

# ON THE CORRECT FORMULATION OF THE LAW OF THE EXTERNAL PHOTOELECTRIC EFFECT

Temur Z. Kalanov

Home of Physical Problems, Pisatelskaya 6a, 100200 Tashkent, Uzbekistan.  
tzk\_uz@yahoo.com, t.z.kalanov@mail.ru, t.z.kalanov@rambler.ru

**Abstract.** The critical and correct scientific analysis of the generally accepted theory of the external photoelectric effect is proposed. The methodological basis for the analysis is the unity of formal logic and of rational dialectics. It is shown that Einstein's formulation of the law of the photoelectric effect is not free from the following objection. The terms of Einstein's formula characterize the quantitative determinacy (i.e., energy) which belongs and is related to the different material objects: "photon", "electron in metal", and "electron not in metal". This signifies that Einstein's formula represents violation of the formal-logical laws of identity and absence (lack) of contradiction. The correct mathematical formulation of the law of the external photoelectric effect within the framework of the system approach is proposed. The correct formulation represents the proportion by relative increments of the energy of the incident photon and the energy of the emitted electron. The proportion describes the linear relationship between the energy of the incident photon and the energy of the emitted electron.

**Keywords:** physics, mathematical physics, electronic physics, quantum chemistry, mathematics, formal logic, dialectics, philosophy of science.

**PACS:** 01.55.+b, 01.40.Fk, 01.65.+g, 01.70.+w, 02.90.+p

## INTRODUCTION

Recently, the progress of sciences, engineering, and technology has given rise to a new problem: the problem of rationalization of the fundamental sciences (for example, theoretical physics and mathematics). Rationalization of sciences is impossible without rationalization of thinking and critical analysis of the foundations of sciences within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics. Therefore, one should call achievements of classics of sciences in question within the framework of the correct methodological basis. As the critical analysis shows [1-27], the foundations of theoretical physics and of mathematics contain formal-logical and dialectical errors. This signifies that any generally accepted theory can be rebutted if it contradicts to the formal-logical and dialectical laws.

As is known, researches of the external photoelectric effect (photoeffect) are one of remarkable scientific achievements in physics of 19-20 centuries. H. Hertz (1887), A. Stoletow (1888-1891), J. J. Thomson (1899), P. Lenard (1899, 1900, 1902), R. Millikan (1916), and other scientists proved experimentally that the energy (the maximum speed) of the photoelectrons does not depend on the intensity of incident light and is directly proportional to the frequency of light [28-33]. A. Einstein [34] proposed the quantum theory of external photoelectric effect. The theory is in accordance with experimental observations of photoelectric effect. However, in my opinion, this does not mean that the theory is free from objection.

The purpose of this work is to propose the critical analysis of the generally accepted theory of external photoelectric effect. The methodological basis for the analysis is the unity of formal logic and of rational dialectics.

## 1. METHODOLOGICAL BASIS

As is known, correct methodological basis of science is the unity of formal logic and of rational dialectics. Use the correct methodological basis is a necessary condition for correct analysis to make distinction between truth and falsehood. However, this fact is ignored by majority of scientists until now. Therefore, the main propositions of formal logic and of materialistic dialectics which are used in the present work must be stated.

1. The system is a set of elements that are in relations and connections with each other, forming certain integrity, unity.

2. The system principle reads as follows: property of system is not a consequence of the properties of its elements; the system determines the properties of the elements; and the properties of elements characterize the system;

3. Structure (construction, arrangement, order) is a set of stable connections (bonds) in object, which ensure its integrity and qualitative self-identity (i.e., ensure conservation of the basic properties) under different external and internal changes;

4. The system analysis of material system represents a task of finding the states of a material system. This task can be reduced to the task of finding quantitative (tabular or analytical) relationships between the characteristics of the elements of the material system under condition of conservation of the structure (i.e., qualitative determinacy) of the system. The correct solution of the task should be based on the following practical operations (steps): (a) one chooses the element which must be subjected to quantitative change (i.e., to movement); selected element undergoes quantitative change without changing in the qualitative determinacy of the system; (b) one finds quantitative changes in other elements stipulated (conditioned) by the change in the selected element; these changes should not lead to a change in the structure of the system (i.e., to a change in the qualitative determinacy of the system); (c) one finds the boundaries of quantitative changes within which the system remains identical to itself; (d) one finds the elements that are not changed; (e) one finds a quantitative (tabular or analytic) relationships between the values and dimensions of variables quantities which characterize elements. However, it should be emphasized that one can obtain an analytical solution of the task only in case of a simple statement of the problem or in the case of simple systems. In these cases, an analytical solution represents a proportion. The proportion represents linear relationship between relative increments of quantities describing the different elements.

5. Property is a philosophical category that designates such aspect of material object, which stipulates (determines) difference or commonality between other objects. Property is one of the aspects of the given object or phenomenon. Some properties express qualitative determinacy of object, others express quantitative determinacy of object;

6. Energy is general quantitative measure of motion and of interaction of all species (kinds) of matter. Energy does not emerge (appear, spring up) from nothing and does not vanish (disappear; exterminate). It can only move from one form to another. Various (different) forms of energy are as follows: mechanical, internal, electromagnetic, chemical, nuclear and other forms. The law of conservation of energy reads as follows: the energy of an isolated system is conserved:

$$E^{(isolated\ system)} = const$$

7. Energy is an inherent (inalienable) physical property, an essential feature of a material object. The energy of a material object represents a physical quantity.

8. The material object is a body, a field, a particle, as well as a system of bodies, of fields, of particles.

9. Physical quantity is the unity of the qualitative determinacy and of the quantitative determinacy of a material object. Mathematics describes the change in the quantitative

determinacy of a material object (physical quantity). In terms of formal logic, mathematics does not describe changes in qualitative determinacy of an object.

10. Mathematics studies the quantitative determinacy belonging to the qualitative determinacy of the object. In accordance with formal logic, the left-hand side and right-hand side of the mathematical expression describing the property of a system should be relate and belong to the qualitative determinacy of this system, i.e.,

$$\begin{aligned} &(\text{qualitative determinacy of system}) = \\ &(\text{qualitative determinacy of system}). \end{aligned}$$

The left-hand side and right-hand side of the mathematical expression describing the property of element should be relate and belong to the qualitative determinacy of this element, i.e.

$$\begin{aligned} &(\text{qualitative determinacy of element}) = \\ &(\text{qualitative determinacy of element}). \end{aligned}$$

11. Both quantitative and qualitative determinacy of an object must obey logical laws. Therefore, according to the logical law of identity, the left-hand and right-hand sides of the mathematical equation must belong to the same physical object (i.e. to the same property of physical object or the physical model of the object). And, according to the logical law of absence (lack) of contradiction, the left-hand and right-hand sides of the mathematical equation must not belong to different physical objects (i.e., to different properties, models).

## 2. THE STARTING POINTS OF THE CORRECT THEORY OF THE EXTERNAL PHOTOELECTRIC EFFECT

1. As is known, emission of electrons from a metal surface into a vacuum under the influence of the incident monochromatic light is called the external photoelectric effect (photoeffect). The experimental device for research of the external photoelectric effect is the evacuated balloon with window. The ultraviolet light falls through a quartz window upon the surface of the cathode (electrode of an alkali metal) and ejects electrons from the surface. The emitted electrons are called photoelectrons. The other electrode – the anode – collects the emitted photoelectrons. The current in the electric circuit of the device arises when the electric field takes action between the anode and the cathode. Voltage-current characteristic (i.e., the dependence of the photocurrent on the voltage between the electrodes) is determined. Also, the stopping voltage under which the value of current is zero is determined. These data lead to the following conclusion: the kinetic energy of the photoelectrons increases linearly with increase of the frequency of light [35].

2. The following assertions are true:

(a) the interaction of monochromatic (UV) light with the alkali metal surface can be described as a set of separated processes (events). Each separated (individual) process (event) is the emergence of “electron not in metal” as a result of the interaction of a photon with the “electron in metal”. “Electron in metal” absorbs the incident photon (i.e., absorbs the energy of the incident photon) and is converted into “electron not in metal”:

$$\text{“electron in metal”} \rightarrow \text{“electron not in metal”}.$$

This signifies that “electrons not in metal” with the energies  $E_1^{(\text{electron not in metal})}$ ,  $E_2^{(\text{electron not in metal})}$ ,  $E_3^{(\text{electron not in metal})}$  ... correspond to photons with energies  $E_1^{(\text{photon})}$ ,  $E_2^{(\text{photon})}$ ,  $E_3^{(\text{photon})}$ , ... ;

(b) the physical system,

$$S(\textit{photon}; \textit{electron in metal}; \textit{electron not in metal})$$

(where the “photon”, “electron in metal”, and “electron not in metal” are the elements of the system), represents a closed system for each separated (individual) process (event);

(c) the qualitative determinacy of the elements “photon”, “electron in metal”, and “electron not in metal” is different. This difference is expressed by the formal-logical law of absence (lack) of contradictions:

$$\begin{aligned} \textit{“electron in metal”} &\neq \textit{“electron not in metal”}, \\ \textit{“photon”} &\neq \textit{“electron”}. \end{aligned}$$

Therefore, the concepts of “electron in metal” and “electron not in metal” are not identical to each other. The relation between these concepts is the relation of disagreement.

(d) the left-hand side and right-hand side of the mathematical expression describing the property of element should be relate and belong to the qualitative determinacy of this element, i.e.

$$\begin{aligned} (\textit{qualitative determinacy of the element}) &= \\ (\textit{qualitative determinacy of the element}); \\ \textit{“photon”} &= \textit{“photon”}, \\ \textit{“electron in metal”} &= \textit{“electron in metal”}, \\ \textit{“electron not in metal”} &= \textit{“electron not in metal”}. \end{aligned}$$

In this case, the mathematical relationship satisfies the formal-logical law of identity;

(e) the left-hand side and the right-hand side of the mathematical expression describing the property of set of unconnected elements,

$$\textit{“photon”} + \textit{“electron in metal”} + \textit{“electron not in metal”},$$

do not belong to the qualitative determinacy of certain (separated) element or of system. Therefore, such mathematical expression does not satisfy the formal-logical law of identity and is wrong.

In accordance with the results of the experiments, and from the above-stated assertions, a new formulation of the problem follows: if the change in the energy of photon results in the change in the energy of photoelectron, then one should find a mathematical relationship between the relative increment of the energy of photon and the relative increment of the energy of photoelectron.

The problem has the following solution. Mathematical relationship between relative increments is as follows:

$$\left( \frac{E^{(\textit{photon})} - E_1^{(\textit{photon})}}{E_1^{(\textit{photon})}} \right) = \left( \frac{E^{(\textit{electron not in metal})} - E_1^{(\textit{electron not in metal})}}{E_1^{(\textit{electron not in metal})}} \right)$$

where  $E_1^{(\textit{electron not in metal})} \neq 0$  and  $E_1^{(\textit{photon})} \neq 0$  are a certain values of the variable quantities  $E^{(\textit{electron not in metal})}$  and  $E^{(\textit{photon})}$ , respectively. This relationship represents the proportion; it describes the mutual changes in the energies of the elements of the system; it satisfies the formal-logical law of identity; it is the unique one. Therefore, this relationship is the only correct formulation of the law of the photoelectric effect:

$$E^{(not\ in\ metal)} = \left( \frac{E_1^{(electron\ not\ in\ metal)}}{E_1^{(photon)}} \right) E^{(photon)}$$

where

$E^{(photon)} = h\nu^{(photon)}$  is the energy of the photon;

$E^{(electron\ not\ in\ metal)} \equiv E^{(free\ electron)} = h\nu^{(electron)}$  is the energy of the free electron;

$h$  is Planck constant;

$\nu^{(photon)}$  is oscillation frequency of photon;

$\nu^{(electron)}$  is oscillation frequency of electron.

It should be emphasized that, in formal-logical point of view, the expression  $h\nu$  is correct if  $h$  is oscillating quantity [8]. Therefore  $\nu^{(photon)}$  represents the photon frequency of oscillation of the quantity  $h$ ;  $\nu^{(electron)}$  represents the electron frequency of oscillation of the quantity  $h$ .

### 3. THE OBJECTION CONCERNING THE GENERALLY ACCEPTED THEORY OF THE EXTRNAL PHOTOELECTRIC EFFECT

As is known, generally accepted formulation of the law of the photoelectric effect represents the mathematical expression of the law of conservation of energy [33]:

$$E^{(electron\ not\ in\ metal)} = E^{(mono-chromatic\ light)} - E^{(electron\ in\ metal)}$$

or, in quantum point of view,

$$E^{(electron\ not\ in\ metal)} = E^{(photon)} - E^{(electron\ in\ metal)}$$

where

$E^{(mono-chromatic\ light)}$  is the energy of the incident monochromatic light;

$E^{(electron\ in\ metal)}$  is the work of exit, i.e., the binding energy of electron in metal;

$E^{(electron\ not\ in\ metal)}$  is the energy of the photoelectron (i.e., emitted electron);

$E^{(photon)} = h\nu^{(photon)}$  is the energy of the photon;

This expression – Einstein’s formula [33] – is a linear relationship between the quantities  $E^{(electron\ in\ metal)}$  and  $E^{(electron\ in\ metal)}$ . The essence of Einstein’s formula is that the terms belong to different physical objects (i.e., to different qualitative determinacy). Therefore, Einstein’s formula is not free from the following objection.

Qualitative determinacy of elements “photon”, “electron in metal”, and “electron not in metal”, as well as the concepts of “photon”, “electron in metal”, and “electron not in metal” are considered to be identical to each other in the Einstein formula. But the qualitative determinacy of elements “photon”, “electron in metal”, and “electron not in metal” of the physical system,

$$S(\text{photon; electron in metal; electron not in metal}),$$

is different (nonidentical). This difference (non-identity) is expressed the formal-logical law of absence (lack) of contradictions:

$$\begin{aligned} \text{"electron in metal"} &\neq \text{"electron not in metal"}, \\ \text{"photon"} &\neq \text{"electron"}; \end{aligned}$$

Also, the concepts of "photon", "electron in metal", and "electron not in metal" are not identical to each other. The relation between these concepts is the relation of disagreement.

Consequently, Einstein's formula contradicts to the formal-logical laws of identity and absence (lack) of contradiction and is incorrect in essence.

**Remark.** As has been shown in [20, 21, 25-27], the graphical representation of mathematical functions is inadmissible (incorrect) operation if: (a) the scales (i.e., abscissa and ordinate) of the coordinate system have different (unlike) dimensions (i.e., different qualitative determinacy): for example, the voltage dimension ("V") and the current dimension ("amp"), the frequency dimension ("1/sec") and the stopping potential ("V"); (b) the scales characterize different material objects. Really:

(1) if the scales (abscissa and ordinate) have different dimensions (i.e., different qualitative determinacy), then the point of intersection of the scales and the point which does not lie on the scale have both dimensions. But this contradicts to the formal logic and practice;

(2) if some scale has the dimension of the energy of photon and the other scale has the dimension of the energy of electron, then this signifies that: (a) the point of intersection of the scales and the point not lying on scale belong both the photon and the electron (i.e., the points belong to the different (non-identity) material objects); (b) two different value of energy characterize a point on the plane; (c) the values of the energy are the projections of the point on the plane; (d) the distance of this point from the zero point represents the length of the line segment. But in this case, the points (which do not lie on the scale), line segments, plane, and the coordinate system do not exist. Consequently, the assertion that the points (which are not on the scales), the line segments, plane, and coordinate system exist is erroneous because it contradicts to formal logic and practice.

#### 4. DISCUSSION

1. As is known, formal logic is the general science of the laws of correct thought. The laws of formal logic represent the theoretical generalization and reflection of practice in human consciousness. Consequently, formal logic exists in human consciousness and practice. Practice is criterion of validity (trueness, truth) of formal logic.

2. Dialectical materialism is the general science of the most common (general) kinds of connections and laws of development of nature, of human society, and of thought. The laws of dialectics represent the theoretical generalization and reflection of practice in human consciousness. Consequently, dialectics exists in human consciousness and practice. Practice is criterion of validity (trueness, truth) of dialectics.

3. The only correct methodological basis of sciences is the unity of formal logic and of rational dialectics. Mathematics and theoretical physics are sciences if and only if its foundations are formulated within the framework of correct methodological basis.

4. As is well known, science originated in the ancient world in connection with the requirements of social practice and had quick development since 16-17-th ages. In the course of historical development, science changed into a productive force and into the most important social institution which has a significant impact on all spheres of society. Today, science is a huge sphere of human activity aimed at obtaining new knowledge and theoretical systematization of objective knowledge about reality. Sum of objective knowledge underlies the scientific picture of the world. The scientific picture of the world plays an important world-outlook role in the development of human society.

5. Science is developed in the inductive way, i.e., in the way of "negation of negation". Therefore, extensive and revolutionary periods are alternated in the development of science.

Scientific revolutions lead to a change in the structure of science, the cognition principles, categories and methods, as well as forms of organization of science.

Inevitability of scientific revolutions was first emphasized by A. Einstein: “progress of science will be the cause of revolution in its foundations” (A. Einstein). Also, the following statement is true one: a critical reassessment of the standard foundations of science leads to the progress of science. These aspects in development of science are characterized, for example, by A. Einstein’s words: “There has been formed a notion that the foundations of physics were finally established and the work of a theoretical physicist should be to bring a theory in correspondence with all the time increasing abundance of the investigated phenomena. Nobody thought that a need for radical rebuilding of the foundations of all physics could arise. Our notions of physical reality never can be final ones”. At present, the validity of Einstein’s assertion is confirmed by the poor states of sciences.

In this connection, the problem of critical analysis of foundations of theoretical physics and of mathematics within the framework of the correct methodological basis (i.e., the unity of formal logic and of rational dialectics) arises. This methodological basis represents the system of logical laws and of general-scientific methods of cognition of reality: observation and experiment, analysis and synthesis, induction and deduction, analogy and hypothesis, logical and historical aspects, abstraction and idealization, generalization and limitation, ascension from concrete concepts to abstract concepts, comparison, modeling, etc.

6. The necessity of application of general-scientific methods for the critical analysis of theoretical physics and of mathematics is also stipulated by the fact that the foundations of theoretical physics and of mathematics contain vagueness which can not become aware and be formulated in the standard physical and mathematical terms because the physics and mathematics do not contain many universal (general-scientific, philosophical) concepts; moreover, origin of vagueness is often stipulated by “thoughtless use of mathematics” (L. Boltzmann). In this case, formal-logical errors come into existence in mathematics and natural-scientific theories. In my opinion, the errors in mathematics and physics are an inevitable consequence of the inductive method of cognition.

This gives possibility to elicit, to reveal, to recognize errors done by the great scientists of the past time and leads to the abolishment (elimination) of set of standard theories. But even the mistakes done by the great scientists contribute to progress in science: “false hypotheses often rendered more services than the true ones” (H. Poincare) because mistakes extend consciousness of scientists. Such is the dialectics of truth and of lie in science.

7. Einstein’s great achievement was recognized internationally by the Nobel Prize award to him in 1921 for “his discovery of the law of the photoelectric effect”. It was recognition of his contribution to world science of that time. But today this fact signifies that one should call great scientific achievements in question within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics.

## CONCLUSION

Thus, the correct scientific analysis of the generally accepted theory of the external photoelectric effect is possible only within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics. The correct scientific analysis of the theory leads to the following results:

1. The mathematical formulation of the law of the external photoelectric effect proposed by Einstein is not free from objection. The objection is as follows.

The terms of Einstein’s formula characterize quantitative determinacy (i.e., energy) which belongs and is related to different material objects: “photon”, “electron in a metal”, and “electron not in metal”. But according to the formal-logical laws of identity and absence (lack) of contradiction: (a) the terms of the mathematical (quantitative) relationship should belong and be related to the qualitative determinacy of only one material object; (b) the terms of the

mathematical (quantitative) relationship should not belong and be related to the qualitative determinacy of different (non-identical) material objects. Therefore, Einstein's formula contradicts to the formal-logical laws of identity and absence (lack) of contradiction.

2. The correct mathematical formulation of the law of external photoelectric effect is proposed. The formulation is based on the use of formal logic and of the system approach. The essence of the formulation is that it represents the proportion by relative increments of the energy of the incident photon and the energy of emitted electron. The proportion describes correctly the linear relationship between the energy of the incident photon and the energy of the emitted electron.

## REFERENCES

- [1] T.Z. Kalanov, "The crisis in theoretical physics: The problem of scientific truth". *Prespacetime Journal*, Vol. 1, No. 5 (2010), pp. 824-842.
- [2] T.Z. Kalanov, "The critical analysis of the foundations of theoretical physics. Crisis in theoretical physics: The problem of scientific truth". LAP Lambert Academic Publishing. ISBN 978-3-8433-6367-9, (2010).
- [3] T.Z. Kalanov, "Critical analysis of Bose–Einstein and Fermi–Dirac statistics". *Elixir (Statistics)*, No. 45 (2012), pp. 7657-7659.
- [4] T.Z. Kalanov, "The Boltzmann distribution: a logical error". *Elixir (Adv. Pow.)*, No 49 (2012), pp. 9935-9936.
- [5] T.Z. Kalanov, "The correct analysis of theory of photon gas". *Elixir (Nuclear & Radiation Physics)*, No. 50 (2012), pp. 10197-10205.
- [6] T.Z. Kalanov. "Where are the logical errors in the theory of Big Bang?". *Bulletin of the Amer. Phys. Soc.*, (April Meeting), Vol. 60, No. 4 (2015).
- [7] T.Z. Kalanov. "Where are the logical errors in the theory of Big Bang?". *Scientific GOD Journal*, Vol. 5, No. 5 (2014), p. 432-433.
- [8] T.Z. Kalanov, "On a new analysis of the problem of Planck constant". *Bulletin of the Amer. Phys. Soc. (April Meeting)*, Vol. 54, No.4 (2009).
- [9] T.Z. Kalanov, "Critical analysis of the foundations of differential and integral calculus". *MCMS (Ada Lovelace Publications)*, (2011), pp. 34-40.
- [10] T.Z. Kalanov, "Logical analysis of the foundations of differential and integral calculus". *Indian Journal of Science and Technology*, Vol. 4, No. 12 (2011).
- [11] T.Z. Kalanov, "Logical analysis of the foundations of differential and integral calculus". *Bulletin of Pure and Applied Sciences*, Vol. 30 E (Math.& Stat.), No. 2 (2011), pp. 327-334.
- [12] T.Z. Kalanov, "The critical analysis of the Pythagorean theorem and of the problem of irrational numbers". *Basic Research Journal of Education Research and Review*, (ISSN 2315-6872, <http://www.basicresearchjournals.org>), Vol. 2, No. 4 (2013), pp. 59-65.
- [13] T.Z. Kalanov, "The logical analysis of the Pythagorean theorem and of the problem of irrational numbers". *Asian Journal of Mathematics and Physics*, (ISSN 2308-3131, <http://scienceasia.asia>), Vol. 2013 (2013), pp. 1-12.
- [14] T.Z. Kalanov, "On the logical analysis of the foundations of vector calculus". *International Journal of Scientific Knowledge. Computing and Information Technology*, Vol. 3, No. 2 (2013) pp. 25-30.
- [15] T.Z. Kalanov, "On the logical analysis of the foundations of vector calculus". *International Journal of Multidisciplinary Academic Research*, Vol. 1, No. 3 (2013).
- [16] T.Z. Kalanov, "On the logical analysis of the foundations of vector calculus". *Journal of Computer and Mathematical Sciences*, Vol. 4, No. 4 (2013), pp. 202-321.
- [17] T.Z. Kalanov, "On the logical analysis of the foundations of vector calculus". *Journal of Research in Electrical and Electronics Engineering (ISTP-JREEE)*, (ISSN: 2321-2667), Vol. 2, No. 5 (2013), pp. 1-5.



- [18] T.Z. Kalanov, "The critical analysis of the Pythagorean theorem and of the problem of irrational numbers". Global Journal of Advanced Research on Classical and Modern Geometries, (ISSN: 2284-5569), Vol. 2, No. 2 (2013), pp. 59-68.
- [19] T.Z. Kalanov, "On the logical analysis of the foundations of vector calculus". Aryabhatta Journal of Mathematics & Informatics, (ISSN: 0975-7139), Vol. 5, No. 2 (2013), pp. 227-234.
- [20] T.Z. Kalanov, "On the system analysis of the foundations of trigonometry". Journal of Physics & Astronomy, ([www.mehtapress.com](http://www.mehtapress.com)), Vol. 3, No. 1 (2014).
- [21] T.Z. Kalanov, "On the system analysis of the foundations of trigonometry". Pure and Applied Mathematics Journal, Vol. 3, No. 2 (2014), pp. 26-39.
- [22] T.Z. Kalanov. "Critical analysis of the foundations of the theory of negative number". International Journal of Informative & Futuristic Research (IJIFR, [www.ijifr.com](http://www.ijifr.com)), Vol. 2, No. 4 (2014), pp. 1132-1143.
- [23] T.Z. Kalanov. "Critical analysis of the foundations of the theory of negative numbers". Aryabhatta Journal of Mathematics & Informatics, Vol. 7, No. 1 (2015), pp. 3-12.
- [24] T.Z. Kalanov. "On the formal-logical analysis of the foundations of mathematics applied to problems in physics". Aryabhatta Journal of Mathematics & Informatics, Vol. 7, No. 1 (2015), pp. 1-2.
- [251] T.Z. Kalanov. "Critical analysis of the foundations of pure mathematics". Mathematics and Statistics (CRESCO, <http://crescopublications.org>), V. 2, No. 1 (2016), MSOA-2-005.
- [26] T.Z. Kalanov. "Critical analysis of the foundations of pure mathematics". International Journal for Research in Mathematics and Mathematical Sciences, V. 2, No. 2 (2016), pp. 15-33.
- [27] T.Z. Kalanov. "Critical analysis of the foundations of pure mathematics". Aryabhatta Journal of Mathematics & Informatics, Vol. 8, No. 1 (2016), pp. 1-14.
- 
- [28] R.D. Knight, "Physics for Scientists and Engineers With Modern Physics: A Strategic Approach". Pearson-Addison-Wesley, 2004. ISBN 0-8053-8685-8.
- [29] K.A. Tsokos, "Cambridge Physics for the IB Diploma". Cambridge University Press, 2010. ISBN 978-0-521-13821-5.
- [30] "Photoelectric effect". Physics 2000. University of Colorado, Boulder, Colorado.
- [31] E.N. Wichmann, "Quantum Physics". Berkeley Physics Course, V. 4. McGraw-Hill Book Company.
- [32] "The Photoelectric effect". Physics 320 Laboratory, Department of Physics, Davidson College, Davidson.
- [33] M. Fowler, "The Photoelectric Effect". Physics 252, University of Virginia.
- [34] A. Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt". Ann. Phys., V. 17 (1905), pp. 132-148.
- [35] R. Millikan, "A direct photoelectric determination of Planck's  $h$ ". Phys. Rev., V. 7, No. 3 (1916), pp. 355-388.