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Terminal Speed of an Electron Accelerated by an Electric Field

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Abstract

An electron of mass *m* and charge –*e* moving at time *t* with velocity **v** and acceleration (*d***v**/*dt*) in a straight line, in an electric field of intensity **E** is under the influence of an impressed force –*e***E**. The moving electron encounters a radiation reaction force –*eE***v**/*c* = *e***E***v*/*c*, so that the accelerating force is –*e***E**(1 – *v/c*) = –*m*(*d***v**/*dt*). It exhibits radiation power as scalar product –**v.**(–*eE***v**/*c*) = eEv^2/c and reaches a terminal speed equal to that of light *c*, whereby the impressed force –*e***E** becomes equal and opposite to the radiation reaction force and the accelerating force reduces to zero.

Keywords: Acceleration, force, electric charge, field, mass, speed, relativity

1. Introduction

In a most remarkable experiment in 1964, William Bertozzi [1] of the Massachusetts Institute of Technology, demonstrated the existence of a universal limiting speed, equal to the speed of light *c* in a vacuum. The experiment showed that electrons accelerated through energies of 15 MeV or over, attain, for all practical purposes, the speed of light *c*.

The theory of special relativity [2] explains the existence of a limiting speed equal to the speed of light c by positing that mass of a moving particle, such as an electron, increases with its speed, becoming infinitely large at the speed of light. Since an infinite mass cannot be accelerated any faster by a finite force, that speed becomes an ultimate limit.

This paper shows that the accelerating force exerted by an electric field on an electron decreases with its speed, reducing to zero at the speed of light *c*. Infinite mass or zero force at the speed of light leads to zero acceleration and constant speed *c* as a limit, in accordance with Newton's laws of motion. It is unfortunate that special relativity proposed *mass* as the variable rather than *force*. The ultimate speed without infinite mass is a more comfortable and more realistic proposition.

Radiation reaction force due to an electron of charge –*e* moving in a straight line with velocity **v** in the opposite direction of an electric field of intensity **E** is –*eE***v**/*c* = *e***E***v*/*c*. The accelerating force is the sum of the impressed force, the electrostatic force –*e***E**, and the radiation reaction force. At the speed of light *c*, the accelerating force, –*e***E** + *e***E***v*/*c*, on the electron becomes zero and it moves with constant speed *c*.

2. Acceleration in rectilinear motion

An electron of mass *m* and charge –*e* moving at time *t* with velocity **v** and acceleration (*d***v**/*dt*) in the opposite direction of an electric field of intensity **E**, encounters a radiation reaction force –*eE***v**/*c* = *e***E***v*/*c*. The accelerating force **F** on the electron, equal to the sum of the impressed force –*e***E** and the radiation reaction force *e***E***v*/*c*, that is –*e***E**(1 – v/c), is given by Newton's second law of motion, as vector equation:

$$\mathbf{F} = -e\mathbf{E}\left(1-\frac{v}{c}\right) = m\frac{d\mathbf{v}}{dt} \tag{1}$$

Since velocity **v**, of magnitude *v*, is in the opposite direction of **E**, the scalar equation is:

$$F = -eE\left(1-\frac{v}{c}\right) = -m\frac{dv}{dt} \tag{2}$$

where *c* is the speed of light in a vacuum and *m* is the mass of the particle, which is considered as independent of speed *v* of the particle.

For acceleration in a uniform field (*E* constant), where $eE/m = a$ is a constant, the solution of equation (2), for an electron accelerated from an initial speed *u*, is:

$$v = c-(c-u)\exp\left(\frac{-at}{c}\right) \tag{3}$$

For an electron accelerated from an initial speed $v = 0$, the solution of equation (2) is:

$$\frac{v}{c} = 1-\exp\left(-\frac{at}{c}\right) \tag{4}$$

In equations (3) and (4), the speed of light *c* is the limit to which a charged particle, such as an electron, can be accelerated by an electric field. A graph of *v/c* against at/c is shown as curve *C1* in Figure 1 below for equation (4).

3. Deceleration in rectilinear motion

For a decelerated electron the differential equation of motion (replacing *v* by –*v* in equation 2) becomes:

$$F = -eE\left(1+\frac{v}{c}\right) = m\frac{dv}{dt} \tag{5}$$

The solution of equation (5), for a charged particle decelerated, by a uniform electric field, from speed *u*, is:

$$v = (c+u)\exp\left(-\frac{at}{c}\right)-c \tag{6}$$

$$\frac{v}{c} = 2\exp\left(-\frac{at}{c}\right)-1 \tag{7}$$

In equations (6) and (7), the particle is decelerated to a stop and then accelerated in the opposite direction to reach a terminal speed equal to –*c*, shown as *C2* of the graphs in Figure 1. This result is not obtainable from the point of view of the theory of special relativity. In special relativity a particle moving at the speed of light will continue to move at that speed, losing potential energy without gaining kinetic energy.

4. Speed-time Equations and radiation of energy.

Figure 1 is a graph of *v/c (speed in units of c)* against *at/c (time in units of c/a)* for an electron of charge –*e* and mass *m* accelerated from zero initial speed or decelerated from the speed of light *c*, by a uniform field of magnitude *E*, where $a = eE/m$; the lines (A1) and (A2) according to classical electrodynamics, the dashed curve (B1) and line (B2) according to relativistic electrodynamics and the dotted curves (C1) and (C2) according to an alternative electrodynamics giving equations (4) and (7).

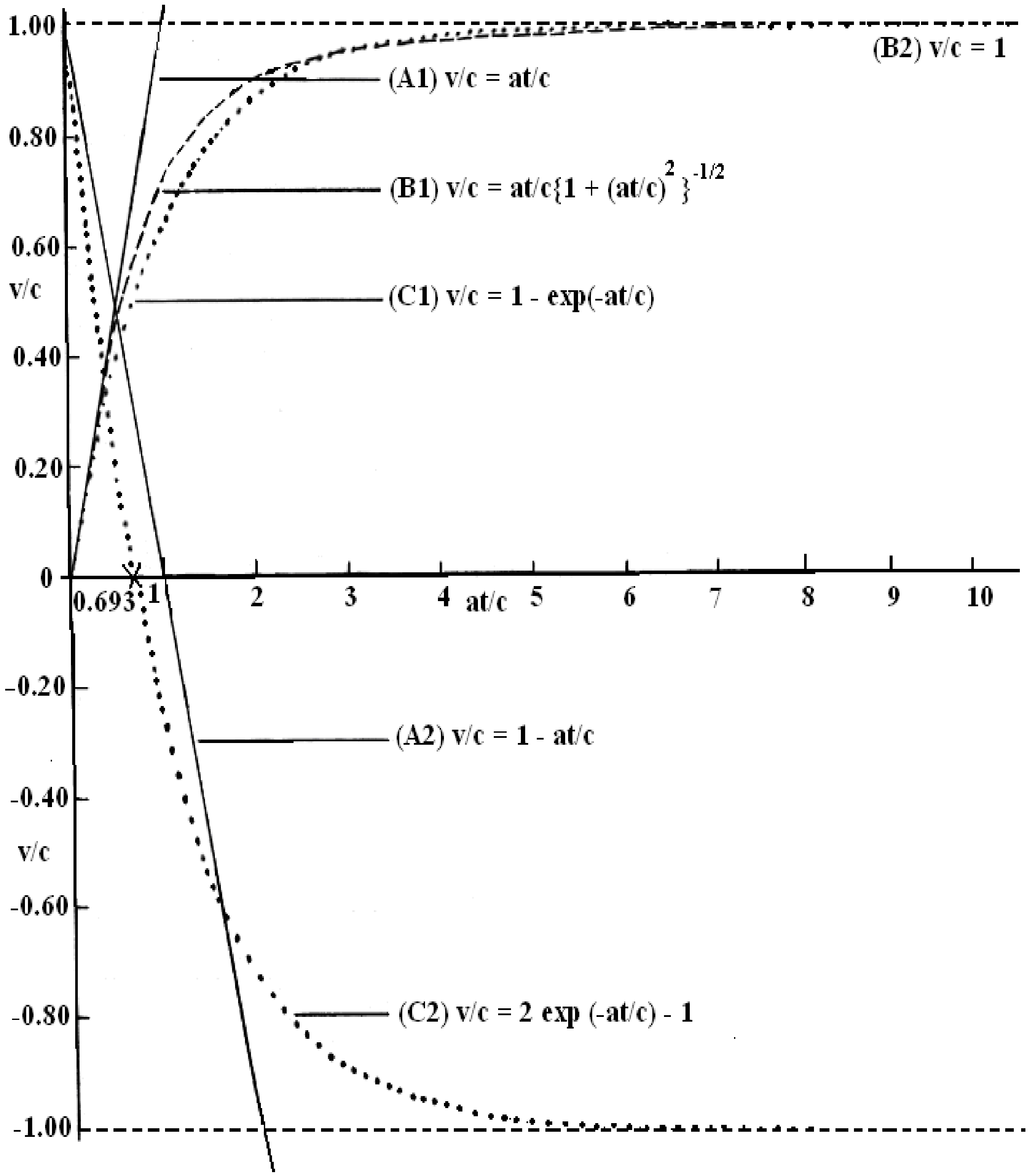

Figure 1: Graph of *v/c (speed in units of c)* against *at/c (time in units of c/a)* for an electron of charge *–e* and mass $m = m_o$ accelerated from zero initial speed or decelerated from the speed of light *c*, by a uniform electrostatic field of magnitude *E*, where $a = eE/m$; the lines (A1) and (A2) according to classical electrodynamics, the dashed curve (B1) and line (B2) according to relativistic electrodynamics and the dotted curves (C1) and (C2) according to equations 4 and 7.

In classical electrodynamics, an electron of charge *–e* and mass *m*, moving at the speed of light *c*, on entering a uniform retarding field of magnitude *E*, should be stopped in time $t = mc/eE$, energy radiation notwithstanding. In relativistic electrodynamics an electron moving at the speed of light *c* should be unstoppable by any force. In the alternative electrodynamics, an electron moving at the speed of light *c* is easily decelerated to a stop, by a uniform retarding field *E*, in time $t = 0.693mc/eE$, with radiation of energy.

5. Conclusions

Equations (2) and (5) are simply extensions of Coulomb's law of electrostatic force taking into consideration the speed of a charged particle in an electric field. Equations (4) and (7) give the speed of light c as the ultimate speed with mass of a moving particle remaining constant as the rest mass. It is energy radiation which makes all the difference between classical, relativistic and the alternative electrodynamics advanced here.

The speed of light c, being an ultimate limit to which a charged particle can be accelerated by an electric field, has nothing to do with the mass of the particle. It is a property of the electric field and radiation reaction force that limit the speed of an accelerated charged particle to that of light c.

5. References

[1] W. Bertozzi; "Speed and Kinetic Energy of Relativistic Electrons", *Am. J. Phys.*, 32 (1964), 551–555.
Also online at: http://spiff.rit.edu/classes/phys314/lectures/relmom/bertozzi.htm

[2] A. Einstein; "On the Electrodynamics of Moving Bodies", *Ann. Phys.,* 17 (1905), 891.