreases with speed of the electron. The accelerating force reduces to zero at the speed of light and the electron continues to move at that speed as a limit.

In rectilinear motion, it was found that electrons cannot be accelerated beyond the speed of light, no matter the magnitude of the accelerating potential in an accelerator [6]. The existence of a limiting speed, equal to the speed of light, was clearly demonstrated in Bertozzi's experiment (1964) [7]. Here, again, the limiting speed was attributed to mass increasing with speed, becoming infinitely large at the speed of light, as per the relativistic equation (1). An accelerating force decreasing with speed, becoming zero at the speed of light, should also lead to that speed being an ultimate limit.

2 Rectilinear Motion in Classical electrodynamics

2.1 Potential energy lost by an accelerated electron

In classical electrodynamics, the accelerating force $\boldsymbol{F}$ on an electron of charge $-e$ and constant mass m, moving with velocity $\boldsymbol{v}$ and acceleration dv/dt at time t, in an electrostatic field of intensity $\boldsymbol{E}$, is given, in accordance with Coulomb's law of electrostatic force and Newton's second law of motion, by the vector equation:

$$\mathbf{F} = -e\mathbf{E} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{2}$$

For rectilinear motion, in the direction of a displacement $\boldsymbol{x}$, equation (2), with E as the magnitude of $\boldsymbol{E}$, becomes:

$$-eE\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} = -mv\frac{dv}{dx}\hat{\mathbf{u}} \tag{3}$$

where $\hat{\boldsymbol{u}}$ is a unit vector in the direction of the electrostatic field $\boldsymbol{E}$ and the displacement $\boldsymbol{x}$. The scalar equation is:

$$eE = mv\frac{dv}{dx} \tag{4}$$

The potential energy P lost by the moving electron or work done on the electron, in being accelerated with constant mass m, through a distance x from an origin $(x = 0)$, to a speed v from rest, is given by the definite integral:

$$P = \int_0^x eE(dx) = m\int_0^v v(dv) \tag{5}$$

Integrating, equation (5) gives:

$$\int_0^x eE\left(dx\right) = P = \frac{1}{2}mv^2$$

This is equal to the kinetic energy of the electron.

$$\frac{P}{mc^2} = \frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{6}$$

Here, with no consideration of energy radiation, the potential energy P lost is equal to the kinetic energy gained

2.2 Potential energy gained by a decelerated electron

In classical electrodynamics, an electron, moving at the speed of light c, can be decelerated to a stop and may be accelerated in the opposite direction to reach a speed greater than $-c$. The potential energy P gained in decelerating an electron from the speed of light c to a speed v, within a distance x in a field E, is:

$$-\int_0^x eE(dx) = P = \frac{1}{2}m(c^2 - v^2)$$

$$\frac{P}{mc^2} = \frac{1}{2}\left(1 - \frac{v^2}{c^2}\right) \qquad (7)$$

3 Rectilinear Motion in Relativistic electrodynamics

3.1 Potential energy lost by an accelerated electron

In relativistic electrodynamics, the kinetic energy K gained by an electron or the work done, in being accelerated by an electric field E, through a distance x, to a speed v from rest, is the potential energy P lost. The kinetic energy K of a particle of mass m and rest mass m_o moving with speed v is given by the relativistic equation:

$$\int_0^x eE(dx) = K = P = mc^2 - m_o c^2$$

$$\int_0^x eE(dx) = P = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_o c^2$$

$$\frac{P}{m_o c^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \qquad (11)$$

where m_o is the rest mass (at $v = 0$) and c the speed of light in a vacuum. Bertozzi's experiment was conducted to verify equation (11) and it did so in a remarkable way.

3.2 Potential energy gained by a decelerated electron

In relativistic electrodynamics, an electron moving at the speed of light c (with infinite mass), cannot be stopped by any decelerating force. The electron continues to move at the same speed of light c, gaining potential energy without losing kinetic energy, contrary to the principle of conservation of energy.

4 Rectilinear Motion in an Alternative electrodynamics

4.1 Motion of an electron in an electric field

Figure 1 depicts an electron of charge $-e$ and constant mass $m = m_o$, moving at a point P with velocity $\boldsymbol{v}$ at time t, in an electric field of intensity $\boldsymbol{E}$ due to a stationary source charge $+Q$ at the origin O. The velocity $\boldsymbol{v}$ is at an angle θ to the accelerating force $\boldsymbol{F}$, which is a force of attraction in the $\boldsymbol{PO}$ direction. The relative velocity between the accelerating force (which is propagated with velocity of light $\boldsymbol{c}$) and the electron moving with velocity $\mathbf{v}$, is $(\boldsymbol{c} - \boldsymbol{v})$. The velocity of light $\boldsymbol{c}$ is inclined at an aberration angle α to the accelerating force $\boldsymbol{F}$, such that
:

$$\sin\alpha = \frac{v}{c}\sin\theta \qquad (12)$$

where v and c are the magnitudes of the velocities $\boldsymbol{v}$ and $\boldsymbol{c}$ respectively.

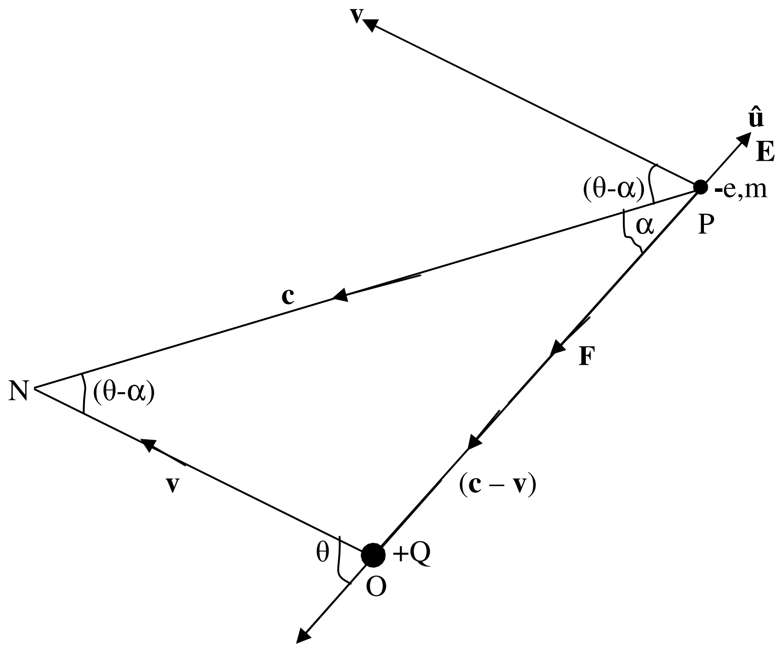

Figure 1. An electron of charge $-e$ and mass m moving, at a point P, with velocity $\boldsymbol{v}$, at an angle θ to the accelerating force $\boldsymbol{F}$. The unit vector $\hat{\boldsymbol{u}}$ is in the direction of the electric field $\mathbf{E}$ due to a positive charge Q at O.

In the alternative electrodynamics, the accelerating force $\boldsymbol{F}$, with reference to Figure 1, is given by the vector equation:

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c} - \mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{13}$$

where E is the magnitude of the electrostatic field of intensity $\boldsymbol{E}$.

Expanding equation (13) by taking the *modulus* of $(\boldsymbol{c} - \boldsymbol{v})$, with respect to the angles θ and α in Figure 1, gives the equation:

$$\mathbf{F} = \frac{-eE}{c}\sqrt{c^2 + v^2 - 2cv\{\cos(\theta - \alpha)\}}\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt} \tag{14}$$

where $(\theta - \alpha)$ is the angle between the vectors $\boldsymbol{c}$ and $\boldsymbol{v}$ and $\hat{\mathbf{u}}$ is a unit vector in the direction of the field $\boldsymbol{E}$**,** opposite to the direction of $(\boldsymbol{c} - \boldsymbol{v})$. The electron can move in a straight line, in the direction of the force, with acceleration where $\theta = 0$ or against the force with deceleration where $\theta = \pi$ radians or it can revolve in a circle, with constant speed v, if $\theta = \pi/2$ radians.

4.2 Potential energy lost by an accelerated electron

For an electron accelerated in a straight line, equations (12) and (14), with $\theta = 0$, give:

$$\mathbf{F} = -eE\left(1 - \frac{v}{c}\right)\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} \tag{15}$$

The scalar equation is:

$$eE\left(1 - \frac{v}{c}\right) = m\frac{dv}{dt} = mv\frac{dv}{dx} \tag{16}$$

The potential energy P lost in accelerating the electron, through a distance x, to a speed v from rest, is given by the integral:

$$P = \int_0^x eE(dx) = \int_0^v mv \frac{dv}{1-\frac{v}{c}} \tag{17}$$

Resolving the right-hand integral into partial fractions, we obtain:

$$P = mc\int_0^v \left(\frac{1}{1-\frac{v}{c}} - 1 \right) dv \tag{18}$$

$$P = -mc^2 \ln\left(1-\frac{v}{c}\right) - mcv \tag{19}$$

$$\frac{p}{mc^2} = -\ln\left(1-\frac{v}{c}\right) - \frac{v}{c} \tag{20}$$

Equation (20) for the alternative electrodynamics, should be compared with equation (11) for relativistic electrodynamics and equation (6) for classical electrodynamics.

4.3 Potential energy gained by a decelerated electron

For a decelerated electron, equations (12) and (14), with $\theta = \pi$ radians, give:

$$\mathbf{F} = -eE\left(1+\frac{v}{c}\right)\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}} \tag{21}$$

$$eE\left(1+\frac{v}{c}\right) = -m\frac{dv}{dt} = -mv\frac{dv}{dx} \tag{22}$$

Potential energy P gained in decelerating the electron through a distance x, from speed of light c to v, is:

$$P = -\int_0^x eE(dx) = \int_c^v -mv\frac{dv}{1+\frac{v}{c}} \tag{23}$$

Resolving the integrand into partial fractions and integrating, we obtain:

$$P = -mc\int_c^v \left(1 - \frac{1}{1+\frac{v}{c}} \right) dv \tag{24}$$

$$P = mc^2 \ln\frac{1}{2}\left(1+\frac{v}{c}\right) + mc^2\left(1-\frac{v}{c}\right) \tag{25}$$

Equation (25) for the alternative electrodynamics should be compared with equation (7) for classical electrodynamics.

5 Bertozzi's experiment

A remarkable demonstration of the speed of light being a universal limiting speed, was in an experiment conducted by William Bertozzi, at the Massachusetts Institute of Technology in 1964 [7]. In this experiment, the speed *v* of high-energy electrons was determined by measuring the time *T* required for them to traverse a distance of 8.4 metres after having been accelerated through a potential energy *P* inside a linear accelerator. Bertozzi's experimental data is reproduced in Table 1. It was clearly demonstrated that electrons accelerated through potential energies over 15 MeV attain, practically, the speed of light *c* as a limit.

TABLE 1 RESULTS OF BERTOZZI'S EXPERIMENTS WITH ELECTRONS ACCELERATED THROUGH ENERGY P IN A LINEAR ACCELERATOR (m_oc^2 = 0.5 MeV, v = 8.4/T m/sec)

P (MeV)	P/m_oc^2	T x 10^{-8} sec.	v x 10^8 m/sec	v/c
0.5	1	3.23	2.60	0.87
1.0	2	308	2.73	0.91
1.5	3	2.92	2.88	0.96
4.5	9	2.84	2.96	0.99
15.0	30	2.80	3.00	1.00

6 Motions of an electron in three systems of electrodynamics

Table 2 is drawn for speed *v* in units of *c* and potential energy *P* in units of mc^2 with respect to an electron of mass *m* accelerated by an electric field, through a distance *x*, in a straight line, from zero initial speed. Table 3 is similarly drawn for an electron decelerated from the speed of light *c*.

A graph of P/mc^2 (potential energy in units of mc^2) against *v/c* (speed in units *c*), is shown in Figure 2; the solid curves (A1 and A2) in accordance with classical electrodynamics (equations 6 and 7), the dashed curve (B1) according to relativistic electrodynamics (equation 11) and the dotted curves (C1) and (C2) according to the alternative electrodynamics (equations 20 and 25). The three solid squares are the results of Bertozzi's experiment

TABLE 2: POTENTIAL ENERGY *P* LOST BY AN ELECTRON OF CHARGE –*e* AND MASS *m* ACCELERATED FROM ZERO INITIAL SPEED BY AN ELECTRIC FIELD IN THREE SYSTEMS OF ELECTRODYNAMICS

Speed v in unit of speed of light *c*	Classical electrodynamics	Relativistic electrodynamics	Alternative electrodynamics	Observation
$\frac{v}{c}$	$\frac{P}{mc^2}=\frac{v^2}{2c^2}$ (Equation 6)	$\frac{P}{m_oc^2}=$ $\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}-1$ (Equation 11)	$\frac{P}{m_oc^2}=$ $-\ln\left(1-\frac{v}{c}\right)-\frac{v}{c}$ Equation 20	Bertozzi's experiment
0.00	0.000	0.000	0.000	-
0.05	0.001	0.001	0.001	-
0.10	0.005	0.005	0.005	-
0.20	0.020	0.021	0.023	-

0.30	0.045	0.048	0.057	-
0.40	0.080	0.091	0.111	-
0.50	0.125	0.155	0.193	-
0.60	0.180	0.250	0.316	-
0.70	0.245	0.400	0.504	-
0.80	0.320	0.667	0.809	-
0.87	0.378	1.028	1.170	1.0
0.91	0.414	1.412	1.498	2.0
0.96	0.461	2.571	2.259	3.0
0.99	0.490	6.089	3.615	9.0
1.00	0.500	∞	∞	30.0
1.10	0.605			-
1.20	0..720			-

TABLE 3: POTENTIAL ENERGY P GAINED BY AN ELECTRON OF CHARGE $-e$ AND MASS m DECELERATED FROM THE SPEED OF LIGHT c BY AN ELECTRIC FIELD IN THREE SYSTEMS OF ELECTRODYNAMICS

Speed v in unit of speed of light c	Classical electrodynamics	Relativistic Electrodynamics	Alternative Electrodynamics
$\frac{v}{c}$	$\frac{P}{mc^2} = \frac{1}{2}\left(1 - \frac{v^2}{c^2}\right)$ (Equation 7)	Not applicable Formula	$\frac{P}{m_o c^2} = \ln \frac{1}{2}\left(1 + \frac{v}{c}\right) + 1 - \frac{v}{c}$ (Equation 25)
1.00	0.000	?	0.000
0.99	0.010	?	0.005
0.98	0.020	?	0.010
0.96	0.040	?	0.020
0.94	0,058	?	0.030
0.92	0.077	?	0.039
0.90	0.095	?	0.049
0.80	0.180	?	0.095
0.70	0.255	?	0.137
0.60	0.320	?	0.177
0.50	0.375	?	0.212
0.40	0.420	?	0.243
0.30	0.455	?	0.269
0.20	0.480	?	0.289
0.10	0.495	?	0.302
0.00	0.500	?	0.307
–0.10	0.495	?	0.301
–0.20	0.480	?	0.284
–0.30	0.455	?	0.250
–0.40	0.420	?	0.196
–0.50	0.375	?	0.114
–0.59	0.326	?	0.005
–0.60	0.320	?	–0.009
–0.70	0.255	?	–0.197
–0.80	0.180	?	–0.503
–0.90	0.095	?	–1.096
–0.92	0.077	?	–1.299

–0.94	0.058	?	–1.567
–0.96	0.040	?	–1.952
–0.98	0.020	?	–2.625
–0.99	0.010	?	–3.308
–1.00	0.000	?	–∞
–1.10	–0.105	?	
–1.20	–0.220	?	

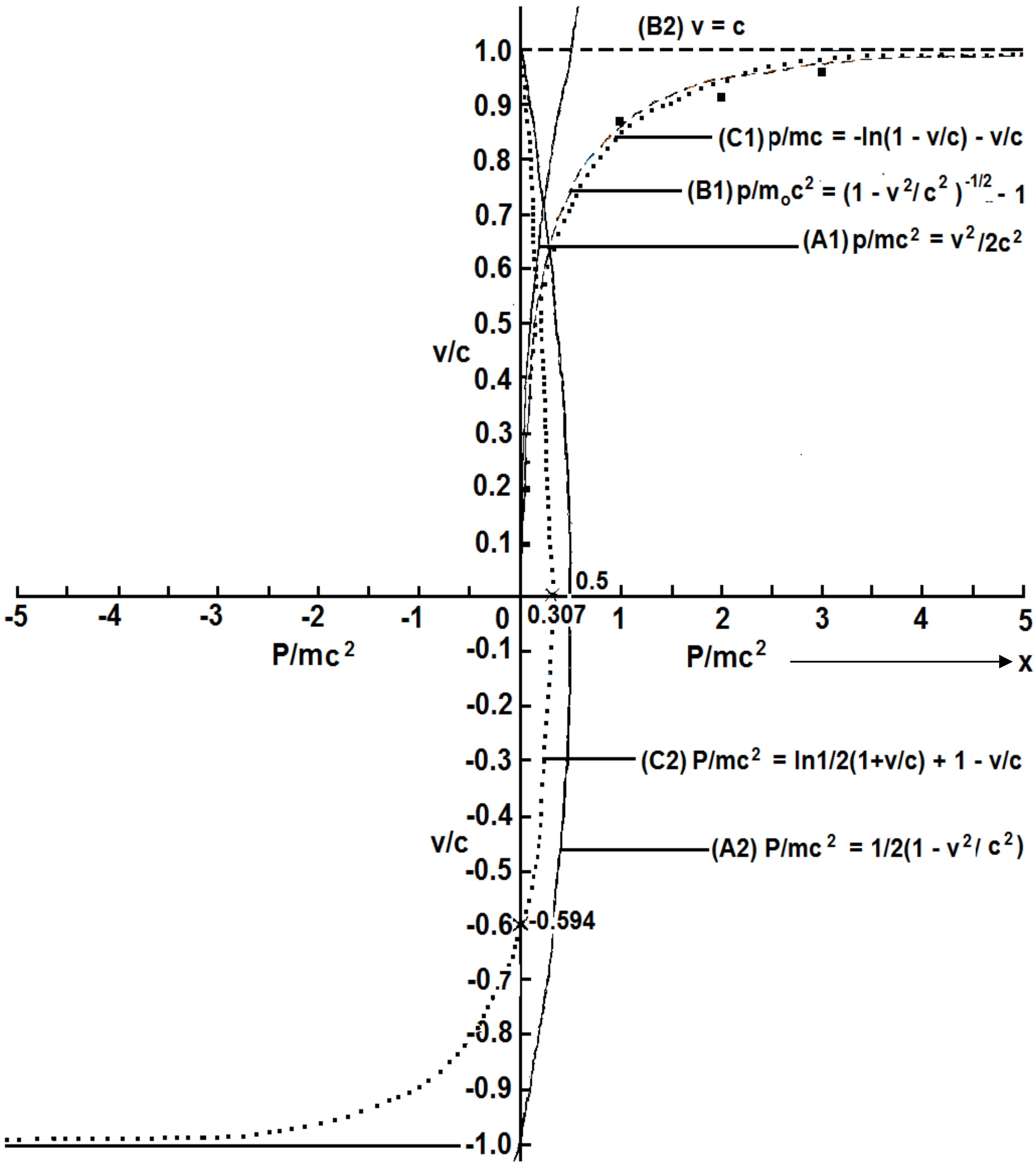

Figure 2 *v/c* (speed in units of *c*) against *P/mc*^{2} (potential energy in units of *mc*^{2}) for an electron of mass *m* accelerated from zero initial speed or decelerated from the speed of light c, the solid lines (A1 and A2) according to classical electrodynamics (equation 6 and 7), the dashed curve (B1) according to relativistic electrodynamics (equation 11) and the dotted curves (C1 and C2) according to the alternative *electrodynamics* (equations 20 and 25). The solid squares are the result of Bertozzi's experiment (Table 1).

8 Observations

Relativistic electrodynamics and the alternative electrodynamics merge to classical electrodynamics at very low speeds, compared to the speed of light. Relativistic electrodynamics and the alternative electrodynamics give zero acceleration at the speed of light. In relativistic electrodynamics, an electron cannot attain the speed of light *c*, no matter the magnitude of accelerating potential. In the alternative electrodynamics an electron is easily accelerated to the speed of light by a potential energy of 15 Mev or over.

Bertozzi's experimental results (Table 1) appear to be in agreement with relativistic electrodynamics (equation 11) and the alternative electrodynamics (equation 20) for an accelerated electron in rectilinear motion, as depicted in Figure. 2. The two systems of electrodynamics demonstrate the speed of light *c* as a limit; relativistic electrodynamics on the basis of mass of a moving particle increasing to become infinitely large at the speed of light and the alternative electrodynamics on the basis of accelerating force reducing to become zero at that speed.

The question now is: "Which one of the electrodynamics is correct?" The answer may be found in the motion of electrons decelerated from the speed of light *c*. According to classical electrodynamics, an electron of mass *m* entering a retarding field at a point (x = 0), with speed *c*, is brought to rest after losing kinetic energy $\frac{1}{2}mc^2$, equal to the potential energy gained, without energy radiation. The electron may then be accelerated backwards to reach the point of entry (x = 0) with speed $-c$ and may reach a speed greater than $-c$ (curve A2 of Figure 2) without radiation of energy.

According to relativistic electrodynamics, an electron moving at the speed of light (with infinitely large mass and energy), cannot be stopped by any force. The electron should continue to move at the speed of light gaining potential energy without losing kinetic energy.

In the alternative electrodynamics, an electron moving at the speed of light, on entering a retarding field at a point (x = 0), is easily brought to rest after gaining potential energy equal to $0.307mc^2$ and radiating energy equal to $0.193mc^2$. The electron may then be accelerated backwards to return to the point of entry with speed -$-0.594c$, losing potential energy equal to $0.307mc^2$, gaining kinetic energy equal to $0.176mc^2$ and radiating energy equal to $0.121mc^2$. The electron may then be accelerated to reach an ultimate speed equal to $-c$ with radiation of energy. An electron moving at the speed of light c acquires the characteristic of light.

9 Conclusion

For accelerated electrons, special relativity and the alternative electrodynamics appear to be in agreement. The picture is completely different for decelerated electrons. It is energy radiation which makes all the difference. In the alternative electrodynamics, an electron moving at the speed of light is easily brought to rest by a decelerating field. An electron moving at the speed of light, being stopped and turned back by a decelerating field, invalidates the theory of special relativity.

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