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Revolution of a Charged Particle Round a Centre of Force of Attraction

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Abstract

A charged particle of mass *nm* revolves in an orbit through angle ψ in time *t*, at a distance *r* from a centre of force of attraction, with constant angular momentum $n\mathbf{L}$, as:

$$n\mathbf{L} = nmr^2 \frac{d\psi}{dt}\mathbf{k}$$

where *n* is an integer greater than 0, *m* is the electronic mass, $\boldsymbol{k}$ is a unit vector perpendicular to the plane of the orbit. The equation of the nth orbit of motion is:

$$\frac{1}{r} = \frac{A}{n}\exp(-b\psi)\cos(\alpha\psi + \beta) + \frac{1}{nr_1}$$

where *A* and β are determined from the initial conditions, *b* and α are constants and r_1 is the radius of the first stable circular orbit. An excited particle revolves in an unclosed ellipse, with emission of radiation at the frequency of revolution, before settling down, after many cycles of ψ, into a stable circular orbit of radius nr_1. A particle of mass *nm* carrying the electronic charge $-e$ revolves in a unipolar orbit, round a positively charged nucleus or two particles of the same mass *nm* and charges *e* and $-e$ revolve, in a bipolar orbit, round a centre of mass. The discrete masses *nm* leads to quantization of the orbits without recourse to Bohr's quantum mechanics.

Keywords: Angular momentum, force, mass, orbit of revolution, radiation.

1 Introduction

Revolution of a body, round a centre of force of attraction, is the most common motion in the universe. This comes with revolution of planets round the Sun, binary stars round their centre of mass, a moon round a planet or an electron round the nucleus of an atom.

The German astronomer, Johannes Kepler [1, 2] early in the 17th century, formulated three laws, named after him, concerning the motions of planets. Kepler based his laws on astronomical data painstakingly collected in 30 years of observations by the Danish astronomer Tycho Brahe [3], to whom he was an assistant. Kepler's proposals broke with a centuries-old belief based on the Ptolemaic system advanced by the Alexandrian astronomer Ptolemy [4], in the 2nd century AD, and the Copernican system put forward by the Polish astronomer, Nicolaus Copernicus [5], in the 16th century.

The Ptolemaic cosmology postulated a geocentric universe in which the Earth was stationary and motionless at the centre of several concentric rotating spheres, which bore (in order of distance away from the earth) the Moon, the planets and the stars. The major premises of the Copernican system are that the Earth rotates daily on its axis and revolves yearly round the Sun and that the planets also circle the Sun. Copernicus's heliocentric theories of planetary motion had the advantage of accounting for the daily and yearly motions of the Sun and stars and it neatly explained the observed motions of the planets. However, the reigning dogma in the 16th century, that of the Roman Catholic Church, was in favour of the Ptolemaic system and it abhorred the Copernican theory.

The Copernican theory had some modifications and various degrees of acceptance in the 16th and 17th centuries. The most famous Copernicans were the Italian physicist Galileo Galilei [6] and his contemporary, the astronomer Johannes Kepler [1, 2].

By December 1609 Galileo [7] had built a telescope of 20 times magnification, with which he discovered four of the moons circling Jupiter. This showed that at least some heavenly bodies move around a centre other than the Earth. By December 1910 Galileo [7] had observed the phases of Venus, which could be explained if Venus was sometimes nearer the Earth and sometimes farther away from the Earth, following a motion round the Sun.

In 1616, Copernican books were subjected to censorship by the Church [8]. Galileo was instructed to no longer hold or defend the opinion that the Earth moved. He failed to conform to the ruling of the Church and after the publication of his book titled *Dialogue on the Two Chief World Systems* [9], he was accused of heresy, compelled to recant his beliefs and then confined to house arrest. Galileo's *Dialogue* was ordered to be burned and his ideas banned.

The ideas contained in the *Dialogue* could not be suppressed by the Roman Catholic Church. Galileo's reputation continued to grow in Italy and abroad, especially after his final work. Galileo's final and greatest work is the book titled *Discourses Concerning Two New Sciences*, published in 1638. It reviews and refines his earlier studies of motion and, in general, the principles of mechanics. The book opened a road that was to lead Sir Isaac Newton [10] to the law of universal gravitation, which linked the planetary laws discovered by astronomer Kepler with Galileo's mathematical physics.

Kepler [1, 2] stamped the final seal of validity on the Copernican planetary system in three laws, viz.:

(i) *The paths of the planets are ellipses with the sun as one focus.*

(ii) *The line drawn from the sun to the planet sweeps over equal areas in equal time.*

(iii) *The square of the periods of revolution (T) of the different planets are proportional to the cube of their respective mean distances (r) from the sun* $(T^2 \propto r^3)$

The import of Kepler's first law is that there is no dissipation of energy in the revolution of a planet, in a closed orbit, round the Sun. Any change of kinetic energy is equal to the change of potential energy. The second law means that a planet revolves round the Sun with constant angular momentum, which is the case if there is no force perpendicular to the radius vector. From the third law $(T^2 \propto r^3)$ and the relationship between the centripetal force and speed $(F \propto v^2/r)$, it can be deduced that the force of attraction F on a planet is inversely proportional to the square of its distance from the Sun as discovered by Newton in 1687 [10]..

Kepler's laws played an important part in the work of the English astronomer, mathematician and physicist, Sir Isaac Newton [10]. The laws are significant for the understanding of the orbital paths of the moon, the natural satellite of the Earth, and the paths of the artificial satellites launched from space stations.

While the orbital path of a satellite is a closed ellipse, the orbit of an electrically charged particle, round a central force of attraction, is an unclosed ellipse or a closed circle. A charged particle revolves in an unclosed orbit with emission or absorption of radiation. The energy radiated is the difference between change in kinetic energy and change in potential energy. Revolution in a circular orbit is without radiation and inherently stable as there is no change in the kinetic energy and potential energy of a revolving particle. Revolution in a circular orbit is the perfect motion as it involves no change in the status of energy.

The purpose of this paper is to derive equations of the orbit of motion of a charged particle revolving round a centre of force of attraction. A charged particle may revolve round a nucleus as a unipolar radiator or two particles may revolve round their centre of mass as a bipolar radiators. The equations are used to show that the orbit of a charged particle is an unclosed (aperiodic) ellipse where it moves with constant angular momentum *nL*, *n* being an integer. The masses *nm*, (*m* being the electronic mass) of revolving particles lead to quantisation of the orbits. A particle revolves with emission or absorption of radiation of discrete frequencies, in many cycles of revolution, before settling into the stable circular orbit.

2 Unipolar motion under a central force

Consider a particle of charge **−***e* and mass *nm* at a point *P* and time *t* revolving round, anticlockwise, in an angle ψ and with velocity ***v*** in an orbit under the attraction of a positive charge *Q* fixed at origin *O*, as shown in Figure 1. Here, *n* is an integer greater than *0*, **−***e* is the electronic charge and *m* the electronic mass. The particle at *P* executes unipolar motion under a central force at *O*. In unipolar revolution, a particle, as the pole of the orbit, revolves round a stationary centre under a force of attraction. The orbit of motion is an unclosed (aperiodic) ellipse with emission or absorption of radiation or a closed circle without radiation.

In Figure 1, the radius vector ***OP*** makes an angle ψ with the ***OX*** axis in space. The position vector ***r*** of the point ***P*,** in the direction of unit vector $\hat{\mathbf{u}}$, (radial direction) and the velocity ***v*** at time *t*, are respectively given by the equations:

$$\mathbf{r} = r\hat{\mathbf{u}} \tag{1}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\hat{\mathbf{u}}}{dt} \tag{2}$$

For orbital motion in the *X-Y* plane of the Cartesian coordinates, $d\hat{\mathbf{u}}/dt$, the angular velocity, is given by the vector (cross) product:

$$\frac{d\hat{\mathbf{u}}}{dt} = \frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \tag{3}$$

The angle ψ is the inclination of the radius vector ***OP*** from ***OX***, ***k*** is a constant unit vector in the **Z**-direction, perpendicular to the orbital plane (out of the page in Figure 1) and $(d\psi/dt)\mathbf{k}$ is the angular velocity. The velocity ***v*** is:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \tag{4}$$

The velocity in the radial direction is:

$$v_r\hat{\mathbf{u}} = \frac{dr}{dt}\hat{\mathbf{u}} \qquad (5)$$

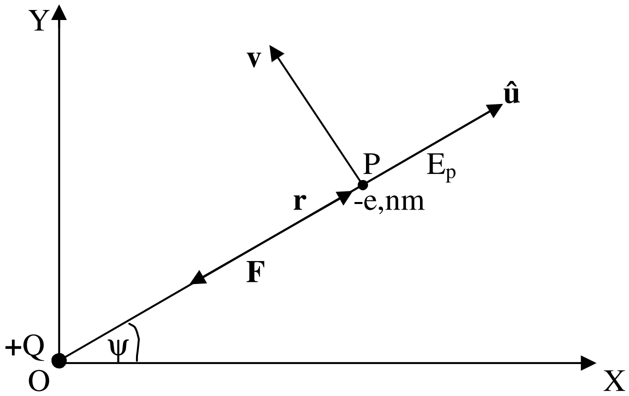

Figure 1. A particle of charge *–e* and mass *nm* at a point *P* revolving anticlockwise, with angular displacement ψ, in an orbit at velocity ***v*** under the attraction of a stationary positive charge *Q* at a centre of attraction *O*.

The acceleration (noting that ***k*** is a constant unit vector) is obtained, a vector in two orthogonal directions, as:

$$\frac{d\mathbf{v}}{dt} = \left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} + \left(2\frac{dr}{dt}\frac{d\psi}{dt} + r\frac{d^2\psi}{dt^2}\right)\mathbf{k}\times\hat{\mathbf{u}} \qquad (6)$$

The acceleration, in the direction of force of attraction, is $a_r\hat{\mathbf{u}}$ in the radial direction only, thus:

$$\frac{d\mathbf{v}}{dt} = \left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} = a_r\hat{\mathbf{u}} \qquad (7)$$

From equation (6), the force perpendicular to the radial direction is zero and, therefore, the acceleration of mass *nm* is also zero in the $\mathbf{k}\times\hat{\mathbf{u}}$ direction, so that the equation gives:

$$nm\left(2\frac{dr}{dt}\frac{d\psi}{dt} + r\frac{d^2\psi}{dt^2}\right)\mathbf{k}\times\hat{\mathbf{u}} = \frac{nm}{r}\frac{d}{dt}\left(r^2\frac{d\psi}{dt}\right)\mathbf{k}\times\hat{\mathbf{u}} = 0$$

This equation can be expressed in terms of angular momentum ***L***, as:

$$\frac{nm}{r}\frac{d}{dt}\left(r^2\frac{d\psi}{dt}\right)\mathbf{k}\times\hat{\mathbf{u}} = \frac{n}{r}\frac{d}{dt}\mathbf{L}\times\hat{\mathbf{u}} = 0$$

where

$$n\mathbf{L} = nmr^2\frac{d\psi}{dt}\mathbf{k} \qquad (8)$$

Here, ***L*** is the constant angular momentum with respect to the first orbit. Equations (7) and equation (8) will be used to derive the equation of the orbit of motion of the particle at *P* in Figure 1 in unipolar oscillation.

In Figure 1, the positively charged particle at O is considered to be very much more massive such that it could be taken as almost stationary. In this case, we have central motion where a particle at P carrying the electronic charge $-e$ and a multiple nm of the electronic mass m, revolves in an orbit, round a stationary particle of charge $+Q$, as nucleus at O. The particle moves in an electrostatic field E_p under a force of attraction (Coulomb force) and a radiation reaction force.

The author [11] showed that the accelerating force $\boldsymbol{F}$ on an electron of charge $-e$ and mass nm moving with velocity $\boldsymbol{v}$ in an electric field of intensity $\boldsymbol{E}_p$, $= E_p\hat{\mathbf{u}}$, is:

$$\mathbf{F} = \frac{eE_p}{c}(\mathbf{c} - \mathbf{v}) = nm\frac{d\mathbf{v}}{dt}$$

where the velocity of light $\boldsymbol{c}$ is at aberration angle α and velocity $\boldsymbol{v}$ is at an angle θ to $\boldsymbol{F}$. In elliptic motion, there is a component of velocity along the radial $\hat{\mathbf{u}}$ direction, so that:

$$\mathbf{F} = \frac{eE_p}{c}(\mathbf{c} - \mathbf{v}) = \frac{eE_p}{c}(\mathbf{c}.\hat{\mathbf{u}} - \mathbf{v}.\hat{\mathbf{u}})\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt}$$

Taking the scalar products gives:

$$\mathbf{F} = \frac{eE_p}{c}(-c\cos\alpha + v\cos\theta)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt}$$

As α is a small angle, $c\cos\alpha \approx c$ and with $v\cos\theta = -v_r$, we get:

$$\mathbf{F} = -eE_p\left(1 + \frac{v_r}{c}\right)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt} \tag{9}$$

where v_r is the speed in the radial direction, $-eE_p\hat{\boldsymbol{u}}$ is the electrostatic force and $-(eE_pv_r/c)\hat{\boldsymbol{u}}$ is the radiation reaction force. The radiation reaction force, akin to a frictional force or damping force in dynamics, results in energy radiation. Radiation always comes into play when a charged particle is accelerated or decelerated by an electrostatic field.

Substituting for v_r from equation (5) into equation (9) gives the accelerating force on a charged particle of mass nm, as:

$$\mathbf{F} = -eE_p\left(1 + \frac{1}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt} \tag{10}$$

Substituting for the acceleration $a_r\hat{\boldsymbol{u}}$ from equations (7) into equation (10), gives the accelerating force $\boldsymbol{F}$ on a charged particle of mass nm, moving in an electrostatic field E_p, as:

$$\mathbf{F} = -eE_p\left(1 + \frac{1}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} \tag{11}$$

Putting $E_p = Q/4\pi\varepsilon_o r^2$, gives the magnitude of the force as:

$$F = \frac{-eQ}{4\pi\varepsilon_o r^2}\left(1+\frac{1}{c}\frac{dr}{dt}\right) = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \tag{12}$$

$$F = \frac{-\chi}{nmr^2}\left(1+\frac{1}{c}\frac{dr}{dt}\right) = \frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2 \tag{13}$$

where $\chi = eQ/4\pi\varepsilon_o$ is a constant. Equation (13) is the mixed differential equation of motion of the particle revolving in an orbit through angle ψ and with instantaneous radius r at time t. We need to reduce it to an equation of r as a function of one variable, ψ.

In equation (13), taking the angle ψ as the variable, making the substitution $r = 1/u$ to give $dr/du = -1/u^2$ and with $(d\psi/dt) = L/mr^2$ (equation 8), we get:

$$\frac{dr}{dt} = \frac{dr}{du}\frac{d\psi}{dt}\frac{du}{d\psi} = \frac{-L}{m}\frac{du}{d\psi} \tag{14}$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\frac{dr}{dt} = \frac{d\psi}{dt}\frac{d}{d\psi}\left(-\frac{L}{m}\frac{du}{d\psi}\right) = \frac{-L^2u^2}{m^2}\frac{d^2u}{d\psi^2} \tag{15}$$

Substituting equations (15) and (14) into equation (13) gives:

$$-\frac{L^2u^2}{m^2}\frac{d^2u}{d\psi^2} - \frac{L^2u^3}{m^2} = -\frac{\chi u^2}{nm}\left(1-\frac{L}{mc}\frac{du}{d\psi}\right)$$

$$\frac{d^2u}{d\psi^2} + \frac{\chi}{ncL}\frac{du}{d\psi} + u = \frac{m\chi}{nL^2} \tag{16}$$

This is a 2nd order differential equation with constant coefficients. A solution for the nth orbit is $u = (A/n)exp(x\psi)$, the *transient*, if the *auxiliary equation*, $x^2 + 2qx + 1 = 0$ and $q = \chi/2ncL$. This gives:

$$x = -q \pm \sqrt{q^2 - 1} = -q \pm j\alpha$$

where α is the "rotation factor" and $\alpha^2 = 1 - q^2$, is positive. The general solution is:

$$u = \frac{1}{r} = \frac{A}{n}\exp(j\alpha - q)\psi + \frac{m\chi}{nL^2} \tag{17}$$

The particular or appropriate solution of equation (16) is:

$$u = \frac{1}{r} = \frac{A}{n}\exp(-q\psi)\cos(\alpha\psi + \beta) + \frac{m\chi}{nL^2} \tag{18}$$

where the excitement amplitude A/n and phase angle β are obtained from the initial conditions and $m\chi/nL^2$ is the *steady state*.

Equation (18) gives the path or the nth unstable orbit of the particle (at P in Fig.1) with O as the fixed centre of revolution. For $q > 0 < 1$ and $\alpha < 1$, the orbit is an unclosed (aperiodic) ellipse whose major axis (line joining the points of farthest separations of the particles) rotates about an axis through the centre, perpendicular to the orbital plane. A particle makes one cycle of $2\pi/\alpha$ radians as the major axis goes through $2\pi/\alpha - 2\pi$ radians.

The exponential decay factor, $exp(-q\psi)$, is due to energy radiation. As a result of radiation of energy, after a great number of revolutions in the angle ψ, the *transient*, $(A/n)\ exp(-q\psi)$, decreases to zero and the radius increases to the *steady state* $nL^2/m\chi$, as long as q is greater than zero. This is the radius of the stable orbit when the radiator settles down from the excited state with the particle revolving in the nth stable orbit, a circle of radius $nL^2/m\chi. = nr_1$. The radius r_1 is with respect to the innermost orbit where $n = 1$.

In the stable orbit, there is only motion in a perfect circle, perpendicular to a radial electric field. No radial motion of the charged particle, no change of potential or kinetic energy and, therefore, no radiation of energy. The author [11] showed that radiation occurs provided there is a component of velocity of a particle in the direction of an electric field.

3 Bipolar motion under a central force

A bipolar orbit consists of two particles of equal mass but oppositely charged, each carrying the electronic charge of magnitude e and a multiple nm of the electronic mass m, under mutual attraction, revolving round their common centre of mass, the common centre of revolution, at a point O as depicted in Figure 2. The centripetal electrostatic force of attraction $\boldsymbol{F}$, on a charged particle, is balanced by the centrifugal force due to acceleration.

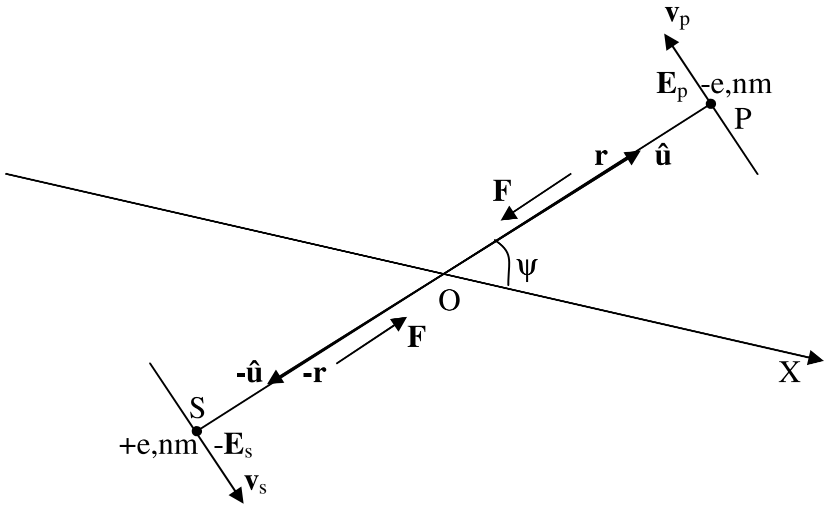

Figure 2. Two equal and oppositely charged particles at P and S having electronic charges $-e$ and $+e$ and the same mass nm (n being an integer and m is the electronic mass) revolving anticlockwise in angle ψ, under mutual attraction, in an orbit of radius r, round the centre of revolution at O.

The two oppositely charged particles at P and S in Figure 2, separated by distance $2r$, make up the two poles of the bipolar orbit, each particle being one pole in the orbit. Thus the bipolar orbit (in contrast to the unipolar orbit) has no nucleus but an empty point as the centre of mass, the centre of revolution, located halfway between the revolving charged particles.

In Figure 2, the particle (of mass *nm* and charge **–***e*) at point *P*, of position vector ***r***, is moving with velocity $\boldsymbol{v}_p$, at an angle ψ in the electrostatic field $\boldsymbol{E}_p$ of the other particle (of mass *nm* and charge **+***e*) at *S*. The particle at *S* of position vector **–*r***, is moving with velocity $\boldsymbol{v}_s$ in the electrostatic field $\boldsymbol{E}_s$ of the particle at *P*. The velocities $\boldsymbol{v}_p$ and $\boldsymbol{v}_s$ are respectively given by the vector equations:

$$\mathbf{v}_p = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \tag{19}$$

$$\mathbf{v}_s = -\frac{d\mathbf{r}}{dt} = -\frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \tag{20}$$

The two particles, of equal mass, move with the same angular velocity, in a plane orbit, but with relative linear velocity in the radial direction. The relative velocity $\boldsymbol{v}_r$ of the moving particle at *P* with respect to the moving particle at *S*, is:

$$\mathbf{v}_r = v_r\hat{\mathbf{u}} = \mathbf{v}_p - \mathbf{v}_s = 2\frac{dr}{dt}\hat{\mathbf{u}} \tag{21}$$

It is shown above that the accelerating force ***F***, due to attraction, on a particle of charge **–***e* and mass *nm* revolving in an ellipse, at time *t*, with speed v_r in the direction of an electrostatic field $\hat{\boldsymbol{u}}E_p$ of magnitude E_p (Figure 1), is given by equation (9):

$$\mathbf{F} = -eE_p\left(1+\frac{v_r}{c}\right)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt} \tag{22}$$

Substituting for v_r from equation (21) into equation (22) gives the accelerating force on a charged particle of mass *nm*, as:

$$\mathbf{F} = -eE_p\left(1+\frac{2}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt} \tag{23}$$

Substituting for the acceleration $a_r\hat{\boldsymbol{u}}$ from equation (7) into equation (23), gives the accelerating force ***F*** on a particle of mass *nm*, similar to equation (11), as:

$$\mathbf{F} = -eE_p\left(1+\frac{2}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} \tag{24}$$

Putting $E_p = e/16\pi\varepsilon_o r^2$, gives the magnitude of the force, similar to equation (12) as:

$$F = \frac{-e^2}{16\pi\varepsilon_o r^2}\left(1+\frac{2}{c}\frac{dr}{dt}\right) = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \tag{25}$$

Re-arrenging equation (25) gives an expression similar to equation (13), as:

$$F = \frac{-\kappa}{nmr^2}\left(1 + \frac{2}{c}\frac{dr}{dt}\right) = \frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2 \tag{26}$$

where $\kappa = e^2/16\pi\varepsilon_o$. Equation (26) is the mixed differential equation of revolution of the particle in an orbit through angle ψ.

In equation (26), taking the angle ψ as the variable and making the substitution $r = 1/u$ and with $(d\psi/dt) = L/mr^2$ (equation 8) we get equations (14) and (15) and the first order differential equation:

$$\frac{d^2u}{d\psi^2} + \frac{2\kappa}{ncL}\frac{du}{d\psi} + u = \frac{m\kappa}{nL^2} \tag{27}$$

If $u = (A/n)exp(y\psi)$ is a solution for the nth orbit, the *auxiliary equation* $y^2 + 2by + 1 = 0$, with $b = \kappa/ncL$. The general solution is:

$$u = \frac{1}{r} = \frac{A}{n}\exp(j\alpha - b)\psi + \frac{mK}{nL^2} \tag{28}$$

The appropriate solution of equation (27) is:

$$u = \frac{1}{r} = \frac{A}{n}\exp(-b\psi)\cos(\alpha\psi + \beta) + \frac{m\kappa}{nL^2} \tag{29}$$

where the amplitude of the excitement A/n and phase angle β are determined from the initial conditions, α *is* the "rotation factor" and $\alpha^2 = 1 - b^2$. The *steady state* is $m\kappa/nL^2$, obtained after many revolutions.

Equation (29) gives the path or the nth unstable orbit of the particles (at *P* or at *S* in Figure 2) with *O* as the centre of revolution. For $b > 0 < 1$ and $\alpha < 1$, the orbit is an unclosed (aperiodic) ellipse where a particle completes one cycle of revolution in $2\pi/\alpha$ radians, with relative motion in the radial direction and with emission of radiation.

The exponential decay factor, $exp(-b\psi)$, is due to radiation. After a great number of revolutions in the angle ψ, the *transient* $(A/n)\ exp(-b\psi)$, decreases to zero and the radius settles at the *steady state* $nL^2/m\kappa$; as long b is greater than zero. This is the radius of the stable orbit when the radiating particle settles down from the excited state with the two particles revolving in the nth stable orbit, a circle of radius $nL^2/m\kappa = nr_1$, shown as *WCYD* in Fig.3.

In the stable bipolar orbit there is only revolution in a perfect circle. There is no motion of a particle along the electric field. Motion is perpendicular to the electric field only and, therefore, no radiation.

4 Free ellipse and stable orbit of revolution of a radiating particle

Equation (29), giving the bipolar orbit of a radiating particle in the nth orbit, with the phase angle β being 0, may be written as:

$$\frac{1}{r} = \frac{A}{n}\exp(-b\psi)\cos(\alpha\psi) + \frac{1}{nr_1}$$

where $r_1 = L^2/m\kappa$ is the radius of the first orbit. If the decay factor b is negligible, $\alpha \approx 1$, the equation of the nth orbit becomes:

$$\frac{1}{r} = \frac{A}{n}\cos\psi + \frac{1}{nr_1} = \frac{1}{nr_1}\left(1 + Ar_1\cos\psi\right) \qquad (30)$$

The orbit, shown as *WXYZ* in Figure 3, an ellipse of eccentricity $\eta = Ar_1 = A/B$, is the **free ellipse**. This ellipse is a hypothetical orbit that the particle would have taken if there were no radiation, i.e. if $b = 0$.

Revolution of a radiating particle is in an unclosed (aperiodic) ellipse, with a decreasing period (increasing frequency). After a great number of revolutions $(\psi \to \infty)$, the nth orbit reduces to a circle, the **stable orbit** of radius nr_1, shown as *CDEF* in Figure 3. The frequencies of revolution of a radiating particle, in the unstable orbits, are very nearly equal to that of revolution in the nth stable orbit. So, radiation from a particle, in a bipolar orbit or unipolar orbit, is a narrow band of frequencies, very nearly equal to the frequency of revolution in the nth stable orbit. This leads to a spread of frequencies of revolution, as discussed in section 6 below.

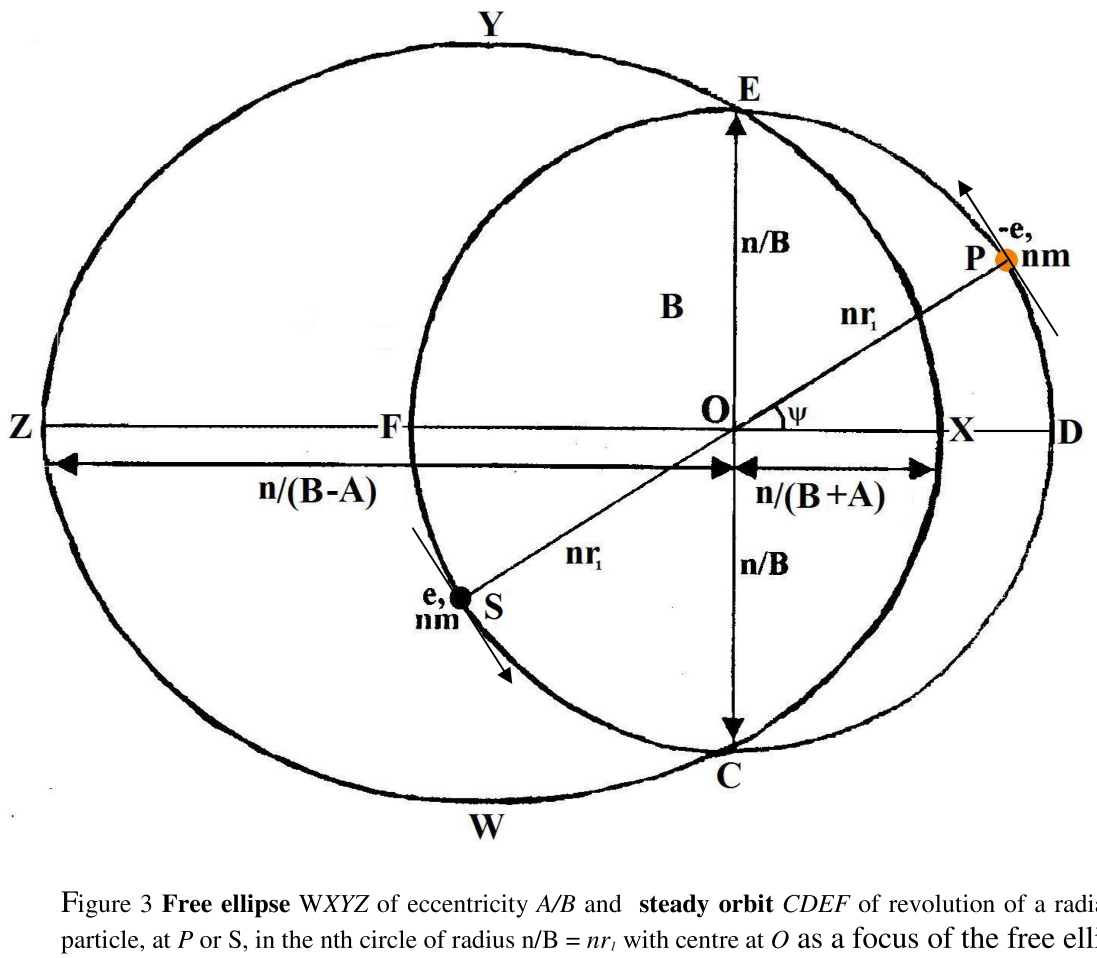

Figure 3 **Free ellipse** *WXYZ* of eccentricity *A/B* and **steady orbit** *CDEF* of revolution of a radiating particle, at *P* or S, in the nth circle of radius n/B = nr_1 with centre at *O* as a focus of the free ellipse

5 Energy radiated by a revolving charged particle

The accelerating force on a particle of mass *nm* and charge *−e* revolving at time *t* and at a point distance *r* from the centre of a force of attraction due to an electric field of magnitude E_p of an electric charge, is given by equation (23). The radiation force is:

$$-\mathbf{R}_f = eE_p\left(\frac{2}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = \frac{2\kappa}{c}\frac{1}{r^2}\left(\frac{dr}{dt}\right)\hat{\mathbf{u}}$$

where $\kappa = e^2/16\pi\varepsilon_o$. Energy radiated is obtained by integrating the radiation force with respect to displacement *(dr)* in a cycle, *s* to *(s+1)*, through $2\pi/\alpha$ radians, of the nth orbit, to give:

$$s_r = \frac{2\kappa}{c}\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \frac{1}{r^2}\left(\frac{dr}{dt}\right)(dr)$$

Substituting for *(dr/dt)* from equation (14) and with $(dr) = -1/u^2(du)$, we obtain:

$$s_r = \frac{2\kappa L}{mc}\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \left(\frac{du}{d\psi}\right)^2 (d\psi) \qquad (31)$$

Substituting for $(du/d\psi)$ from equation (28), gives the integral in complex form. The energy radiated in the (s+1)th cycle of the nth orbit, is given by the [Real Part] of the integral:

$$s_r = \frac{2\kappa L}{mc}\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \frac{A^2}{n^2}\exp 2\{(j\alpha - b)\psi\}(j\alpha - b)^2(d\psi) \qquad (32)$$

$$s_r = \frac{2\kappa LA^2}{mcn^2}\left[\frac{\exp 2\{(j\alpha - b)\psi\}(j\alpha - b)^2}{2(j\alpha - b)}\right]_{\psi=\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}$$

$$s_r = \frac{\kappa LA^2}{mcn^2}\left[\frac{\exp(-2b\psi)\{\cos(2\alpha\psi) + j\sin(2\alpha\psi)\}(j\alpha - b)}{1}\right]_{\psi=\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}$$

The [Real Part] is obtained as:

$$s_r = \frac{-\kappa LA^2}{mcn^2}\left[\frac{\exp(-2b\psi)\{b\cos(2\alpha\psi) + \alpha\sin(2\alpha\psi)\}}{1}\right]_{\psi=\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}$$

$$s_r = \frac{\kappa L A^2 b}{mcn^2}\exp\left(\frac{-4\pi bs}{\alpha}\right)\left\{1-\exp\left(\frac{-4\pi b}{\alpha}\right)\right\} \tag{33}$$

In the final cycle *(s → ∞)*, the energy radiated is 0. The total energy radiated E_r, after many cycles, is the sum of geometric series:

$$E_r = \sum_{s=0}^{\infty} s_r = \frac{\kappa L A^2 b}{mcn^2} = \frac{A^2\kappa^2}{mc^2n^3} \tag{34}$$

Here A/n is the excitement amplitude in the nth orbit and $b = \kappa/ncL$.

6 Period and Spread of frequency of oscillation of a radiator

Equation (29) gives the bipolar orbit of a charged particle of mass nm revolving through angle ψ, in the nth orbit, with constant angular momentum nL and with phase angle $\beta = 0$, as:

$$\frac{1}{r} = \frac{A}{n}\exp(-b\psi)\cos(\alpha\psi) + \frac{m\kappa}{nL^2} \tag{35}$$

$$\frac{1}{r} = \frac{m\kappa}{nL^2}\left\{1 + \frac{AL^2}{m\kappa}\exp(-b\psi)\cos(\alpha\psi)\right\}$$

$$\frac{1}{r} = \frac{B}{n}\left\{1 + \frac{A}{B}\exp(-b\psi)\cos(\alpha\psi)\right\}$$

where $B = m\kappa/L^2$.

$$\frac{1}{r} = \frac{B}{n}\{1 + \eta\exp(-b\psi)\cos(\alpha\psi)\} \tag{36}$$

This is the equation of an unclosed (aperiodic) ellipse, in the polar coordinates, with eccentricity $\eta = A/B$ as the eccentricity of the free ellipse. Equation (36) gives r as:

$$r = \frac{n}{B}\{1 + \eta\exp(-b\psi)\cos(\alpha\psi)\}^{-1} \tag{37}$$

The angular momentum nL of the particle, of mass nm, gives:

$$dt = \frac{mr^2}{L}(d\psi)$$

$$dt = \frac{mn^2}{LB^2}\{1 + \eta\exp(-b\psi)\cos(\alpha\psi)\}^{-2}(d\psi) \tag{38}$$

The period of revolution $T_{(s+1)}$ in the (s + 1)th cycle (s = 0, 1, 2, 3...∞) of the nth orbit (n = 1, 2, 3...N_h), is obtained by integrating equation (38) for ψ through an angle $2\pi/\alpha$ radians, to obtain:

$$T_{(s+1)} = \frac{mn^2}{LB^2} \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \left\{1+\eta\exp(-b\psi)\cos(\alpha\psi)\right\}^{-2}(d\psi) \tag{39}$$

Expanding the integrand, in equation (39), into an infinite series, by the binomial theorem, we obtain:

$$\left\{1+\eta\exp(-b\psi)\cos(\alpha\psi)\right\}^{-2}$$
$$= \sum_{p=0}^{\infty}(-p)^p(1+p)\eta^p\exp(-pb\psi)\cos^p(\alpha\psi)$$

where p is a positive integer, 0 – ∞. Equation (39) then becomes:

$$T_{(s+1)} =$$
$$\frac{mn^2}{LB^2} \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \sum_{p=0}^{\infty}(-p)^p(1+p)\eta^p\exp(-pb\psi)\cos^p(\alpha\psi)(d\psi) \tag{40}$$

$$T_{(s+1)} = \frac{mn^2}{LB^2}\sum_{p=0}^{\infty}Q_p$$

where

$$Q_p = (-1)^p(1+p)\eta^p \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi\{s+1\}}{\alpha}} \exp(-pb\psi)\cos^p(\alpha\psi)(d\psi) \tag{41}$$

Expressing $cos^p(\alpha\psi)$ as a sum of cosines of multiples of $(\alpha\psi)$, let us take the first five terms of Q_p.

$$Q_0 = \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} (d\psi) = \frac{2\pi}{\alpha} \tag{42}$$

$$.\, Q_1 = -2\eta \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-b\psi)\cos(\alpha\psi)(d\psi)$$

Putting $exp(-b\psi)cos(\alpha\psi)$ = [Real Part] of $\{exp(j\alpha - b)\psi\}$, the integral, Q_1, is obtained as:

$$Q_1 = -2\eta\left[\frac{\exp(-b\psi)\{-b\cos\alpha\psi + \alpha\sin(\alpha\psi)\}}{a^2 + b^2}\right]_{\psi=\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}$$

Noting that $\alpha^2 + b^2 = 1$, we get:

$$Q_1 = -2\eta b\exp\left(\frac{-2\pi bs}{\alpha}\right)\left\{1 - \exp\left(\frac{-2\pi b}{\alpha}\right)\right\} \tag{43}$$

$$Q_2 = -3\eta^2 \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-2b\psi)\cos^2(\alpha\psi)(d\psi)$$

$$Q_2 = \frac{-3\eta^2}{2} \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-2b\psi)\{1 + \cos(2\alpha\psi)\}(d\psi)$$

$$Q_2 = \frac{3\eta^2}{2}\left(\frac{1}{2b} + \frac{b}{2}\right)\exp\left(\frac{-4\pi bs}{\alpha}\right)\left\{1 - \exp\left(\frac{-4\pi b}{\alpha}\right)\right\} \tag{44}$$

$$Q_3 = -4\eta^3 \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-3b\psi)\cos^3(\alpha\psi)(d\psi)$$

$$Q_3 = -\eta^3 \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-3b\psi)\{3\cos(\alpha\psi) + \cos(3\alpha\psi)\}(d\psi)$$

$$Q_3 = \eta^3\left(\frac{9b}{\alpha^2 + 9b^2} + \frac{b}{3}\right)\exp\left(\frac{-6\pi bs}{\alpha}\right)\left\{1 - \exp\left(\frac{-6\pi b}{\alpha}\right)\right\} \tag{45}$$

$$Q_4 = 5\varepsilon^4 \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-4b\psi)\cos^4(\alpha\psi)(d\psi)$$

Expressing $cos^4(\alpha\psi)$ in terms of $cos(2\alpha\psi)$ and $cos(4\alpha\psi)$, gives:

$$Q_4 = \frac{5\eta^4}{8} \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-4b\psi)\{3+\cos(4\alpha\psi)+4\cos(2\alpha\psi)\}(d\psi)$$

$$Q_4 = \frac{5\eta^4}{8}\left(\frac{3}{4b}+\frac{b}{4}+\frac{4b}{\alpha^2+4b^2}\right)\exp\left(\frac{-8\pi bs}{\alpha}\right)\left\{1-\exp\left(\frac{-8\pi b}{\alpha}\right)\right\} \tag{46}$$

Note that as $s \rightarrow \infty$ or $b = 0$, only Q_0 remains, since $Q_1 = Q_2 = Q_3 = ...Qp = 0$. Where $b \neq 0$, the period of the (s + 1)th cycle in the nth orbit of revolution, is obtained as the sum:

$$\begin{aligned} T_{(s+1)} &= \frac{mn^2}{LB^2}\sum_{p=0}^{\infty} Q_p \\ &= \frac{mn^2}{LB^2}\left(Q_0 + Q_1 + Q_2 + Q_3 + ... + Q_p + ... + Q_\infty\right) \end{aligned} \tag{47}$$

Since the eccentricity η is small and b is smaller, neglecting powers of η greater than 2 and powers of b greater than 1, we obtain $Q_1 \approx Q_3 \approx Q_5 \approx ... \approx Q_{2p+1} \approx 0$. An approximate expression for the period of revolution in the first cycle (with $s = 0$), in the nth orbit, is obtained as the sum:

$$\begin{aligned} T_1 &= \frac{mn^2}{LB^2}\sum_{p=0}^{\infty} Q_p \approx Q_o + Q_2 + ... + Q_{2p} + ... + Q_\infty \approx Q_0 + Q_2 \approx \\ &\frac{mn^2}{LB^2}\left[\frac{2\pi}{\alpha} + \frac{3\eta^2}{2}\left(\frac{1}{2b}+\frac{b}{2}\right)\left\{1-\exp\left(-\frac{4\pi b}{\alpha}\right)\right\}\right] \end{aligned} \tag{48}$$

With $\alpha^2 = 1 - b^2$, we obtain $\alpha \approx 1$ and:

$$T_1 \approx \frac{mn^2}{LB^2}\left(\frac{2\pi}{\alpha} + 3\eta^2\frac{2\pi}{\alpha}\right) \approx \frac{mn^2}{LB^2}\left(2\pi + \frac{3\eta^2}{2}2\pi\right)$$

$$T_1 \approx \frac{2\pi mn^2}{LB^2}\left(1+\frac{3\eta^2}{2}\right) \tag{49}$$

Equation (49) gives the time taken, in the first cycle, for the particle, in the nth orbit, to go through $2\pi/\alpha$ radians, due to the rotation of the major axis that goes through $(2\pi/\alpha-\pi)$ radians. After a great number of cycles $(s \rightarrow \infty)$, the period of revolution, through 2π radians, in the steady circle of the nth orbit, is:

$$T_n = \frac{2\pi mn^2}{LB^2} \tag{50}$$

T_n is the period of revolution of the particle in the steady orbit, a circle of radius $r_n = n/B = 16\pi n\varepsilon_o L^2/me$ with speed $v_n = BL/nm = e^2/16\pi n\varepsilon_o L$ and frequency $f_n = LB^2/2\pi mn^2 = me^4/2\pi L^3(16\pi n\varepsilon_o)^2$.

The period of the first cycle T_1, (equation (49), is the highest while that in the steady orbit T_n (equation 50) is the least. The wavelength, λ_1 of radiation, during the 1st cycle of the nth orbit, is:

$$\lambda_1 = cT_1 = \frac{2\pi mn^2 c}{LB^2}\left(1 + \frac{3\eta^2}{2}\right) \tag{51}$$

where c is the speed of light in a vacuum. The separation, splitting or increment of the wavelengths, $\Delta\lambda$, is:

$$\Delta\lambda = \frac{3\eta^2 \pi mn^2 c}{LB^2} \tag{52}$$

The ratio of the separation of wavelengths and the wavelength, with respect to revolution in the nth stable orbit, the same as the magnitude of separation of frequencies, is:

$$\frac{\Delta\lambda}{\lambda_n} = -\frac{\Delta f}{f_n} = \frac{3\eta^2}{2} \tag{53}$$

This ratio is related to the splitting, spread or "fine structure" of a spectral line due to frequency of radiation from the hydrogen atom. The radiation is not of precise frequencies, but has a spread around the frequency of revolution of a particle in the nth stable orbit.

7 Conclusion

The orbit of revolution of a neutral body, round a gravitational force of attraction, is a closed ellipse, without radiation of energy. Such a periodic motion may continue *ad infinitum*. On the other hand, a charged particle revolves in an unclosed (aperiodic) elliptic orbit, with emission of radiation at the frequency of revolution, before settling into a stable circular orbit. This explains the source of atomic radiation and stability of atoms.

The unipolar revolution of an electron round a nucleus, as discussed in section 2, leads to the development of unipolar model or nuclear model of the hydrogen atom. Similarly, bipolar motion of two oppositely charged particles of the same mass round a centre of revolution, as discussed under section 3, leads to the development of bipolar model or non-nuclear model of the hydrogen atom.

It is shown by the author [11] that the mass of a particle is independent of its speed. Therefore, in the treatment of the motion of electrons round a centre of revolution, in the unipolar or bipolar model of the hydrogen atom, relativistic effects were not taken into consideration. Neither was the spin of a revolving particle regarded in the emission of radiation from the hydrogen atom.

The paper concludes that the revolution of a charged particle in a circular orbit, round a centre of force of attraction, is inherently stable. Radiation takes place only if the particle is excited by being dislodged from the stable circular orbit. An excited particle revolves in an aperiodic elliptic orbit, emitting energy, as given by equation (34), before reverting back into the stable circular orbit.

The narrow spread of frequencies, with respect to revolution in the nth orbit, as given by equation (53), may explain the “fine structure” [12] of the spectral lines of radiation from the hydrogen atom, without considering relativistic effects, electron spin or quantum mechanics. .

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