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Gravitational Charge

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ABSTRACT: This article aims to put the gravitational equations in the same format as the electromagnetic equations to use the same mathematical tools we use in electromagnetism. It will be evident, by the analogy between gravitational and electrical forces, that masses are like gravitational charges.

KEYWORDS: gravitational equations, gravitational charge, gravitational current.

Contents

1 Symbology

In this text we will use the following symbols with its abbreviated unit of measurements:

N = Newton, kg = kilogram, m = meter, s = second, V = Volt, C = Coulomb, A = Ampere.

E = Electric field intensity [N C^{-1}] [V m^{-1}];
D = Surface density of electric charge [C m^{-2}];
V_E = Electric potential [V] [Wb s^{-1}];
Φ_E = Electric flux [N m^2 C^{-1}] [V m];
q_E = Electric charge [C];
I_E = Electric current [C s^{-1}] [A];
J_E = Surface density of electric current [A m^{-2}] [C s^{-1} m^{-2}];
ε_0 = Vacuum electric permittivity [C^2 N^{-1} m^{-2}] [C V^{-1} m^{-1}];
k_E = Electrostatic constant [N m^2 C^{-2}] [m^3 C^{-1} s^{-2}];
G = Gravitational field intensity [N kg^{-1}] [m s^{-2}];
M = Surface density of gravitational charge [kg m^{-2}];
V_G = Gravitational potential [N m kg^{-1}] [m^2 s^{-2}];

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Φ_G = Gravitational flux [N m^2 kg^{-1}];
q_G = Gravitational charge [kg];
I_G = Gravitational current [kg s^{-1}] [N s m^{-1}];
J_G = Surface density of gravitational current [kg s^{-1} m^{-2}];
ρ_G = Volume density of gravitational charge [kg m^{-3}];
γ_0 = Vacuum gravitational permeability [kg^2 N^{-1} m^{-2}] [kg s^2 m^{-3}];
k_G = Gravitostatic constant = 6.6739*10^{-11} [N m^2 kg^{-2}] [m^3 kg^{-1} s^{-2}];
F = Force [N] [kg m s^{-2}];
r = Radial length [m];
l = Length [m];
S = Area [m^2];
t = Time [s].

2 Introduction

After Newton's Universal Gravitation Law, the gravitational interaction between two bodies may be expressed by a central force proportional to the bodies' masses and inversely proportional to the square of the distance between them.[1]

$$\vec{F}=G\frac{m_1 m_2}{r^2}\hat{r}$$

Here G is the Universal Gravitational Constant. The equation that represents this law have intrigued for many decades for its similarity with the force equation between two electric charges:

$$\vec{F}=k_E\frac{q_{E1}q_{E2}}{r^2}\hat{r}=\frac{1}{4\pi\varepsilon_0}\frac{q_{E1}q_{E2}}{r^2}\hat{r}$$

Comparing both force equations, we see:

- A direct relation between G and k_E.
- A direct relation between m_1, m_2 and q_{E1}, q_{E2}.
- In the electric force equation, the electrostatic or dielectric constant k_E, called Coulomb's constant, has an inverse relation with the electric permittivity ε_0.
- Both equations have an inverse relation with the square of the distance.

This lead us to consider the universal gravitational constant G like a gravitostatic constant k_G related to the gravitational permeability γ_0 in the same way that the electrostatic constant k_E is related to the electric permittivity ε_0. This way, the unit of measurement of γ_0 [kg^2 N^{-1} m^{-2}] is like its electric counterpart ε_0 [C^2 N^{-1} m^{-2}] and we may equate the gravitational permeability γ_0 considering $G\rightarrow k_G$:

$$k_G=G=\frac{1}{4\pi\gamma_0} \qquad \gamma_0=\frac{1}{4\pi G}=\frac{1}{4\pi k_G}=1.1924*10^9 kg^2 N^{-1} m^{-2}$$

This is the gravitational permeability of vacuum of space.

3 Gravitational Charge

We may see too that the masses are in the place of the electrical charges and this lead us to consider the masses like gravitational charges q_G. With this considerations, the actual and the new gravitational force equations (considering $m\rightarrow q_G$, $G\rightarrow k_G$) are:

$$\vec{F}=G\frac{m_1 m_2}{r^2}\hat{r} \quad \rightarrow \quad \vec{F}=k_G\frac{q_{G1}q_{G2}}{r^2}\hat{r}=\frac{1}{4\pi\gamma_0}\frac{q_{G1}q_{G2}}{r^2}\hat{r}$$

Now we have a gravitational force equation totally similar to its electric counterpart. From here on we will use the character G for gravitational field and k_G for the Universal Gravitational Constant, now called gravitostatic constant.

4 Gravitational Field

The gravitational field equation may be obtained in the same way that the electric field is obtained by the Gauss’ law, considering a punctual gravitational charge enclosed by a spherical surface that surrounds it:

$$\oint_S \vec{G}\cdot d\vec{S}=\int_0^{2\pi}\int_0^{\pi} G r^2 sen\,\theta\, d\theta\, d\phi=4\pi r^2 G=\frac{q_G}{\gamma_0}$$

Isolating the field we see that the gravitational field equation may be defined like its electric field counterpart:

$$\vec{G}=\frac{1}{4\pi\gamma_0}\frac{q_G}{r^2}\hat{r}=k_G\frac{q_G}{r^2}\hat{r}$$

By the Newton’s second law $\vec{F}=m\vec{a}$, we define gravitational acceleration by the gravitational force that acts on a particle of gravitational charge q_G that is at a distance r from a uniform spherical body of gravitational charge Q_G, with $\vec{F}=q_G\vec{a}_G$. Isolating a_G, we may see that it is the same gravitational field **G**.

$$\vec{a}_G=\frac{\vec{F}}{q_G}=k_G\frac{Q_G}{r^2}\hat{r}=\frac{1}{4\pi\gamma_0}\frac{Q_G}{r^2}\hat{r}=\vec{G}$$

The unit of measurement of **G** [N kg^{-1}] is like its electric counterpart **E** [N C^{-1}], and the unit of measurement of acceleration [m s^{-2}] is the same as [N kg^{-1}] because [N] = [kg m s^{-2}]. Then, it is useful to remember that:

1. The unit of measurement of gravitational field is the same as the acceleration. It is the acceleration experienced by the particle at a distance r.
2. When an object is put in a gravitational field, its gravitational acceleration is numerically the same as the gravitational field.
3. When an object is accelerated, it is the same as creating a gravitational field for the object.

5 Surface Density of Gravitational Charge

Now we may define the surface density of gravitational charge **M** like the surface density of electric charge **D**. In the vacuum of space, there are simple relations between gravitational field **G** and surface density of gravitational charge **M**: $\vec{M}=\gamma_0\vec{G}$.

$$\vec{M}=\gamma_0\vec{G}=\frac{q_G}{4\pi r^2}\hat{r}$$

Here we may confirm that **M** is really a surface density of gravitational charge, like its electric counterpart, because $4\pi r^2$ is the spherical area that surrounds the charge q_G.

6 Gravitational Flux

The gravitational flux Φ_G in space that pass through a closed surface S that surrounds a gravitational charge may be equated like its electric flux counterpart Φ_E:

$$\Phi_G = \oint_S \vec{G} \cdot d\vec{S} = \frac{q_G}{\gamma_0} \qquad q_G = \gamma_0 \Phi_G$$

When the integral is over an open area, the gravitational flux indicates the quantity of lines of gravitational force field that pass through the area. This is, a limited area that has a surface density of gravitational charge has a total quantity of charge that may be equated by:

$$\Phi_G = \int_S \vec{G} \cdot d\vec{S} \qquad q_G = \gamma_0 \int_S \vec{G} \cdot d\vec{S} = \int_S \vec{M} \cdot d\vec{S} = \gamma_0 \Phi_G$$

7 Gravitational Current

Electric current I_E is the motion of electric charges with unit of measurement [C s^{-1}], then the gravitational current I_G is the motion of gravitational charges and its unit of measurement is [kg s^{-1}]. It defines the quantity of gravitational charge that passes by unit of time:

$$I_G = \frac{d q_G}{d t} = \frac{d}{d t} \int_S \vec{M} \cdot d\vec{S} = \gamma_0 \frac{d}{d t} \int_S \vec{G} \cdot d\vec{S} = \gamma_0 \frac{d \Phi_G}{d t}$$

Here we may see the two forms of gravitational current: conduction current, determined by the time variation of charge, and displacement current, determined by the time variation of flux. The surface density of gravitational charge J_G is defined like its electrical counterpart J_E:

$$\vec{J}_G = \frac{d \vec{M}}{d t} = \gamma_0 \frac{d \vec{G}}{d t}$$

8 Gravitational Potential

The gravitational potential measured at a distance R from a gravitational charge q_G, with $V_G = 0$ when $r = \infty$, is given like its electric counterpart by:[1]

$$V_G = -\int_\infty^R \vec{G} \cdot d\vec{r} = -\frac{q_G}{4\pi\gamma_0} \int_\infty^R \frac{1}{r^2} d r = \frac{q_G}{4\pi\gamma_0} \left[\frac{1}{r}\right]_\infty^R = \frac{1}{4\pi\gamma_0} \frac{q_G}{R} = k_G \frac{q_G}{R}$$

9 Gravitational Poisson’s Equation

Applying the Gauss’ divergence theorem for a continuous distribution of gravitational charge we may define the total charge q_G by:[1]

$$q_G = \int_V \rho_G \, d V = \gamma_0 \oint_S \vec{G} \cdot d\vec{S} = \gamma_0 \int_V \nabla \cdot \vec{G} \, d V$$

And from it we may extract $\rho_G = \gamma_0 \nabla \cdot \vec{G}$. The gravitational field may be obtained by the gradient of the gravitational potential $\vec{G} = -\nabla V_G$ and its divergent gives a Poisson's equation that is similar to its electric counterpart:

$$\nabla \cdot \vec{G} = -\nabla^2 V_G = \frac{\rho_G}{\gamma_0}$$

10 Gravitational Continuity Equation

The current of gravitational charge that flows out a closed surface boundary that limits a gravitational charge, invoking the divergence theorem, is:

$$I_G = \frac{d\, q_G}{d\, t} = \int_V \frac{\partial \rho_G}{\partial t} d\, V = -\oint_S \vec{J}_G \cdot d\, \vec{S} = -\int_V \nabla \cdot \vec{J}_G d\, V$$

Since this is true for any volume, we may extract the gravitational continuity equation, the mathematical statement of local conservation of gravitational charge (mass):

$$\frac{\partial \rho_G}{\partial t} = -\nabla \cdot \vec{J}_G$$

11 Conclusion

The mass of any object may be treated like a gravitational charge q_G, which has unit of measurement [kg]. The Universal Gravitational Constant may be related with a gravitational permeability γ_0 by an inverse relation $G = (4\pi \gamma_0)^{-1}$, so a more convenient symbol for it is k_G, the gravitostatic constant, like its electric counterpart k_E, the electrostatic constant.

Units of measurement of the new gravitational quantities are: charge [kg], current [kg s^{-1}], flux [N m^2 kg^{-1}], surface charge density [kg m^{-2}]; all are similar to their electric counterparts.

We have now similar equations for electric and gravitational quantities and we may use the same mathematical tools that we use for electromagnetism in calculating gravitational quantities.

	Electric		**Gravitational**	
Greatness	**Equation**	**Unit**	**Equation**	**Unit**
Field from a charge	$E(r) = \frac{1}{4\pi \varepsilon_0} \frac{q_E}{r^2}$	[N C^{-1}] [V m^{-1}]	$G(r) = \frac{1}{4\pi \gamma_0} \frac{q_G}{r^2}$	[N kg^{-1}] [m s^{-2}]
Field from a scalar potential	$\vec{E} = -\nabla V_E$	[N C^{-1}] [V m^{-1}]	$\vec{G} = -\nabla V_G$	[N kg^{-1}] [m s^{-2}]
Surface charge density	$\vec{D} = \varepsilon_0 \vec{E}$	[C m^{-2}]	$\vec{M} = \gamma_0 \vec{G}$	[kg m^{-2}]
Flux	$\Phi_E = \int_S \vec{E} \cdot d\vec{S}$	[N m^2 C^{-1}] [V m]	$\Phi_G = \int_S \vec{G} \cdot d\vec{S}$	[N m^2 kg^{-1}] [m^3 s^{-2}]
Charge	$q_E = \int_S \vec{D} \cdot d\vec{S} = \varepsilon_0 \Phi_E$	[C]	$q_G = \int_S \vec{M} \cdot d\vec{S} = \gamma_0 \Phi_G$	[kg]
Current	$I_E = \frac{d\, q_E}{d\, t} = \varepsilon_0 \frac{d\, \Phi_E}{d\, t}$	[C s^{-1}] [A]	$I_G = \frac{d\, q_G}{d\, t} = \gamma_0 \frac{d\, \Phi_G}{d\, t}$	[kg s^{-1}] [N s m^{-1}]
Scalar potential	$V_E = -\int_L \vec{E} \cdot d\vec{l}$	[V] [Wb s^{-1}]	$V_G = -\int_L \vec{G} \cdot d\vec{l}$	[N m kg^{-1}] [m^2 s^{-2}]

Bibliography

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