

Gravitational Inertia

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Abstract

The paper addresses gravitational inertia and its source and the postulate of numerical equality of gravitational mass and inertial mass. It infers that time intervals may be finite in the gravitational field of a mass.

Keywords

Inertia; Gravitational mass; Inertial mass; Time quantum.

1. Introduction

According to Ernst Mach, the inertia of a material object – the object’s resistance against being accelerated – is not an intrinsic property of matter, but a measure of its interaction with the entities of the universe.

The literature is awash with papers and discourses on inertia; however, there is no single universally accepted theory which explains the source of inertia.

Even though confirmed experimentally to a very high degree of accuracy, there is no single convincing explanation of as to why gravitational mass is numerically equal to inertial mass.

2. Assumptions

We make three assumptions:

- (1) An interaction between two objects is communicated by fields along a space-time path.
- (2) The path of the interaction is continuous.
- (3) The interaction propagates at a finite speed.

3. Inertia and its source

Figure 1 shows an object of mass m initially at rest at O in a community of objects. The objects in the community are effectively distributed to the six “end” points of the x , y , and z axes – each point has mass M . According to Assumption (1), gravitational interaction between m and an M is communicated along a path, say, O-C-Y.

An impulse is imparted to mass m at O along O-X for duration Δt , which is undetermined at this time. As a result, the mass gains acceleration a at A. Per Assumption (3), the interaction travels at a fixed speed, say, b , the speed of gravitational radiation. Masses beyond a sphere of radius $b\Delta t$ will feel the effects of the impulse after Δt . The impulse displaces segment O-C of the interaction path O-C-Y to A-C. Consequently, according to Assumption (2), path O-C-Y turns into A-C-Y in duration Δt .

Interaction path segment A-C is resolved into A-B, which is the gravitational force F_g , and A-O, which is the force F_i in opposition to the acceleration. (As the force F_i

opposes the acceleration a , we name it the force of inertia – in deference to Ernst Mach.)

Similarly, segment A-D is resolved into A-E, which is the gravitational force F_g , and A-O, which adds to the force of inertia F_i in opposition to the acceleration.

The impulse affects the two interaction paths in the x - z plane similarly.

Gravitational forces along AB and AE cancel each other out, so do similar gravitational forces in the x - z plane. These gravitational forces play no part here in accelerating the mass (m).

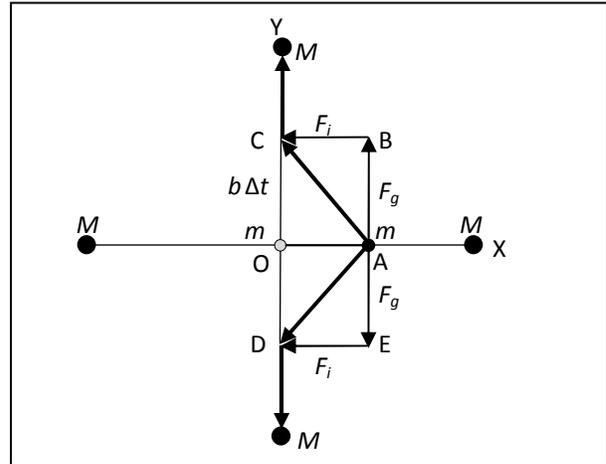


Figure 1. Mass m under the gravitational forces of the masses M in the community

The geometrical ratio of F_i to F_g is:

$$\frac{F_i}{F_g} = \frac{AO}{AB} = \frac{(1/2) a (\Delta t)^2}{b \Delta t} = \frac{a \Delta t}{2b}, \quad (1)$$

where the gravitational force F_g is given by:

$$F_g = \frac{GMm}{(AB)^2} = \frac{GMm_g}{b^2 (\Delta t)^2}, \quad (2)$$

where G is the classical gravitational constant, and M and m ($\equiv m_g$) are *gravitational* masses to match gravitational force F_g . Gravitational mass participates in gravitational interactions and, so, is an intrinsic property of matter.

Substituting (2) into (1) and adding force A-D and the forces in the x - z plane, we get:

$$F_i = \left(\frac{2GMm_g}{b^3 \Delta t} \right) a, \quad (3)$$

where the dimension of the bracket is of mass, which is designated *inertial* mass (m_i) to match inertial force F_i :

$$m_i = \frac{2GM m_g}{b^3 \Delta t} \quad (4)$$

Inertial mass does not participate in gravitational interactions, and, so, is fictitious and *not* an intrinsic property of matter.

Rearranging (4), we get:

$$\frac{m_i}{m_g} = \frac{2G M}{b^3 \Delta t}, \quad (5)$$

that is, the ratio m_i / m_g varies with $M / \Delta t$ but is independent of acceleration a .

From (3) and (4), we make the inferences on inertia:

(1) The impulse which causes acceleration is resisted by the force of inertia.

(2) A body has inertia because: it is participating in a community of interacting bodies; accelerations adjust lines of interactions; and interactions and associated adjustments propagate at a finite speed.

(3) The inertia of a particle is proportional directly to its gravitational mass, the community's mass, and acceleration imparted and inversely to the duration of acceleration and cube of the propagation speed of gravitational interaction.

(4) As $b \rightarrow \infty$, $m_i \rightarrow 0$ and $F_i \rightarrow 0$. That is, if interactions were to propagate at infinite speed, there would be *no* inertia. (In a chamber containing gas, the time it takes for the information to travel the mean free path is negligible. Therefore, a particle has virtually no inertia by other gas particles in the chamber.)

The inferences are consistent with the Mach's principle.

We emphasize that, in this section, we did *not* invoke Newton's laws of motion to introduce the concept of inertia and inertial mass.

4. Gravitational mass and inertial mass

We address the numerical equality of gravitational mass and inertial mass.

A body of gravitational mass m_g and inertial mass m_i is at a distance r from the center of the earth (mass M). The body is falling toward the earth with acceleration a due to gravitational force F_g , which from the Newtonian mechanics, is given by:

$$F_g = \frac{GM m_g}{r^2} = m_i a \quad (6)$$

Rearranging (6), we get:

$$a = \left(\frac{GM}{r^2} \right) \frac{m_g}{m_i}, \quad (7)$$

that is, at any point $r = r$ during the fall, the acceleration a of a body of gravitational mass m_g and inertial mass m_i is proportional to m_g/m_i . From (5), we just inferred that m_g/m_i is independent of acceleration a . These two

seemingly contradictory statements may be reconciled if m_g/m_i is a numerical constant. By choosing the same unit for both m_g and m_i , we make the constant equal to 1. It then follows that:

$$m_g \equiv m_i, \quad (8)$$

this is the postulate of numerical equality of gravitational mass and inertial mass.

5. Time and acceleration

We have from (5) and (8):

$$\Delta t = \frac{2G}{b^3} M \quad (9)$$

As long as M is fixed, Δt is fixed. That is, Δt is the finite interval of time in which impulses may be imparted and accelerations produced in the gravitational field of a mass. That is, time intervals may be finite (quantum) in gravitational fields.

Substituting in (9) the magnitudes of G and b from the Appendix, we have:

$$\Delta t \approx (4.512 \times 10^{-34}) M \quad (10)$$

That is, it takes longer to accelerate a mass in a stronger gravitational field. Near a mass of 1 kg, $\Delta t \approx 10^{-34}$ secs.; near Earth, $\Delta t \approx 10^{-9}$ sec; near Sun, $\Delta t \approx 10^{-3}$ sec; and near the black hole at the center of Milky Way, $\Delta t \approx 10^3$ secs.

6. Remarks

If gravitational radiation were to propagate at the speed of light (if $b = c$), we would infer from (9) that: near a mass of 1 kg, $\Delta t \approx 10^{-36}$ sec; near Earth, $\Delta t \approx 10^{-11}$ sec; near Sun, $\Delta t \approx 10^{-5}$ sec; and near the black hole at the center of the Milky Way, $\Delta t \approx 31$ secs.

Eq. (9) may be used to estimate b , provided Δt can be measured. We are unable to suggest an experiment to measure Δt .

Rotational inertia may be addressed along similar lines.

Electromagnetic inertia may be addressed along similar lines.

References

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Appendix

Speed of gravitational radiation (b): 6.661×10^7 m/s;^[1]

Speed of light (c): 2.998×10^8 m/s;^[2]

Newton's constant (G): 6.672×10^{-11} nt-m²/kg²;^[2]

Sun's mass: 1.989×10^{30} kg;^[3]

Earth's mass: 5.976×10^{24} kg;^[3]

Mass of the Milky Way's black hole: 6.3×10^{36} kg.^[1]

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