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A Constructive Model of Newtonian Gravitation

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Abstract. The paper explores whether Newtonian gravitation is mediated by force fields, and, if so, whether it can explain known gravitational phenomena. In conformity with the historical practice, the paper presents a physical model in which gravitational forces between objects are mediated by those force fields which are associated with the mass and momentum properties of the objects.

The model explains: Gravitational forces between masses, between mass and light, between photons, and between electromagnetic waves; and Gravity's effect on the time periods of atomic clocks, shifts in spectral lines, rays of light, and lengths of material rods. It addresses Mercury's orbital precession rate and the Pioneer anomaly. It derives Larmor's formula for gravitational radiation power from accelerating masses and applies it to estimate gravitational radiation power emissions from the moon, the planets, and the Hulse-Taylor binary pulsars. It advances suggestions for detecting gravitational radiation and measuring its speed.

The model makes *new* predictions: Gravitational forces can be attractive or repulsive; Electromagnetic waves gravitationally interact, so do photons; the Classical gravitational constant is not universally constant; Rods get longer closer to the mass; and Accelerating masses generate gravitational radiation power with angular distribution of a four-lobed quadrupole pattern and which propagates at less than the speed of light.

Keywords. Gravitation; Gravitational interaction; Gravitational force; Gravitational constant; Mass field; Momentum field; Gravitational field; Gravitational wave; Gravitational radiation.

Terms. 'Object' stands for an item of matter or energy. 'Charge' without a qualifier means electrical charge. 'Current' without a qualifier means electrical current.

Appendices. Appendix A contains relevant physical data, which are referenced as datum A(x) or data A(x, y). Appendixes B through E present specific derivations and details. Appendix F lists the results and predictions from the model. Appendix G has the nomenclature. Appendix H presents the readers' comments and the author's responses.

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1. Introduction

Newton discovers the law that gravitational attraction between two bodies is proportional directly to their masses and inversely to the square of their separation distance. This action-at-distance force law conveys how gravity behaves, but not how it is mediated. [1]

The strong, the weak, and electromagnetic interactions are mediated respectively by the strong, the weak, and electromagnetic fields associated respectively with the color, the weak, and electrical charge properties of the objects. The strong and the weak interactions are mediated at the microscopic levels; electromagnetic interactions are mediated at the microscopic through macroscopic levels; and gravitational interactions are known to occur at the macroscopic levels. At microscopic levels, the fields are fundamentally discrete (quanta); at macroscopic levels, the fields are effectively continuous (with values at each space-time point).

The aforementioned theme will be used as a guide to explore the mediation of Newtonian gravitation. [2]

The model will be developed in parts, of which this paper is the first:

- Part 1: Classical gravitation theory. Gravitational forces between objects will be formulated in terms of their pertinent static and dynamic properties and associated fields – at the macroscopic level.

Mass, as *gravitational charge*, is an inherent property of objects. Momentum, as *gravitational current*, is a dynamic property of objects.

Analogy with classical electrodynamics will be invoked. Special relativity will be considered in principle. Quantum theory will be used elementarily. Gauge symmetries will not be extracted.

Non-gravitational and other extraneous agents and effects will be ignored.

Coordinate systems will be used for the convenience of representing and analyzing gravitational phenomena. [3]

- Part 2: Gravitational Inertia.
- Part 3: Classical gravitational field theory.
- Part 4: Quantum gravitational field theory.

2. Assumptions

In analogy with classical electrodynamics, according to which electric charges have electric fields and electric currents create magnetic fields, we make two assumptions regarding masses (gravitational charges) and momenta (gravitational currents):

(1) Mass has an envelope of *mass field*.

(2) Momentum creates an envelope of *momentum field*.

3. The gravitation model

We define mass field and momentum field of Assumptions (1) and (2) and use them to formulate gravitational forces between objects.

The mass field $\boldsymbol{M}$ at a distance r from an object of mass m is defined as:

$$\vec{M} = S\,\frac{m}{r^3}\,\vec{r}\,, \tag{1}$$

where S is *mass-field coefficient*, which is the strength of the mass field of an object of 1 kg mass at 1 m. The mass field extends out to infinity and is uniform in all directions.

The momentum field $\boldsymbol{P}$ at a distance r from an object of momentum $\boldsymbol{p}$ is defined as:

$$\vec{P} = D\,\frac{\vec{p}\cdot\vec{r}}{r^4}\,\vec{r}\,, \tag{2}$$

where D is *momentum-field coefficient*, which is the strength of the momentum field of an object of 1 kg-m/s momentum at 1 m in the direction of momentum. The momentum field is not uniform in all directions.

The effective *momentum field range* q of an object with momentum p is defined as:

$$q = \sigma p \tag{3}$$

where σ is *momentum-field range coefficient*. An object with momentum of 1 kg-m/s has an effective momentum field range of σ m.

The sum of momentum field range q_1 of object-1 and q_2 of object-2 is thus:

$$q_{12} = q_1 + q_2 \tag{4}$$

Gravitational force F_s between object-1 and object-2 with mass m_1 and mass m_2 respectively is mediated by their mass fields as defined in (1):

$$F_s = S\,\frac{m_1 m_2}{r^2} \tag{5}$$

In analogy with electrodynamics, as like charges repel, so would like gravitational charges. That is, F_s is repulsive.

Gravitational force F_d between object-1 and object-2 with momentum $\boldsymbol{p_1}$ and momentum $\boldsymbol{p_2}$ respectively is mediated by their momentum fields as defined in (2):

$$F_d = D\,\frac{\vec{p}_1\cdot\vec{p}_2}{r^2} \tag{6}$$

In analogy with electrodynamics, as parallel currents attract, so would parallel gravitational currents. That is, F_d is attractive or repulsive as the angle between $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ is acute or obtuse.

The dimension of S/D is of the square of speed, which we denote by b. In electrodynamics, the speed of light depends on the constants of the media: $1/(\varepsilon_0\mu_0) = c^2$, where ε_0 is electric permittivity, μ_0 is magnetic permeability, and c is the speed of electric-magnetic (electromagnetic) wave in vacuum. By analogy, as the constants of the media in this model are S and D, the speed (b) of mass-momentum (gravitational) wave may be expressed by:

$$\frac{S}{D} = b^2 \tag{7}$$

Constants S, D, and b are independent of σ. Measurements thereof will be suggested later.

3.1 Universal gravitation

Figure 1 shows a sector of the universal sphere with center at the *Primordial Point* O, the space-time "point" where the universe originated. The Primordial Point is the primary space-time reference point for all objects in the universe. Relative to O, object A_1 at r_1 has momentum $\boldsymbol{p}_1$, and object A_2 at r_2 has momentum $\boldsymbol{p}_2$. The angle between $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ at O is α. Distance between A_1 and A_2 is r.

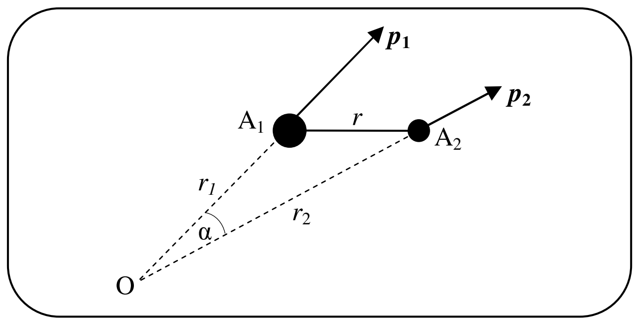

Figure 1. Objects A_1 and A_2, at r_1 and r_2, with their momenta $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ relative to the Primordial Point O.

At separation $r \le q_{12}$, momentum fields are effective. From (6), gravitational force between objects A_1 and A_2 is:

$$F_{12} = \left(D\cos\alpha\right)\frac{p_1 p_2}{r^2};\quad r \le q_{12} \tag{8}$$

For nonzero-mass objects, we set $p = mu$, where m and u respectively denote mass and speed, and re-express (8) as:

$$F_{12} = \left(D\,u_1\,u_2\cos\alpha\right)\frac{m_1 m_2}{r^2};\quad r \le q_{12} \tag{9}$$

At separation $r > q_{12}$, mass fields are predominant. From (5), (6), and (7), gravitational force between (nonzero-mass) objects A_1 and A_2 is:

$$F_{12} = S\left(1 - \frac{u_1 u_2}{b^2}\cos\alpha\right)\frac{m_1 m_2}{r^2};\quad r > q_{12} \tag{10}$$

Eqs. (8), (9), and (10) are the Universal laws of gravitation; the terms multiplying $(p_1 p_2)/r^2$ or $(m_1 m_2)/r^2$ are the universal gravitational coefficients, which are varying with velocities $\boldsymbol{u}$ of the interacting objects. That is, *the universal gravitational forces have been evolving across space and with time since the universe began*.

We simplify (8), (9), and (10), because the present-day velocity $\boldsymbol{u}$ of the solar system, or of the Milky Way galaxy, is not known. The age of the (observable) universe is large (about 14 BY). So, in Figure 1, $r_1 \to \infty$, $r_2 \to \infty$, and $\alpha \to 0$.

There are no evidences of the solar system or the Milky Way galaxy flying apart at present, so, we set $u_1 \approx u \approx u_2$. Under these present conditions, Eqs. (8), (9), and (10) are respectively expressed as:

$$F_{12} = D\,\frac{p_1\,p_2}{r^2};\quad r \le q_{12} \qquad (11)$$

$$F_{12} = D\,u^2\,\frac{m_1 m_2}{r^2};\quad r \le q_{12} \qquad (12)$$

$$F_{12} = S\left(1 - \frac{u^2}{b^2}\right)\frac{m_1 m_2}{r^2};\quad r > q_{12} \qquad (13)$$

Eqs. (11), (12), and (13), represent three forms of Newtonian gravitational forces holding between objects.

Eq. (11) represents the general form of Newtonian gravitational forces between objects separated by $r \le q_{12}$.

Eq. (12) is the familiar Newton's law for nonzero-mass objects, where Newton's constant G is:

$$G \equiv D\,u^2\,;\ \ r \le q_{12} \qquad (14)$$

Eq. (13) is for nonzero-mass objects separated by $r > q_{12}$, where the gravitational "constant" Γ is:

$$\Gamma \equiv S\left(1\ -\ \frac{u^2}{b^2}\right);\ \ r > q_{12} \qquad (15)$$

G and Γ vary with u^2. On the human space-time scale, as u is virtually constant, so are G and Γ.

Based on (12) and (13), Table 1 shows the signs of gravitational forces between nonzero-mass objects.

Table 1. Signs of gravitational forces between nonzero-mass objects

u	$r \le q_{12}$	$r > q_{12}$
$u < b$	attraction	repulsion
$u = b$	attraction	zero
$u > b$	attraction	attraction

3.2 Matter-energy gravitational force

We take a matter object of mass m and an energy object of energy E. (An example of energy object would be a vibrating particle.) The matter object and the energy object gravitationally interact via their momentum fields. Using (11), with $p_1 = mu$ and $p_2 = E/c$, the gravitational force between them is given by:

$$F = \kappa\frac{mE}{r^2}\,, \qquad (16)$$

where κ is *mass-energy gravitational coefficient*:

$$\kappa = \frac{Du}{c} = \frac{G}{uc}\,, \qquad (17)$$

this is analogous to Einstein's constant [4].

3.3 Gravitational force between energy objects

The gravitational force between energy objects ($p = E/c$) is mediated by their momentum fields and, in accordance with (11), is given by:

$$F = \left(\frac{D}{c^2}\right)\left(\frac{E_1\,E_2}{r^2}\right) \qquad (18)$$

We take photons of $E \sim 10^6$ MeV or electromagnetic waves of $\lambda \sim 10^{-18}$ m, separated by $r \sim 10^{-18}$ m. From (18), gravitational force between the photons or between the electromagnetic waves is of the order 10^{-22} nt.

3.4 Eqs. (8) - (17) hold at arbitrary velocities ($\boldsymbol{v}$) and momenta ($\boldsymbol{p}$) as well.

3.5 'Nonzero-mass object' and 'mass' will be used interchangeably.

4. Vibrating particle in gravitational field

We derive the change in frequency (ν) of vibration of a particle as its position relative to a mass (m) changes. The mass, at $r = 0$, is a sphere of radius R. The particle's energy E is proportional to ν. Their momentum fields mediate the gravitational force between the mass and particle in accordance with (16).

4.1 Vibrating particle outside the mass

As the particle is moved from $r = r \ge R$ to $r = (r + x)$, the change in its energy is given by:

$$E_x - E_r = -\int_r^{r+x} \vec{F}\cdot d\vec{r}\ =\ \int_r^{r+x} \kappa\frac{m\,E}{r^2}\,dr \qquad (19)$$

Carrying out the integration, we have:

$$\nu_x = \frac{1+\dfrac{\kappa m}{r}}{1+\dfrac{\kappa m}{r+x}}\ \nu_r \qquad (20)$$

The fractional term > 1, thus $\nu_r < \nu_x$. *The particle vibrates at lower frequency closer to the mass.*

Using (20), we estimate κ in Appendix B, which shows:

$$\kappa = 1.552 \times 10^{-27}\ \text{nt-s}^2/\text{kg}^2 \qquad (21)$$

As $x \to \infty$, Eq. (20) reduces to:

$$\nu_\infty = \left(1\ +\ \frac{\kappa m}{r}\right)\nu_r \qquad (22)$$

If the vibrating particle serves as an atomic clock, its time period (τ) at the earth and at infinity, from (22), (21), and data A(e, f), are related by:

$$\tau_R = (1 + 1.454 \times 10^{-9})\ \tau_\infty\,, \qquad (23)$$

or, $(\tau_R - \tau_\infty)/\tau_\infty = 1.454 \times 10^{-9}$. That is, the time periods of atomic clocks at the earth are dilated by about 1.454×10^{-9} of the time periods at infinity. *The run of time is slower closer to the mass.*

If the vibrating particle serves as an emitter of light, its wavelength (λ) at the sun and at infinity, from (22), (21), and data A(c, d), are related by:

$$\lambda_R = (1 + 4.433 \times 10^{-6})\ \lambda_\infty\,, \qquad (24)$$

or, $(\lambda_R - \lambda_\infty)/\lambda_\infty = 4.433 \times 10^{-6}$. That is, spectral lines produced at the sun are redshifted by about 4.433×10^{-6} of the wavelengths produced at infinity. *Light as emitted has longer wavelength closer to the mass.*

4.2 Vibrating particle inside the mass

As the particle is moved from $r = 0$ to $r = r \le R$, the change in its energy is given by:

$$E_r - E_0 = -\int_0^r \vec{F} \cdot d\vec{r} = \int_0^r \kappa \frac{m_r E}{r^2} dr , \qquad (25)$$

where m_r is the portion of m contained within $r = r \le R$.

Carrying out the integration, we have:

$$\nu_0 = \left(1 - \frac{\kappa m_r}{2r}\right) \nu_r \qquad (26)$$

The bracketed term > 0 but < 1; so, $\nu_0 < \nu_r$. *The particle vibrates at lower frequency closer to the center of the mass.*

An atomic clock's time period at the earth's surface and at center, from (26), (21), and data A(e, f), are related by:

$$\tau_R = (1 - 7.27 \times 10^{-10})\, \tau_0 , \qquad (27)$$

The time periods of atomic clocks at the earth's center are dilated by about 7.27×10^{-10} of the time periods at the surface. *The run of time is slower closer to the center of the mass.*

Light wavelength at the surface and at the center of the sun, from (26), (21), and data A(c, d), are related by:

$$\lambda_R = (1 - 2.22 \times 10^{-6})\, \lambda_0 , \qquad (28)$$

Spectral lines produced at the sun's center are redshifted by about 2.22×10^{-6} of the wavelengths produced at its surface. *Light as emitted has longer wavelength closer to the center of the mass.*

4.3 Material rod near the mass

We address the change in the length of a rod of mass m' as its position relative to another mass m ($\gg m'$) changes from $r = r \ge R$ to $r = \infty$.

We consider an ideal rod constituted of atoms of mass $\delta m'$ ($\ll m'$) and charge e spaced equally by d. Such an atom, under the electrostatic forces of its neighboring atoms, undergoes oscillations with period τ as given by:

$$\tau^2 \propto d^n , \qquad (29)$$

where n is a parameter characteristic of the rod. (As an example, Appendix C shows $n = 3$ for a one-dimensional rod.)

The gravitational field of mass m affects the oscillations of $\delta m'$ according to (22). Substituting (29) in (22), we get:

$$d_r = \left(1 + \frac{\kappa m}{r}\right)^{2/n} d_\infty \qquad (30)$$

In (30), $d_r > d_\infty$. *The rod is longer closer to the mass.*

A thin wire ($n = 3$) is longer at the earth's surface by about 9.7×10^{-10} of its length at infinity.

4.4 Vibrating particle and point-dense mass

A mass of infinitely high point-density may be indicated by $m/R \to \infty$. An example of such a mass would be a so-called black hole.

From (22), as $m/R \to \infty$, $\tau_R/\tau_\infty \to \infty$. Time periods near the surface tend to infinity; time virtually stops running.

From (22), as $m/R \to \infty$, $\lambda_R/\lambda_\infty \to \infty$, subject to $\lambda_R\, \nu_R = c$. Near the surface, light travels at c with nearly flat waveform.

From (30), as $m/R \to \infty$, $d_R/d_\infty \to \infty$. Near the surface, rods flatten to the point where they disintegrate.

4.5 Vibrating particle and no mass

For a vanishing mass, its point density $m/R \to 0/0$. From (22), as $m/R \to 0/0$, $(\tau_R - \tau_\infty)/\tau_\infty \to 0/0$. One interpretation would be $\tau_R \to \tau_\infty \to 0$; that is, the time periods of atomic clocks virtually vanish at the vanishing mass. (Time still runs due to the presence of energy fields.)

4.6 The mass and the run of time

From (20) and (23), at a point closer to (farther from) the mass, the run of *time* is slower (faster). At that point, physical, chemical, and biological processes slow down (speed up). Select examples: decays of unstable particles and nuclei are slower (faster); atoms emit/absorb light of longer (shorter) wavelength; chemical reactions are slower (faster); and evolutions of organisms are slower (faster).

4.7 Effects of the strong, the weak, and electromagnetic fields on time are not known.

5. Constants and parameters

We estimate S, D, b, σ, Γ, κ, and u.

(1) Appendix B has the estimates on κ, u and D:

$\kappa = 1.552 \times 10^{-27}$ nt-s^2/kg^2 (present-day) (31)

$u = 1.433 \times 10^8$ m/s (present-day) (32)

$D = 3.249 \times 10^{-27}$ nt-s^2/kg^2 (33)

(2) Appendix D has the details on the estimation of b:

$b = 6.661 \times 10^7$ m/s (34)

(3) From (7), (15), (32), (34), and datum A(a), we get:

$S = 1.442 \times 10^{-11}$ nt-m^2/kg^2 (35)

Γ = (-) 5.231×10^{-11} nt-m^2/kg^2 (present-day) (36)

$u/b = 2.15$ (present-day) (37)

$u/c = 0.478$ (present-day) (38)

$b/c = 0.2222$ (39)

The speed (b) of gravitational radiation is about 22.22% of the speed (c) of light.

The present-day primordial speed (u) is about 48% of the speed (c) of light and about 2.15 times the speed (b) of gravitational radiation. That is, from Table 1, *the present-day gravitational forces between nonzero-mass objects are attractive*.

(4) We estimate σ using the sun's momentum field range. From (3), (32), and data A(c, g), we have:

$$\sigma = 5.26 \times 10^{-25} \text{ s/kg} \qquad (40)$$

That is, an object with momentum of 1 kg-m/s has an effective momentum field range of the order 10^{-24} m.

From (3), (32), (40), and datum A(h), the black hole at the Milky Way's center has a mass of 3.2 million suns.

6. Gravitational deflection of light

We address the deflection of light in the gravitational field of a mass.

Figure 2 (a) shows light ray (v) at impact parameter d from mass m and deflected by angle θ. Their momentum fields mediate the gravitational force $\boldsymbol{F}$ between the light ray and the mass according to (16). The light ray has initial momentum $\boldsymbol{p}_i$ and final momentum $\boldsymbol{p}_f = \boldsymbol{p}_i + \Delta\boldsymbol{p}$. Figure 2 (b) shows the vectorial relationships among the momenta.

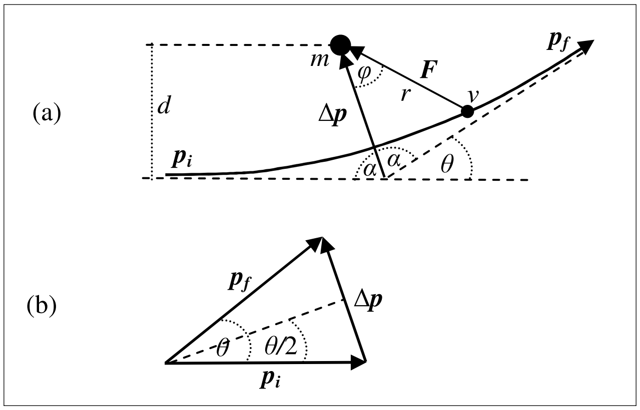

Figure 2. (a) Deflection of a light ray (v) by a mass (m). (b) Momentum vectors of the light ray.

Change (Δp) in momentum (p) of the light ray, from Figure 2 (b), is given by:

$$\Delta p = 2p\sin(\theta/2) \qquad (41)$$

where p is the magnitude of initial and final momenta.

Angular momentum (J) of the light ray of energy E_v before and after the deflection is given by:

$$\frac{E_v\, d}{c} = pd = J = \frac{E_v}{c^2}\, r^2 \frac{d\varphi}{dt} \qquad (42)$$

or,

$$\frac{1}{r^2} = \frac{1}{cd}\frac{d\varphi}{dt} \qquad (43)$$

By definition, force ($\boldsymbol{F}$) is given by:

$$\Delta\vec{p} = \int \vec{F}\, dt\,, \qquad (44)$$

where, in this case, F will be the gravitational force between mass (m) and light ray (E_v), as given by (16):

$$F = \frac{\kappa m E_v}{r^2} \qquad (45)$$

From Figure 2 (a) and Eq. (44), we get:

$$\Delta p = \int_{-\alpha}^{\alpha} F\cos\varphi\, dt \qquad (46)$$

Substituting (45) and (43) in (46), and noting that $\alpha = (\pi - \theta)/2$ in Figure 2 (a), we get:

$$\Delta p = \frac{\kappa m E_v}{cd}\int_{-(\pi-\theta)/2}^{(\pi-\theta)/2} \cos\varphi\, d\varphi$$

or,

$$\Delta p = \frac{2\kappa m E_v}{cd}\cos\frac{\theta}{2} \qquad (47)$$

Equating (47) to (41) and using $E_v = pc$, we get:

$$\tan\left(\frac{\theta}{2}\right) = \frac{\kappa m}{d} \qquad (48)$$

Rearranging (48), we get:

$$\theta = 2\tan^{-1}\left(\frac{\kappa m}{d}\right) \qquad (49)$$

To a first approximation:

$$\theta \approx \frac{2\kappa m}{d} \qquad (50)$$

For a ray of light grazing the sun, we have, from (49), (31), and data A(c, d), the *gravitational* deflection angle $\theta \approx 1.83$ arc-secs.

The 1919 expedition determined the deflection to be 1.75 arc-secs[5]; the 1929 expedition yielded 2.2 arc-secs[6]; later measurements ranged from 1.5 to 3 arc-secs[7]; and recent experiments support 1.75 arc-secs[8].

From (49) we may infer that light with impact parameter $d \le R$ at a black hole might turn back toward its source.

6.1 Escape radius for light near a mass

To escape a mass m, light rays must be outside a critical impact parameter R_e , which, from (49), is given by:

$$R_e = \kappa m\,, \qquad (51)$$

this is equivalent to the Schwarzschild radius of mass m. [9]

From (51) and (31), the black hole at the center of the Milky Way has $R_e \approx 10^{10}$ m. As if they were points, the sun and the earth would have $R_e \approx 3$ km and 1 cm respectively.

7. Mercury's orbital precession rate

Mercury's orbital precession rate is 575 arc-secs/century, which have been accounted for:

- Price and Rush show that the cumulative Newtonian gravity of planets Venus through Saturn contributes about 532 arc-secs/century. Each planet is replaced by a ring of uniform linear mass density to get a fairly accurate time-averaged effect of the moving planets. The force exerted by each planet on Mercury is directed outward, opposite to the force exerted by the sun. [10]

- Biswas shows that a Lorentz covariant modification of the Newtonian potential contributes about 43 arc-secs/century. A second-rank symmetric tensor is introduced into special relativity as a potential rather than a metric. [11]

- Barwacz shows that using an object's total energy rather than its rest mass in a Newtonian gravitational field predicts the orbital precession rate as observed. [12]

8. The Pioneer anomaly

The masses in the universe do not have exactly the same primordial velocity $\boldsymbol{u}$; that is, G and Γ are not constant across space and over time. Even an infinitesimal change in $\boldsymbol{u}$ changes G and Γ, which leads to the perturbation of trajectories and orbits. A change in $\boldsymbol{u}$ may be caused by a local force, such as of gravitational, electromagnetic, thermal or other origin.

From (12), a small change Δu in u of a mass at r from another mass leads to a small change ΔG in G and a small change Δa in a (acceleration) as given by:

$$\frac{\Delta a}{a} = \frac{\Delta G}{G} = \frac{2\Delta u}{u}; \quad r \leq q_{12} \tag{52}$$

We now address the Pioneer anomaly. The Pioneer-10 and Pioneer-11 spacecraft, after they passed about 20 AU on their trajectories, were observed to have an additional acceleration of $\Delta a = (8.74 \pm 1.33) \times 10^{-10}$ m/s^2 toward the sun.[13] [14] From (52), an increase of 0.00296% in u of the spacecraft leads to an increase of 0.00592% in G at their sites resulting in additional acceleration of $\Delta a = 8.74 \times 10^{-10}$ m/s^2 toward the sun. That is, even a small variation in u (or G) at the site can account for *all or a portion* of the observed additional acceleration.

9. Gravitational radiation

9.1 Gravitational radiation from accelerating mass

We derive gravitational Larmor's formula. Conceptually, we follow J. J. Thomson's derivation of electromagnetic radiation power from accelerating charges.[15]

Figure 3 shows a mass m at rest at A at time $t = 0$. The mass m accelerates at $\boldsymbol{a}$ for an infinitesimal duration δt and thereby gains velocity $\boldsymbol{\upsilon}$, momentum $\boldsymbol{p}$, and momentum field $\boldsymbol{P}$ at B at $t = \delta t$. We examine the effects of acceleration $\boldsymbol{a}$ on momentum field $\boldsymbol{P}$ and its components. Field vectors are continuous and changes to them propagate at a finite speed (b); that is, the farther the point is on a field vector, the later it feels the change. So, the one particular momentum-field component emerging at A along AD grows to BD at B. (With only uniform velocity $\boldsymbol{\upsilon}$, that component would be BC at B.) The momentum-field component BD is now resolved into radial momentum field BC (say, $\boldsymbol{P_r}$) and transverse momentum field CD (say, $\boldsymbol{P_t}$). Radial BC drifts at $\boldsymbol{\upsilon}$ but propagates radially at b. Transverse CD changes from zero to amplitude P_t and back to zero in δt; this is *gravitational radiation pulse*, which detaches and propagates outwardly at speed b.

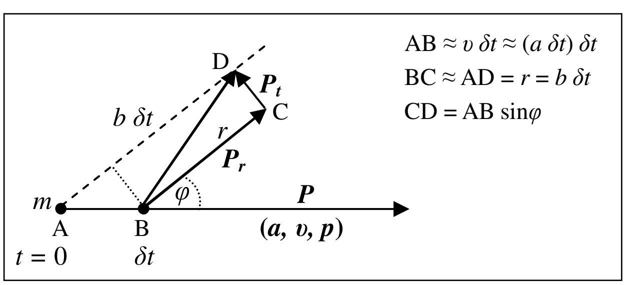

Figure 3. Mass m with its momentum $\boldsymbol{p}$ and momentum-field vector $\boldsymbol{P}$.

The ratio of transverse to radial momentum fields is:

$$\frac{P_t}{P_r} = \frac{CD}{BC} = \frac{(a\,\delta t)\,\delta t\,\sin\varphi}{b\,\delta t} = \frac{a\,r\sin\varphi}{b^2}, \tag{53}$$

where the radial momentum field P_r is given by (2):

$$P_r = D\frac{p}{r^2}\cos\varphi \tag{54}$$

Substituting (54) in (53), we get transverse momentum field P_t:

$$P_t = \frac{D\,p\,a\sin 2\varphi}{2\,b^2\,r} \tag{55}$$

Radial P_r varies with $1/r^2$, and transverse P_t with $1/r$. That is, transverse P_t survives over radial P_r at greater distances.

We define *gravitational radiation intensity I* associated with a momentum field P as:

$$I = \frac{b}{D}P^2 \tag{56}$$

Substituting (55) in (56), we get gravitational radiation intensity I_t due to propagating transverse P_t:

$$I_t = \frac{D\,p^2\,a^2}{4b^3}\frac{\sin^2 2\varphi}{r^2} \tag{57}$$

Gravitational radiation intensity I_t falls off as $1/r^2$ and its angular variation is shown in Figure 4 (a), which is a four-lobed quadrupole pattern. Figure 4 (a), in turn, shows that gravitational radiation accelerates a mass at 45^0 to its propagation direction.

In contrast, the angular variation of electromagnetic radiation intensity is shown in Figure 4 (b), which, in turn, shows that electromagnetic radiation accelerates a charge at 90^0 to its propagation direction.

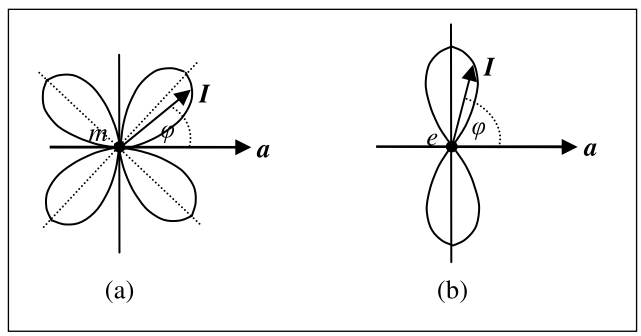

Figure 4. (a) Variation with φ in the intensity I of gravitational radiation. (b) Variation with φ in the intensity I of electromagnetic radiation.

Gravitational radiation power (Ω) emitted is given by integrating (57) over all directions as follows:

$$\Omega = 2\pi\int_0^{\pi} I_t\, r^2\sin\varphi\, d\varphi \tag{58}$$

If $\boldsymbol{p}$ and $\boldsymbol{a}$ are orthogonal, the magnitude of $\boldsymbol{p}$ stays constant, but its direction changes uniformly; that is, no momentum-field pulses develop. That is, masses in circular orbits do *not* emit gravitational radiation. That is, circular orbits are gravitationally stable.

Thus, carrying out the integration in (58), we get *Larmor's formula for gravitational radiation power*:

$$\Omega = \left(\frac{8\pi D}{15\,b^3}\right)\left(\vec{p}\cdot\vec{a}\right)^2 \tag{59}$$

A mass with motive power of $pa \approx 10^{25}$ watts emits gravitational radiation power of about one watt.

Charges emit electromagnetic radiation power proportional to $(ea)^2$. In contrast, charges even in circular orbits emit electromagnetic radiation.

9.2 Gravitational radiation from mass in elliptical orbit

Figure 5 shows mass m_2 in an elliptical orbit around mass m_1. The elliptical orbit is given by semimajor axis A and eccentricity ε. Mass m_2 is at a point (r, θ) from mass m_1 and has tangential velocity $\boldsymbol{v}$ and momentum $\boldsymbol{p}$.

We will use the reduced-mass frame for general binary systems. The reduced mass at m_2 is $\mu = (m_1 m_2)/(m_1 + m_2)$; the compensated mass at m_1 is $m = (m_1 + m_2)$. Masses participating in interactions are not adjusted.

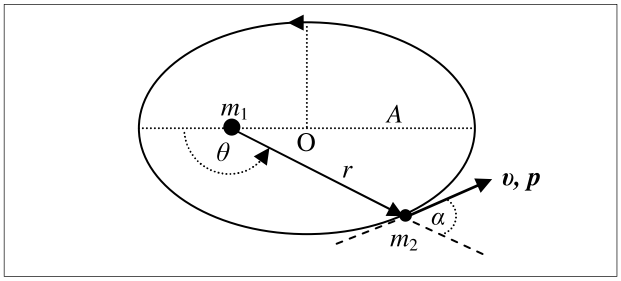

Figure 5. Mass m_2 in an elliptical orbit around mass m_1.

From the geometry of the ellipse, we have:

$$r = \frac{A(1-\varepsilon^2)}{1+\varepsilon\cos\theta} \tag{60}$$

$$\sin^2\alpha = \frac{A^2(1-\varepsilon^2)}{r(2A-r)} \tag{61}$$

$$\tan^2\alpha = \frac{A^2(1-\varepsilon^2)}{A^2\varepsilon^2-(A-r)^2} \tag{62}$$

Angular momentum vector ($\boldsymbol{J}$), by definition, is given below, where $\boldsymbol{p}$ is momentum vector:

$$\left|\vec{J}\right| = \left|\vec{r}\times\vec{p}\right| = rp\sin\alpha \tag{63}$$

or,
$$\sin^2\alpha = \frac{J^2}{r^2p^2} \tag{64}$$

The angular momentum (J) of m_2 in an elliptical orbit is a constant of motion as given by [16]:

$$J^2 = \frac{G m_1^2 m_2^2 A(1-\varepsilon^2)}{m_1+m_2} \tag{65}$$

From (59), the gravitational Larmor's formula, we have:

$$\cos^2\alpha = \frac{15b^3\Omega}{8\pi D p^2 a^2}, \tag{66}$$

where acceleration $a = Gm_1/r^2$.

With (64), (65), and (66), we get from physics:

$$\tan^2\alpha = \frac{8\pi D}{15b^3}\frac{G^3 m_1^4 m_2^2 A(1-\varepsilon^2)}{\Omega(m_1+m_2)\,r^6} \tag{67}$$

Using (67), (62), and (60), we get gravitational radiation power emission from a point (r, θ) on the orbit:

$$\Omega(\theta) = \left[\frac{8\pi D}{15b^3}\right]\left[\frac{G^3 m_1^4 m_2^2}{m_1+m_2}\right]\left[\frac{\varepsilon^2}{A^5(1-\varepsilon^2)^5}\right] \times (1+\varepsilon\cos\theta)^4\sin^2\theta \tag{68}$$

From (68), we get peak gravitational radiation power (Ω_{max}) from the points on the orbit at angles θ_+ and θ_- from the periastron:

$$\theta = \pm\cos^{-1}\left(\frac{-1+\sqrt{1+24\varepsilon^2}}{6\varepsilon}\right); \quad 0<\varepsilon<1 \tag{69}$$

From (68) and (69), Figure 6 shows the variation of Ω with θ. The gravitational radiation power is emitted in a pair of pulses with peaks Ω_{max} at θ_+ and θ_- in an orbital period. The peaks are $(1-\theta_+/\pi)\tau$ apart, then $(\theta_+/\pi)\tau$ apart.

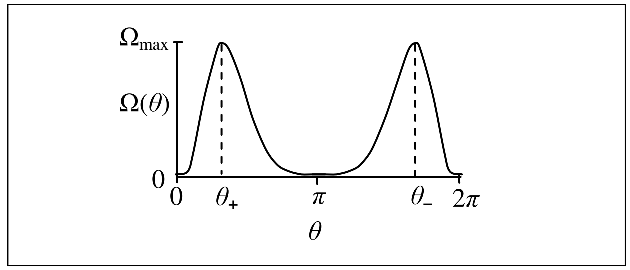

Figure 6. Variation of gravitational radiation power emission Ω with θ.

Gravitational radiation energy E_o emitted in an orbital period (τ) is given by integrating $\Omega(\theta)$ in (68) as follows:

$$E_o = \frac{\tau}{2\pi}\int_0^{2\pi}\Omega(\theta)\,d\theta \tag{70}$$

or,
$$E_o = \frac{1}{2}\tau\left[\frac{8\pi D}{15b^3}\right]\left[\frac{G^3 m_1^4 m_2^2}{m_1+m_2}\right]\left[\frac{\varepsilon^2}{A^5(1-\varepsilon^2)^5}\right] \times\left[1+\frac{3\varepsilon^2}{2}+\frac{\varepsilon^4}{8}\right], \tag{71}$$

this is analogous to the Peters' formula [17].

Substituting $p = \mu v$ in (64), we get speed v of m_2 at a point (r, θ):

$$v^2(\theta) = \frac{J^2}{\mu^2 r^2\sin^2\alpha} \tag{72}$$

From (61), (65), and (72), we get:

$$v^2(\theta) = \frac{G(m_1+m_2)}{A}\frac{(2A-r)}{r} \tag{73}$$

We use (60) in the r-term of (73) to get the θ-term:

$$\frac{2A-r}{r} = \frac{1+\varepsilon^2+2\varepsilon\cos\theta}{1-\varepsilon^2} \tag{74}$$

From (73) and (74), the speed (v) of m_2 at a point (r, θ) on the ellipse is given by:

$$v^2(\theta) = \frac{G(m_1+m_2)}{A(1-\varepsilon^2)}(1+\varepsilon^2+2\varepsilon\cos\theta) \tag{75}$$

Kinetic energy (T) of m_2 per orbital period is given below, where $v^2(\theta)$ comes from (75):

$$T = \int_0^{2\pi}\frac{1}{2}\mu v^2(\theta)\,d\theta \tag{76}$$

Carrying out the integration in (76), we have:

$$T = \frac{\pi G m_1 m_2 (1+\varepsilon^2)}{A(1-\varepsilon^2)} \tag{77}$$

Emission of gravitational radiation power (Ω) reduces kinetic energy (T). Changes ΔT, ΔJ, ΔA, and $\Delta\varepsilon$ in one orbital period (τ) are:

$$\Delta T = \frac{\partial T}{\partial A}\Delta A + \frac{\partial T}{\partial \varepsilon}\Delta\varepsilon = -E_o \tag{78}$$

$$\Delta J = \frac{\partial J}{\partial A}\Delta A + \frac{\partial J}{\partial \varepsilon}\Delta\varepsilon = 0 \tag{79}$$

Substituting (65) and (77) in (78) and (79), we get changes ΔA and $\Delta\varepsilon$ per orbital period:

$$\Delta A = \frac{-E_o}{\pi G m_1 m_2} A^2 \tag{80}$$

$$\Delta\varepsilon = \frac{(1-\varepsilon^2)\Delta A}{2\varepsilon A}, \tag{81}$$

Kepler's third law applies to binary systems as well[18]:

$$\tau^2 = \frac{4\pi^2 A^3}{G(m_1+m_2)} \tag{82}$$

From (82), first-order change in the orbital period ($\Delta\tau$) per orbital period (τ) is given by:

$$\Delta\tau = \frac{6\pi^2 A^2}{G(m_1+m_2)\tau} \Delta A, \tag{83}$$

where ΔA comes from (80).

9.3 Gravitational radiation from the moon

We will use the following data: [19]
Mass of the earth: m_1 = 5.976 x 10^{24} kg;
Mass of the moon: m_2 = 7.348 x 10^{22} kg;
Semimajor axis of lunar orbit: A = 3.844 x 10^8 m;
Eccentricity of lunar orbit: ε = 0.055; and
Lunar orbital period: τ = 27.322 days.

From (68) and (69), the gravitational radiation power is emitted in a pair of pulses at peaks Ω_{max} = 2,306 watts at $\theta = \pm 83^0.8$ away from the perihelion at approximately 14.6 days apart and then at 12.7 days apart per orbital period of 27.3 days.

From (71), gravitational radiation energy emitted in an orbital period is E_o = 2.702 x 10^6 joules.

From (80), the semimajor axis is decreasing at 4.338 x 10^{-15} m an orbital period.

From (83), the orbital period is decreasing at 4.0 x 10^{-17} sec in an orbital period.

Even though the moon has low orbital eccentricity, its mass is as much as 0.0123 times that of the earth. That is, the moon barely manages to emit gravitational radiation power at peaks of low but appreciable 2,306 watts when it is at $\pm 83^0.8$ away from its perihelion. The peak emission of 2,306 watts falls off to intensity of the order 10^{-15} watts/m^2 at the earth, which may be too weak to be detectable. However, this 2,306 watts power could be detectable by a detector in an artificial satellite at a calculated distance from the moon.

9.4 Gravitational radiation from Mercury

We will use the following data: [19]
Mass of the sun: m_1 = 1.989 x 10^{30} kg;
Mass of Mercury: m_2 = 3.5856 x 10^{23} kg;
Semimajor axis of Mercury's orbit: A = 5.791 x 10^{10} m;
Eccentricity of Mercury's orbit: ε = 0.206.

From (68) and (69), the gravitational radiation power is emitted in a pair of pulses at peaks Ω_{max} = 519 watts at $\theta = \pm 70^0.1$ away from the perihelion. The orbital period is decreasing at 5.3 x 10^{-28} sec a period.

9.5 Gravitational radiation from other planets

Planets Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune have very low orbital eccentricities ($\varepsilon < 0.09$) and small masses compared to the sun's mass. That is, each planet emits peak gravitational radiation power at about one watt or less.

Pluto, having higher orbital eccentricity ($\varepsilon = 0.25$) but very small mass relative to sun, emits peak gravitational radiation power at only 3.2 x 10^{-9} watt.

9.6 Gravitational radiation from Hulse-Taylor pulsars

The neutron stars are orbiting each other around their center of mass. Their physical and orbital data are: [20]
Mass of the first star: m_1 = 2.866 x 10^{30} kg;
Mass of the second star: m_2 = 2.759 x 10^{30} kg;
Semimajor axis of the orbits: A = 1.95 x 10^9 m;
Eccentricity of the orbits: ε = 0.617131;
Orbital period: τ = 2.791 x 10^4 secs (7.752 hours); and
Distance from Earth: 21,000 light years.

From (68) and (69), the gravitational radiation power is emitted in a pair of pulses at peaks Ω_{max} = 1.674 x 10^{26} watts at $\theta = \pm 53^0.85$ away from the periastron at approximately 5.43 hours apart and then at 2.32 hours apart per orbital period of 7.752 hours.

From (71), gravitational radiation energy emitted in one orbital period is E_o = 1.645 x 10^{30} joules.

From (80), due to the emission of gravitational radiation, the semimajor axis is decreasing at 3.71 mm per orbital period. The observed decrease rate is 3.1 mm a period.[20]

From (81), the eccentricity is decreasing at 9.71 x 10^{-13} per orbital period.

From (83), the orbital period is decreasing at 7.987 x 10^{-8} sec a period. The observed decrease rate is 6.759 x 10^{-8} sec a period.[20]

The peak gravitational radiation power emission of 1.674 x 10^{26} watts falls off to intensity 10^{-16} watts/m^2 at the earth, which may be too weak to be detectable.

10. Antimatter gravitation

Antimatter has the same mass property as its counterpart matter but equal and opposite value of some other property. Negative mass is considered to be nonexistent. That is, the model applies as is to matter-antimatter and to antimatter-antimatter gravitational forces.

(If masses of negative sign were to exist, the model would not apply as is to such masses and the model would need to be extended.)

11. Measurements

We suggest measurements for b, $\boldsymbol{u}$, and σ:

(a) Appendix E has the outlines for measuring b.

(b) Appendix B presents two approaches for determining the magnitude of velocity $\boldsymbol{u}$.

(c) We are unable to make a suggestion for determining the *direction* of $\boldsymbol{u}$ of the solar system or the Milky Way.

(d) Measurements of q_m of a mass m yield the value of σ. From (3), $\sigma = q_m/(mu)$, where u comes from (b) above. (The farthest extent of the solar system, perhaps beyond the Oort Cloud, should be considered as the sun's q_m.)

(e) The rest may be calculated using b, u, and G: $D = G/u^2$; $S = Db^2$; $\kappa = Du/c$; and $\Gamma = S\,(1 - u^2/b^2)$.

Gravity elongates a rod. Measurements may be made of Bragg's reflection/diffraction of X-rays from a crystal at the earth and at an altitude. (The same crystal and equipment should be used at the sites.) The spacings between the atoms of the crystal would be relatively larger at the earth. Eqs. (20) and (29) and the Bragg's law apply.

12. Conclusions

Newtonian gravitation can be mediated by mass fields and momentum fields. There are three forms of Newtonian gravitational force, which apply to zero-mass and nonzero-mass objects. The second form is the familiar classical Newton's law of gravitational force between nonzero-mass objects.

Directionally correctly and within an order of magnitude, the model agrees with the literature on gravity's effects on the time periods of atomic clocks, shifts in spectral lines, deflections of light rays, and orbital periods of the Hulse-Taylor pulsars. Gravity, by slowing down the run of time, slows down physical, chemical, and biological processes. Mercury's anomalous orbital precession rate may be explained away with modified Newtonian gravity. One or more effects, including minute variations in the Newton's constant at the sites, may be contributing to the Pioneer anomaly. The model differs from the literature on the signs and mediation of gravitational forces, the constancy of Newton's constant, and, more significantly, the structure, propagation, and speed of gravitational radiation.

Appendix F lists the results and predictions.

13. Remarks

Faraday introduced the concept of *field* in physics. Classical physics introduced gravitational field and electromagnetic field. Modern physics introduced the strong nuclear field and the weak nuclear field. General relativity introduced space-time geometry field for gravity. This model introduces mass-momentum field for gravity.

Mass (momentum) fields are about 10^{30} times weaker than electric (magnetic) fields. Momentum fields are no more than 10^8 times weaker than mass fields. Mass and momentum fields are *not* as noticeable as are electric and magnetic fields.

Besides other fundamental fields and associated objects, the universe contains mass fields and momentum fields and associated objects.

Electromagnetic and gravitational forces are similar in some aspects. They are repulsive and attractive and mediated respectively by the electric-magnetic and mass-momentum fields associated respectively with the charge-current and mass-momentum properties of objects. Charge fields extend out to infinity, so do mass fields.

Electromagnetic and gravitational forces are *not* similar in some other aspects. Charge is positive or negative, but mass is known to be only positive. Current has its magnetic field extending out to infinity; momentum has its momentum field effectively limited in range and direction. Electromagnetic interactions occur when charges are present somewhere; gravitational interactions may occur even when masses are not present anywhere, because even zero-mass objects have momentum fields. Charges and magnetic poles of both signs but masses of only positive sign exist; so, electric and magnetic forces can be shielded, but gravitational forces may not be.

There are evidences of the existence of gravitational radiation but little evidences so far on its structure, polarization, propagation, speed, emission and absorption. Measurements of the speed of gravitational radiation are as essential to understanding gravitation as were the measurements of the speed of light to understanding electrodynamics. The model estimates that the speed of gravitational radiation is less than the speed of light ($b = 0.2222\,c$). The literature considers the speed of gravitational radiation to be numerically equal to the speed of light ($b \equiv c$). The model's results follow the literature's closely if $b = 0.2222\,c$, but poorly if $b = c$. There exists little consensus on the polarization of gravitational radiation.

Table 2 is revealing. With the model's $\theta = 1.83$ arc-secs, $u = c/2.09$; with the observed $\theta = 1.75$ arc-secs, $u = c/2$. With the former, the model is close to the literature; with the latter, the model is closer. Examples follow for the latter. Eq. (17) yields $\kappa = 2G/c^2$; Einstein's constant $\kappa = 8\pi G/c^2$.[4] Eq. (20) yields relative spectral shift of $\Delta = 4.91 \times 10^{-15}$ between the earth and a point 22.5 m high.[21] Eq. (50), to a first approximation, reduces to $\theta \approx (4Gm)/(dc^2)$.[22] [23] Eq. (51) yields $R_e = 2Gm/c^2$, which is the Schwarzschild radius of mass m.[9] Eq. (83) yields that the orbital period of the Hulse-Taylor binary pulsars is decreasing at 6.403×10^{-8} sec a period – the observed value is 6.759×10^{-8} sec a period.[20]

In the literature, thermal recoil has the most support for the Pioneer anomaly.[14] But, no theory considers minute variations in G at the spacecraft.

We present the rationale for the boundaries of the model:

(1) Non-gravitational and other extraneous agents were ignored. Measurements of several gravitational phenomena have been performed with high accuracy. However, it has been *not* possible to place complete confidence in the accounting and contributions of such agents to those measurements – especially when the magnitudes are very small and in astronomical settings. Examples follow. The

agents which affect spectral shifts on stellar surfaces include Doppler shifts in high temperature gas; intense electromagnetic radiation due to gas ionization; high electric and magnetic fields; vertical currents; etc. Mass, diameter, and density of a star would affect the deflection of light grazing it. Magnetic moments of binary stars would affect their orbital periods. However, it would be challenging to estimate a star's mass, diameter, density, and magnetic moments as accurately.

(2) Special relativity was considered in principle only. Because the speed of gravitational radiation has not been measured. Routine relativistic corrections to time and length were not made. Mass-energy equivalence principle was invoked judiciously. For light we used $p = E/c$, not $m = E/c^2$, to formulate light-mass and light-light gravitational forces. In dynamics, we used E/c^2 for the as-if mass of light rays. [24]

(3) Quantum theory was used elementarily. The relationship that the energy of a vibrating particle is proportional to its frequency, as advanced by Planck to explain black-body radiation, sufficed.

(4) Gauge symmetries were not extracted. These are useful in developing field equations, which is an objective in the next parts of the model. Gauge covariance may or may not make a theory more (or less) physical.

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[2] The next evolution of the theory of gravitation occurs in 1915. Albert Einstein publishes a general relativity theory, according to which gravitation is due to the *curvature* which ponderable matter creates in the field of spacetime geometry. The field of spacetime geometry is the gravitational field. The theory obviates the concept of (gravitational) *force* and its conventional mediation.

The linearized version of general relativity is strikingly similar to classical electromagnetism. Charges are not known to induce any distortions in the field of spacetime geometry. In reality, electromagnetic forces are mediated by electromagnetic fields. That is, in this limiting case, general relativity presents a non-force, which is very similar to the mediated force! In the similar limiting case where the spacetime geometry field is weak, Newtonian gravity works very well. Thus, it is natural (not arbitrary) to explore whether Newtonian gravity could be mediated as well by gravitationally pertinent fields.

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[Milne holds that geometry can be selected primarily by the nature of underlying phenomenon and the convenience of representing and analyzing that phenomenon, and transformations of coordinates alone are but translations of language and have not necessarily much to do with phenomena. Milne's argument further encourages us to explore whether gravity could be understood outside of coordinates and geometries.]

[4] A. Einstein, *The Meaning of Relativity* (Eqs. 96 and 105a), Princeton University Press, 1955.

[Remarked later: Eq. (17) reduces to $\kappa \sim 2G/c^2$. That is, Einstein's constant (κ) = $4\pi\kappa$.]

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[15] M. S. Longair, *High Energy Astrophysics*, vol. 1, p. 64. Cambridge University Press, 1981.

[Pre-Maxwell, Thomson derived an equation for electric power emission from accelerating charges. Post-Maxwell, Longair revived it for its simplicity and insight.]

[16] H. Goldstein, *Classical Mechanics* (p. 79), Addison-Wesley Publishing Company, 1959.

[17] P. C. Peters and J Mathews, Gravitational Radiation from Point Masses in a Keplarian Orbit, *Physical Review*, 131 (435), 1963.

[The linearized version of general relativity is strikingly similar to classical electrodynamics, according to which accelerating charges radiate electromagnetic energy. With that understanding, the authors formulate gravitationally radiated energy from point masses in a Keplarian orbit. The resulting Peters' formula predicts the decrease rate of the major axes of the Hulse-Taylor binary pulsars well. The authors ignore the physical meaning, if any, of such gravitational radiation.]

[18] Reference 16 (p. 80).

[19] I. Ridpath, Ed., *Oxford Dictionary of Astronomy*, Oxford University Press, 2003.

[20] Data on the PSR B1913+16 system; www.johnstonsarchive.net/relativity/binpulsar.html; 2013.

[21] Reference 7 (p. 130, paragraph 3).

[22] Reference 4 (p. 93).

[23] Reference 7 (p. 193).

[24] Even elementary classical treatment of electromagnetic radiation force and pressure shows that electromagnetic waves have linear momentum $p = E/c$ in the direction of propagation. Then the as-if mass of electromagnetic waves would be E/c^2.

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Appendix A: Physical data

The following data [19] [25] are used in the calculations. They are referenced as datum A(x) or data A(x, y).

(a) Speed of light (c): 2.998 x 10^8 m/s

(b) Newton's constant (G): 6.672 x 10^{-11} nt-m^2/kg^2

(c) Sun's mass: 1.989 x 10^{30} kg

(d) Sun's radius: 6.963 x 10^8 m

(e) Earth's mass: 5.976 x 10^{24} kg

(f) Earth's radius: 6.378 x 10^6 m

(g) Farthest Kuiper Belt bodies from the sun: ~ 10^3 AU

(h) Diameter of the Milky Way galaxy: ~ 10^5 ly

(i) Coulomb's constant (Q): 8.988 x 10^9 nt-m^2/coul^2

(j) Ratio of electrical to gravitational force: 10^{40}.

Appendix B: Estimation of *κ*, *u*, and *D*

We present two approaches to estimating κ, u, and D.

B-1 Estimation using gravitational spectral shift

From (20): light produced at the surface of the earth is redshifted compared to light produced at a height.

In the Pound-Rebka experiment [26] [27], gamma rays emitted from Fe^{57} at the bottom of a 22.5-m tower travel to absorber Fe^{57} at the top; the emitter is moved upward at a just speed so that a compensating Doppler shift is produced which allows resonant absorption by the absorber. That is:

$$\lambda_R = (1 + \Delta)\, \lambda_x \qquad \text{(B.1)}$$

where x and R indicate the tower's height and the earth's radius respectively, and the relative shift $\Delta = 5.13 \times 10^{-15}$.

From (20), we have:

$$\lambda_R = \frac{1+\dfrac{\kappa m}{R}}{1+\dfrac{\kappa m}{R+x}}\, \lambda_x \qquad \text{(B.2)}$$

Comparing (B.1) with (B.2), we get:

$$1+\Delta = \frac{1+\dfrac{\kappa m}{R}}{1+\dfrac{\kappa m}{R+x}} \qquad \text{(B.3)}$$

Solving (B.3) for κ, we get:

$$\kappa = \frac{R+x}{m\left(\dfrac{x}{R\Delta}-1\right)} \qquad \text{(B.4)}$$

Substituting in (B.4) $x = 22.5$ m and $\Delta = 5.13 \times 10^{-15}$ from the experiment and m and R from data A(e, f), we get:

$$\kappa = 1.552 \times 10^{-27} \text{ nt-s}^2/\text{kg}^2 \qquad \text{(B.5)}$$

Eqs. (14), (17), and (B.5) and data A(a, b) yield:

$$u = 1.433 \times 10^{8} \text{ m/s} \qquad \text{(B.6)}$$

$$D = 3.249 \times 10^{-27} \text{ nt-s}^2/\text{kg}^2 \qquad \text{(B.7)}$$

B-2 Estimation using gravitational deflection of light

From (49), we express κ in terms of gravitational deflection θ of light, impact parameter d, and mass m :

$$\kappa = \frac{d}{m}\tan\left(\frac{\theta}{2}\right) \qquad \text{(B.8)}$$

We calculate κ, u, D, b, and Δ for the salient values of θ.

From (B.8), if m is the sun's mass, d the impact parameter of light grazing the sun, and θ the deflection of light, we get κ. Substituting this κ in (17), we get u and D.

In Appendix D, Eq. (D.3), with D from above, gives b.

From (B.3), if m and R are Earth's mass and radius, $x = 22.5$ m for the height of the tower in the Pound-Rebka experiment, and κ as calculated previously, we get Δ.

The results of calculations are listed in Table 2 below.

Table 2. Magnitudes of κ, u, D, b, and Δ for salient θ arc-secs.

θ	$\kappa \times 10^{-27}$	$u \times 10^{8}$	$D \times 10^{-27}$	$b \times 10^{7}$	$\Delta \times 10^{-15}$
1.75	1.485	1.498	2.974	6.962	4.91
1.83	1.552	1.433	3.249	6.661	5.13
2.20	1.867	1.191	4.704	5.536	6.17

The observed $\theta = 1.75$ arc-secs and theoretical $\Delta = 4.91 \times 10^{-15}$ are mutually consistent, so are the theoretical $\theta = 1.83$ arc-secs and observed $\Delta = 5.13 \times 10^{-15}$.

B-3 We opted for Appendix B-1 over B-2, because non-gravitational and other extraneous effects could be better accounted for and controlled in terrestrial experiments than in astronomical environments.

Appendix C: One-dimensional rod

We derive a physically simple relationship between the frequency (v) of oscillations of atoms which constitute a rod and the spacings (d) between them.

Figure 7 shows a one-dimensional rod of mass m constituted of atoms of mass δm ($<< m$) and charge e spaced equally by d. Such an atom at O, under the electrostatic forces of its neighboring atoms at O_- and O_+, oscillates between points i and j with frequency v and displacement ε ($< d$).

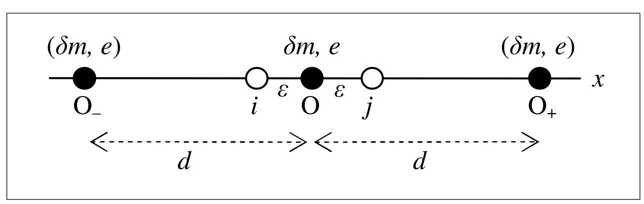

Figure 7. An atom (δm, e) at O under the electrostatic forces of the neighboring atoms at O_- and O_+.

Electrostatic force on δm when it is at point i is given below, where Q is the Coulomb's constant:

$$\vec{F}_i = Q\frac{e^2}{(d-\varepsilon)^2}\hat{x} - Q\frac{e^2}{(d+\varepsilon)^2}\hat{x} = \frac{4Qe^2}{d^3}\varepsilon\,\hat{x} \qquad \text{(C.1)}$$

Similarly, the electrostatic force on δm at point j is:

$$\vec{F}_j \approx -\frac{4Qe^2}{d^3}\varepsilon\,\hat{x} \qquad \text{(C.2)}$$

From (C.1) and (C.2), atom δm has acceleration a directed toward the neutral point O, as given by:

$$\vec{a} = -\frac{4Qe^2}{\delta m\, d^3}\,\varepsilon\,\hat{x} \qquad \text{(C.3)}$$

The equation for a simple harmonic motion is: $a = -(2\pi\nu)^2 x$. Comparing this with (C.3), we get:

$$\nu^2 = \left(\frac{Qe^2}{\pi^2\,\delta m}\right)\left(\frac{1}{d^3}\right) \qquad \text{(C.4)}$$

Thus, for a rod of a given constitution (δm and d), we get:

$$\nu^2 \propto \frac{1}{d^3} \qquad \text{(C.5)}$$

That is, ν^2 is *inversely* proportional to d^3.

Appendix D: Estimation of *b*

We consider two particles, separated by distance r, each of mass m and charge e.

Static gravitational force between the particles, from (5) and (7), is given by:

$$F_g = \frac{S\,m^2}{r^2} = \frac{D\,b^2 m^2}{r^2} \qquad \text{(D.1)}$$

Static electrical force between the particles is given by:

$$F_e = \frac{Q\,e^2}{r^2}\ , \qquad \text{(D.2)}$$

where Q is the Coulomb's constant.

From (D.1) and (D.2), we get:

$$b^2 = \left(\frac{Q}{D}\right)\left(\frac{e}{m}\right)^2\left(\frac{F_g}{F_e}\right) \qquad \text{(D.3)}$$

We take electrical force (F_e) and gravitational force (F_g) between two particles of charge e (1.602×10^{-19} coul) and mass m (4.0×10^{-29} kg), intermediate between a proton and an electron. Using (33) and data A(i, j) in (D.3), we get:

$$b = 6.661 \times 10^7 \text{ m/s} \qquad \text{(D.4)}$$

Appendix E: Measurement of *b*

Figure 8 shows a mass m_2 around another mass m_1 in an elliptical orbit of semimajor axis A and eccentricity ε.

Figure 6 shows variation with θ of gravitational radiation power Ω from m_2. Peak gravitational radiation power Ω_{max} is emitted when m_2 is either at (r_+, θ_+) or (r_-, θ_-). Angles $\theta_\pm$ are given by (69).

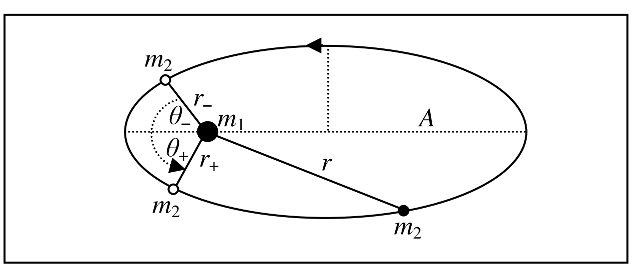

Figure 8. Mass m_2 around mass m_1 in elliptical orbit.

We outline the steps to help carry out the measurement:

(1) Determine the distance of m_2 from the detector: d.

(2) Detect by electromagnetic signals as m_2 arrives at point (r_+, θ_+) and note the time (t_c).

(3) Note the time (t_g) when *peak* gravitational power Ω_{max} from (r_+, θ_+) is detected. The peak power emission could have come from one of the previous orbital periods.

(4) If the electromagnetic signal comes from a specific orbital period, and the peak gravitational power signal comes from a prior n^{th} orbital period (τ), we have:

$$t_g - t_c = \left(n\,\tau + \frac{d}{b}\right) - \frac{d}{c} \qquad \text{(E.1)}$$

(5) Repeat steps 1 – 4 for the next *peak* gravitational power Ω_{max} from point (r_-, θ_-).

We now apply (E.1) to the two astronomical binary systems: the Earth-Moon system and the Hulse-Taylor pulsars.

(1) Hulse-Taylor pulsars

The pulsars are 21,000 light-years away. They are highly magnetic neutron stars. Not much could be known about their surrounding environments. Therefore, not all non-gravitational and other extraneous agents are knowable and accountable.

Each star has an orbital period of 2.791×10^4 secs. That is, there are at least 2.0×10^7 orbital periods during the time light travels from the stars to the detector. Therefore, it is not possible to ascertain n. That is, Eq. (E.1) is not usable.

From Section 9.6, peak gravitational radiation power emitted by the orbiting pulsars is 1.674×10^{26} watts. When received at the earth, it would be about 10^{-16} watts/m^2, which may be too weak to be detectable.

(2) Earth-Moon binary system

The space between the earth and the moon should well be knowable. That is, most non-gravitational and other extraneous agents should be accountable.

The moon is 1.2813 light-seconds away; its orbital period is 27.322 days. That is, the moon has barely moved along its orbit ($\approx 10^{-7}$ of a period) in the time light travels to a detector on the earth. Therefore, Eq. (E.1) is usable here as $n = 0$. Eq. (E.1) then becomes:

$$t_g - t_c = \frac{d}{b} - \frac{d}{c} \qquad \text{(E.2)}$$

or,

$$b = \frac{d}{t_g - t_c + \dfrac{d}{c}} \qquad \text{(E.3)}$$

From section 9.3, the moon emits peak gravitational radiation power of about 2,306 watts when it is at $\pm\, 83^0.8$ away from its perihelion. The peak power arriving at the earth is of very low intensity ($\sim 10^{-15}$ watts/m^2). Therefore, the detector should be placed in an artificial satellite in a calculated orbit around the moon.

Eq. (E.3) could be used to reveal whether $b = c$.

Appendix F: Results and predictions

Table 3 lists the results and predictions from the model. They do *not* include any non-gravitational and other extraneous effects. The observed values include some non-gravitational and other extraneous effects.

Table 3. Results and predictions from the model

Phenomena, etc.	Results and Predictions
Newton's constant G – Eq. (14)	Locally constant. Universally *not* constant.
Gravitational force – Table 1	Attractive; Repulsive.
Gravitational force between photons (electromagnetic waves) – Eq. (18)	Exists (< 10^{-22} nt)
Time-period dilation and Redshift at Earth relative to infinity – Eq. (23)	1.45×10^{-9}
Time-period dilation and Redshift at Sun relative to infinity – Eq. (24)	4.43×10^{-6}
Elongation of rod at Earth relative to infinity – Eq. (30)	1.0×10^{-9}
Elongation of rod at Sun relative to infinity – Eq. (30)	2.96×10^{-6}
Time run at black hole – Sec. 4.4	Virtually stops
At a black hole: Transitions in excited atoms, Decays of unstable nuclei. – Sec. 4.4	Virtually stop
Light waveform at black hole – Sec. 4.4	Nearly flat
Rods at black hole – Sec. 4.4	Elongate to disintegration
Time periods and spectral shifts in the absence of mass – Sec. 4.5	Virtually zero
Transitions in excited atoms. Decays of unstable nuclei. Evolutions of biological systems. – Sec. 4.6	Gravity slows them down.
Mass of the black hole at the Milky Way's center – Sec. 5(4)	3.2 million suns
Deflection of light at Sun – Eq. (49) Observed: 1.75 arc-secs/century [8]	1.83 arc-secs/century
Escape distance for light near the black hole at Milky Way's center – Sec. 6.1	10^{10} m
Escape distance for light at Sun-as-a-point – Sec. 6.1	3 km
Escape distance for light at Earth-as-a-point – Sec. 6.1	1 cm
Mercury's orbital precession – Sec. 7 Observed: 575 arc-secs/century	532 + 43 arc-secs/century accounted for. [10] [11] [12]
The Pioneer anomaly – Sec. 8	Part or all due to small variation in G at the site.
Gravitational radiation – Sec. 9.1 - Emission: - Consists of: - Angular distribution of power: - Accelerates a mass: - Speed – Eq. (D.4):	 by accelerating mass momentum-field pulses quadrupole, four-lobed at 45^0 to propagation $b \approx 0.2222\ c$
Moon – Sec. 9.3 - Gravitational radiation emission: - Peak gravitational radiation emission: - Decrease rate in orbital period:	 2.702×10^6 joules/period 2,306 watts 4.0×10^{-17} sec/period
Hulse-Taylor pulsars – Sec. 9.6 - Gravitational radiation emission: - Peak gravitational radiation emission: - Decrease rate in semimajor axis: Observed: 3.1 mm/orbital period [20] - Decrease rate in orbital period: Observed: 6.759×10^{-8} sec/period [20]	 1.645×10^{30} joules/period 1.674×10^{26} watts 3.71 mm/period 7.987×10^{-8} sec/period
Antimatter-antimatter gravitational forces – Sec. 10	The model applies.
Matter-antimatter gravitational forces – Sec. 10	The model applies.

The model makes *new* predictions:

(1) Gravitational forces are attractive or repulsive.

(2) Newton's constant G is not universally constant.

(3) Electromagnetic waves gravitationally interact; so do photons.

(4) Rods get longer closer to mass.

(5) Accelerating masses generate gravitational radiation power with angular distribution of a four-lobed quadrupole pattern and propagating at less than the speed of light.

[Some inferences could not be made, even though the model hints at them. They are advanced here as *hypotheses*: The degree of polarization of gravitational radiation (momentum-field pulses) is four; Mass fields and momentum fields are the so-called dark matter and dark energy; and The universe is neither static nor forever expanding.]

Appendix G: Nomenclature

$\boldsymbol{a}$	acceleration
A	semimajor axis
b	speed of gravitational radiation
c	speed of electromagnetic radiation
d	impact parameter; space interval.
D	momentum-field coefficient
e	electrical charge
E	energy
$\boldsymbol{F}$	force
G	Newton's constant (classical gravitational constant)
$\boldsymbol{I}$	gravitational radiation intensity (power/area)
$\boldsymbol{J}$	angular momentum
m	mass (gravitational charge)
$\boldsymbol{M}$	mass-field vector
$\boldsymbol{p}$	momentum (gravitational current)
$\boldsymbol{P}$	momentum-field vector
q	momentum field range
q_{12}	sum of q_1 of object-1 and q_2 of object-2
Q	Coulomb's constant
r	distance; separation distance
R	radius
R_e	escape radius for an object near a mass
S	mass-field coefficient
$\boldsymbol{u}$	velocity relative to the Primordial Point
Γ	$\Gamma = S\,(1 - u^2/b^2)$
ε	eccentricity; infinitesimal quantity.
κ	mass-energy gravitational coefficient
λ	wavelength
μ	reduced mass
ν	frequency
σ	momentum field range coefficient
τ	period
$\boldsymbol{v}$	arbitrary velocity
Ω	power (energy/sec).

The author requests the readers to send comments and suggestions. E-mail address: raghu.singh@verizon.net.

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