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RADIO AND LIGHT RADIATIONS FROM A MOVING CHARGED PARTICLE

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Abstract

The paper identified two types of electromagnetic radiations or waves. The first is light radiation due to a moving electric charge having a component of its velocity in the direction of an electric field, thereby encountering a resistive force, which gives rise to power radiation independent of frequency, as in the radiation from atoms. The second is radio radiation, due to acceleration of an electric charge producing a reactive electric field, which gives rise to power radiation proportional to the square of frequency, as in the radiation from an antenna. While light radiation is transmissible in seawater, as an electric current conductor, radio radiation is not

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1. INTRODUCTION

In a medium with electric field **E** and a current of intensity **J** flowing, creating a magnetic field of intensity **H**, the power dissipated and/or stored in a volume *V* or passing through area *S*, is given by the integrals:

$$P = -\int_V \mathbf{E}.\mathbf{J}(dV) = \int_S \mathbf{E}\times\mathbf{H}.(d\mathbf{S}) = \int_S \left(\mathbf{E}_f + \mathbf{E}_a\right)\times\mathbf{H}.(d\mathbf{S}) \tag{1}$$

The Poynting vector is **E×H** [1]. The problem of radiation is solved by determining the radiation reaction field $\mathbf{E}_f$, due motion of an electric charge in an electric field and the reactive electric field E_a due to acceleration.

For an electric charge *q*, at a point *R*, (see Fig. 1) moving with velocity **v** along an electric field of magnitude E, the radiation reaction field is:

$$\mathbf{E}_f = -\frac{E\mathbf{v}}{c} \tag{2}$$

where *c* is the speed of light in a vacuum. The radiation reaction force $-qE\mathbf{v}/c$ is similar to a frictional or resistive force opposing motion, against which work done appears as light radiation..

The reactive electric field, at time t, is:.

$$\mathbf{E}_a = -\frac{d\mathbf{A}_r}{dt} \tag{3}$$

where $\mathbf{A}_r$ is the magnetic vector potential [2] at a point *P* distance *r* from the charge at *R*. For a particle of charge q, work done against the reactive force qE_a is stored as kinetic energy of the particle.

The magnetic field **H** is obtained from the magnetic flux intensity **B**, as:

$$\mathbf{B} = \mu_o \mathbf{H} = \nabla \times \mathbf{A}_r = \hat{\mathbf{u}} \times \frac{d\mathbf{A}_r}{dr} \tag{4}$$

where **ű** is a unit vector in the **RP** direction and ∇× denote the curl of a vector. As a result of time *r/c* taken for a cause at *R* to reach *P* distance *r* away, with the speed of light *c*, the retarded vector potential has to be used. This brings out a second term in the expression for **H**, which is responsible for electromagnetic radiation.

The paper identifies two types of electromagnetic radiations or waves [3] which may be called light radiation and radio radiation. Light radiation or wave is due to a moving electric charge having a component of its velocity in the direction of an electric field. The result is a resistive or frictional electric field, which gives rise to power radiation independent of frequency, as in the radiation from atoms. Radio radiation, is due to acceleration of an electric charge producing a reactive electric field, which gives rise to power radiation proportional to the square of frequency, as in the radiation from an antenna [4]..

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2. AN ELECTRICALLY CHARGED PARTICLE UNDER FREE OSCILLATION

2.1 Velocity and Magnetic Vector Potential

Consider a particle of charge *q* set in oscillation, in the absence of any external electric field, with angular frequency ω and velocity **v**, about an origin *O*. in the direction of unit vector **â**, as shown in Fig.1. The oscillating charge has a potential φ, sets up a magnetic field **H** and generates a reactive electric field $\mathbf{E}_a$ at a point *P* distance *r* from *O*. The line **OP**, in the direction of unit vector **û**, makes an angle θ with the velocity **v**. Let the velocity of the particle, at time *t,* be:

$$\mathbf{v} = \mathbf{a}\omega \cos \omega t = \mathbf{v}_m \cos \omega t \tag{5}$$

where *a* is the displacement amplitude and $\mathbf{v}_m$ is the amplitude of velocity. The magnetic vector potential $\mathbf{A}_o$**,** at *R***,** for a charged particle moving with velocity **v** is:

$$\mathbf{A}_o = \mu_o \varepsilon_o U \mathbf{v} = \mu_o \varepsilon_o U \mathbf{v}_m \cos \omega t \tag{6}$$

where U is the constant electric potential inside the charge. Magnetic vector potential $\mathbf{A}_r$**,** at a point *P* distance *r* outside the charge**,** is given by the equation:

$$\mathbf{A}_r = \mu_o \varepsilon_o \varphi \mathbf{v} = \mu_o \frac{q}{4\pi r} \mathbf{v}_m \cos \omega \left(t - \frac{r}{c} \right) \tag{7}$$

:At a point *P* distance *r* from the charge, *present* time *t* must be replaced by the *retarded* time *(t – r/c)*, as the effect of any displacement about *O*, transmitted with speed of light *c*, takes time *r/c* to reach *P*. Since the amplitude of oscillation is very small, distance of the point *P* from the oscillating charge remains practically at *r*.

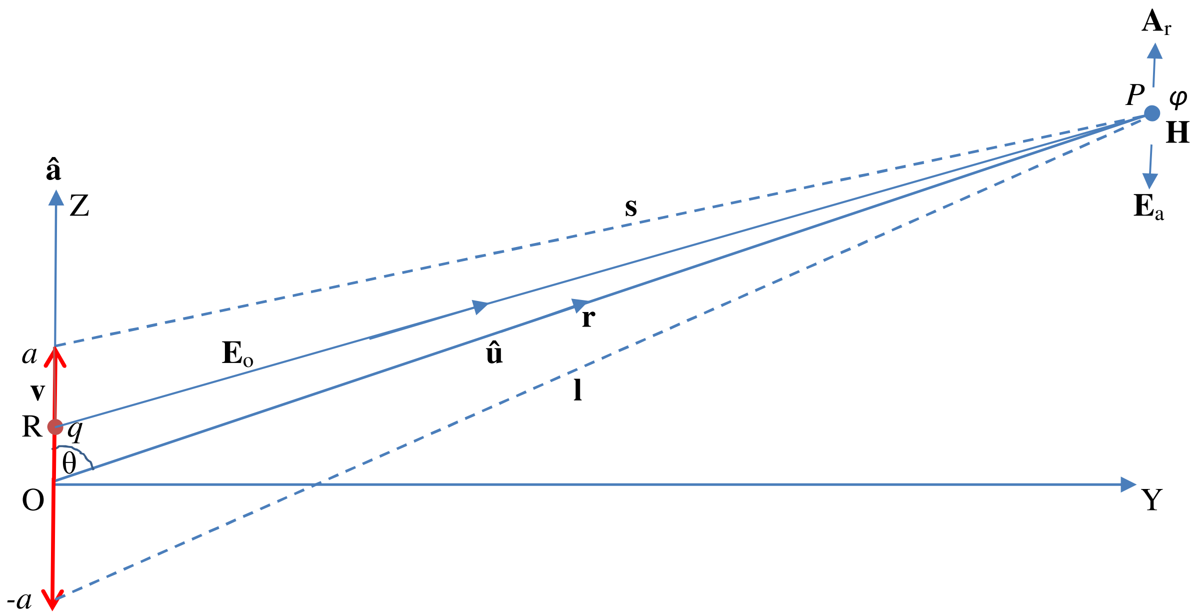

Fig.1 A particle of charge q, oscillating with velocity **v**, in the direction of unit vector **a,** about an origin O and generating a magnetic field **H** and reactive electric field $\mathbf{E}_a$ at a point P distance $\boldsymbol{r}$ from O.

2.2. Magnetic Field Intensity

Magnetic flux intensity **B** set up at a point P, distance r from the oscillating charge, is given by:

$$\mathbf{B} = \nabla \times \mathbf{A}_r = \hat{\mathbf{u}} \times \frac{d\mathbf{A}_r}{dr}$$

Magnetic field intensity **H**, is:

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_o} = \frac{\hat{\mathbf{u}}}{\mu_o} \times \frac{d\mathbf{A}_r}{dr} = \hat{\mathbf{e}}\varepsilon_o v E_o \sin\theta \tag{8}$$

where $\mathbf{E}_o$ is the electrostatic field of the charge and **ű**, **ê**, **î** are unit vectors of the spherical coordinates.

2.3. Reactive Electric Field.

The reactive electric field $\mathbf{E_a}$ generated by the accelerated at a point P, distance r from the charge, is given by equation (7) as:

$$\mathbf{E}_a = -\frac{d\mathbf{A}_r}{dt} = \frac{\mu_o q \omega \mathbf{v}_m}{4\pi r} \sin \omega\left(t - \frac{r}{c}\right) \tag{9}$$

The cross (vector) product of the electric field $\mathbf{E}_a$ and magnetic field **H** is the Poynting vector, as the power transfer across unit area of space.

2.4. Poynting Vector

The Poynting vector is obtained from equations (8) and (9) as:

$$\mathbf{E}_a \times \mathbf{H} = -\frac{d\mathbf{A}_r}{\mu_o (dt)} \times \left(\hat{\mathbf{u}} \times \frac{d\mathbf{A}_r}{dr}\right) \tag{10}$$

2.5. Power Generation

Power transfer across a spherical surface area of radius *r,* is given by the integral:

$$\int_S \mathbf{E}_a \times \mathbf{H}.(d\mathbf{S}) = -\int_S \frac{d\mathbf{A}_r}{\mu_o (dt)} \times \left(\hat{\mathbf{u}} \times \frac{d\mathbf{A}_r}{dr}\right) . \hat{\mathbf{u}}(dS) \tag{11}$$

where $\mathbf{A}_r$ is given by equation (7) and

$$\mathrm{B} = \nabla \times \mathbf{A}_r = \hat{\mathbf{u}} \times \frac{d\mathbf{A}_r}{dr} = -\mu_o \frac{q}{4\pi r^2} \hat{\mathbf{u}} \times \mathbf{v}_m \cos \omega\left(t - \frac{r}{c}\right) + \mu_o \frac{q\omega}{4\pi rc} \hat{\mathbf{u}} \times \mathbf{v}_m \sin \omega\left(t - \frac{r}{c}\right)$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_o} = -\frac{q}{4\pi r^2} \hat{\mathbf{u}} \times \mathbf{v}_m \cos \omega\left(t - \frac{r}{c}\right) + \frac{q\omega}{4\pi rc} \hat{\mathbf{u}} \times \mathbf{v}_m \sin \omega\left(t - \frac{r}{c}\right) \tag{12}$$

The Poynting vector (equation 10) then becomes:

$$\mathbf{E}_a \times \mathbf{H} = \mu_o \left(\frac{q}{4\pi r}\right)^2 \mathbf{v}_m \times (\hat{\mathbf{u}} \times \mathbf{v})_m \sin \omega\left(t - \frac{r}{c}\right) \left\{-\frac{\omega}{r} \cos \omega\left(t - \frac{r}{c}\right) + \frac{\omega^2}{c} \sin \omega\left(t - \frac{r}{c}\right)\right\}$$

$$\mathbf{E}_a \times \mathbf{H} = \mu_o \left(\frac{q}{4\pi r}\right)^2 \mathbf{v}_m \times (\hat{\mathbf{u}} \times \mathbf{v}_m) \left\{-\frac{\omega}{2r} \sin 2\omega\left(t - \frac{r}{c}\right) + \frac{\omega^2}{c} \sin^2 \omega\left(t - \frac{r}{c}\right)\right\}$$

$$\mathbf{E}_a \times \mathbf{H} = \mu_o \left(\frac{q}{4\pi r}\right)^2 \mathbf{v}_m \times (\hat{\mathbf{u}} \times \mathbf{v}_m) \left[-\frac{\omega}{2r} \sin 2\omega\left(t - \frac{r}{c}\right) + \frac{\omega^2}{2c} \left\{1 - \cos 2\omega\left(t - \frac{r}{c}\right)\right\}\right]$$

$$\mathbf{E}_a \times \mathbf{H} = \left(\frac{q}{4\pi r}\right)^2 \mu_o \omega \mathbf{v}_m \times (\hat{\mathbf{u}} \times \mathbf{v}_m) \sin\omega\left(t - \frac{r}{c}\right)\left\{-\frac{1}{r}\cos\omega\left(t - \frac{r}{c}\right) + \frac{\omega}{c}\sin\omega\left(t - \frac{r}{c}\right)\right\} \quad (13)$$

Average power generation P, is:

$$P = \int_S \mathbf{E}_a \times \mathbf{H}.\hat{\mathbf{u}}(dS) = \int_S \mu_o \left(\frac{q}{4\pi r}\right)^2 \mathbf{v}_m \times (\hat{\mathbf{u}} \times \mathbf{v}_m).\hat{\mathbf{u}}\left[-\frac{\omega}{2r}\sin 2\omega\left(t - \frac{r}{c}\right) + \frac{\omega^2}{2c}\left\{1 - \cos 2\omega\left(t - \frac{r}{c}\right)\right\}\right](dS) \quad (14)$$

In equation (14) there is a steady term, which is responsible for energy radiation. The oscillation terms, in time t, give zero energy dissipation over one cycle. Average power P_r radiated is given by the integral:

$$P_r = \int_S \frac{\mu_o{}^2 \omega^2}{2c}\left(\frac{q}{4\pi r}\right)^2 \hat{\mathbf{u}} \times \mathbf{v}_m . \hat{\mathbf{u}} \times \mathbf{v}_m (dS)$$

$$P_r = \int_S \frac{\mu_o v_m^2 \omega^2}{2c}\left(\frac{q}{4\pi r}\right)^2 \sin^2\theta(dS) \quad (15)$$

$$P_r = \int_S \frac{\mu_o v_m^2 \omega^2}{2c}\left(\frac{q}{4\pi r}\right)^2 \sin^2\theta(dS) = \int_\pi^0 \frac{\mu_o v_m^2 \omega^2}{2c}\left(\frac{e}{4\pi r}\right)^2 2\pi r^2 \sin^3\theta(d\theta) \quad (16)$$

where $(dS) = 2\pi r^2 \sin\theta(d\theta)$. Integrating equation (16), we obtain:

$$P_r = \int_\pi^0 \frac{\mu_o v_m^2 \omega^2}{2c}\left(\frac{q}{4\pi r}\right)^2 2\pi r^2 \sin^3\theta(d\theta) = \frac{\mu_o v_m^2 \omega^2 q^2}{12\pi c} \quad (17)$$

where $\int_\pi^0 \sin^3\theta(d\theta) = \frac{4}{3}$

2.6. Directivity of Radiation

Maximum power intensity (power per unit area) R_m, given by equation (15) with $\theta = \pi/2$ radians, is:

$$R_m = \frac{\mu_o v_m^2 \omega^2 q^2}{32\pi^2 r^2 c} \quad (18)$$

Power intensity R emitted by an ideal isotropic radiator, over a spherical surface, gives:

$$4\pi r^2 R = \frac{\mu_o v_m^2 \omega^2 q^2}{12\pi c} --- and --- R = \frac{\mu_o v_m^2 \omega^2 eq}{48\pi^2 r^2 c} \quad (19)$$

Directivity is defined as R_m/R, which gives 1.5. The direction of maximum radiation is perpendicular to the velocity

2.7. Accelerating Force and Kinetic Energy of aan Oscillating Charged Particle.

For a particle of charge q and mass m moving at time t with velocity v and acceleration $d\mathbf{v}/dt$, the accelerating force is:

$$` q\mathbf{E}_a = -q\frac{d\mathbf{A}_e}{dt} = \mu_o \frac{q^2\omega}{4\pi r}\mathbf{v}_m \sin\omega\left(t-\frac{r}{c}\right) = m\frac{d\mathbf{v}}{dt} = m\omega\mathbf{v}_m \sin\omega\left(t-\frac{r}{c}\right)$$

The kinetic energy of the charge is contained in the magnetic field **H** (equation 7) and dynamic electric field $\mathbf{E}_v$, given by:

$$\mathbf{E}_v = \frac{v\mathbf{E}_o}{c}\cos\theta \qquad (20)$$

$$K = \frac{1}{2}\int_V \mu_o H^2(dV) + \frac{1}{2}\int_V \varepsilon_o E_v^2(dV) = \frac{1}{2}\int_V \mu_o\varepsilon_o^2 v^2 E_o^2 \sin^2\theta(dV) + \frac{1}{2}\int_V \varepsilon_o \frac{v^2}{c^2}E_o^2\cos^2\theta(dV) = \frac{1}{2}\int_V \mu_o\varepsilon_o^2 v^2 E_o^2(dV) = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}\int_V \mu_o\varepsilon_o^2 v^2 E_o^2(dV) = \mu_o\varepsilon_o W v^2 = \frac{1}{2}mv^2 \qquad (21)$$

$$W = \frac{m}{2\mu_o\varepsilon_o} = \frac{1}{2}mc^2 \qquad (22)$$

where W is the electrostatic energy of the charge, the work done in creating the charge and $c = (\mu_o\varepsilon_o)^{-1/2}$ is the speed of light in a vacuum..

3. A CHARGED PARTICLE UNDER ACCELERATION BY A CONSTANT ELECTRIC FIELD

Let the particle of charge q (Fig. 1) move up, in the direction of unit vector $\hat{\mathbf{a}}$, with velocity $\mathbf{v}$ and acceleration $d\mathbf{v}/dt$ at time t, in an electric field of constant magnitude E. The radiation reaction field is $\mathbf{E}_f = -\frac{E}{c}\mathbf{v}$, the reactive electric field is $\mathbf{E}_a = -\mu_o\varepsilon_o\varphi\frac{d\mathbf{v}}{dt}$ and the magnetic field is $\mathbf{H} = \varepsilon_o\mathbf{v}\times\mathbf{E}_o$ where E_o is the electrostatic field due to the moving charge. The Poynting vector **P**, is

$$\mathbf{P} = (\mathbf{E}_v + \mathbf{E}_a)\times\mathbf{H} = -\left(\frac{E}{c}\mathbf{v} + \mu_o\varepsilon_o\varphi\frac{d\mathbf{v}}{dt}\right)\times(\varepsilon_o\mathbf{v}\times\mathbf{E}_o) = \frac{\varepsilon_o E}{c}\left(v^2\mathbf{E}_o - \mathbf{v}.\mathbf{E}_o\mathbf{v}\right) + \mu_o\varepsilon_o^2\varphi\left(\mathbf{v}.\frac{d\mathbf{v}}{dt}\mathbf{E}_o - \mathbf{E}_o.\frac{d\mathbf{v}}{dt}\mathbf{v}\right) \qquad (23)$$

Power transfer across surface area S, is:

$$P = \int_S \left\{ \frac{\varepsilon_o E}{c}\left(v^2\mathbf{E}_o - \mathbf{v}.\mathbf{E}_o\mathbf{v}\right) + \mu_o\varepsilon_o^2\varphi\left(\mathbf{v}.\frac{d\mathbf{v}}{dt}\mathbf{E}_o - \mathbf{E}_o.\frac{d\mathbf{v}}{dt}\mathbf{v}\right)\right\}.(d\mathbf{S})$$

$$P = \int_S \left\{ \frac{\varepsilon_o E}{c}\left(v^2\mathbf{E}_o\right) + \mu_o\varepsilon_o^2\varphi\left(\mathbf{v}.\frac{d\mathbf{v}}{dt}\mathbf{E}_o\right)\right\}.(d\mathbf{S}) \quad (24)$$

$$\int_S \left\{ \frac{\varepsilon_o E}{c}\left(\mathbf{v}.\mathbf{E}_o\right) + \mu_o\varepsilon_o^2\varphi\left(\mathbf{E}_o.\frac{d\mathbf{v}}{dt}\right)\right\}\mathbf{v}.(d\mathbf{S}) = 0$$

$$\mathrm{P} = \int_S \frac{\varepsilon_o E}{c} v^2\mathbf{E}_o.(d\mathbf{S}) + \int_S \mu_o\varepsilon_o^2\varphi\mathbf{v}.\frac{d\mathbf{v}}{dt}\mathbf{E}_o.(d\mathbf{S}) = \frac{qEv^2}{c} + \mu_o\varepsilon_o q\varphi v\frac{dv}{dt} \quad (25)$$

where $\int_S \varepsilon_o\mathbf{E}_o.(d\mathbf{S}) = q$. The first term on the right hand side of equation (25); power radiated P_{r} due a charged particle moving along an electric field of intensity **E**. is:

$$P_r = \frac{qEv^2}{c} \quad (26)$$

The time integral of the second term gives the kinetic energy of the charge, thus:

$$K = \int \mu_o\varepsilon_o q\varphi v\frac{dv}{dt}(dt) = \frac{1}{2}\mu_o\varepsilon_o q\varphi v^2 = W\mu_o\varepsilon_o v^2 = \frac{1}{2}mv^2 \quad (27)$$

$$W = \frac{m}{2\mu_o\varepsilon_o} = \frac{1}{2}mc^2 \quad (28)$$

The charged particle can be accelerated to the speed of light c and it moves with constant kinetic energy and radiating power qEc, with maximum intensity perpendicular to the direction of motion.

4. A CHARGED PARTICLE OSCILLATING IN A CONSTANT ELECTRIC FIELD

Let the particle of charge q in Fig. 1 oscillate with angular frequency ω and velocity **v** in an electric field of constant magnitude E in the direction of velocity.. The radiation reaction field is:

$$\mathbf{E}_f = -\frac{E}{c}\mathbf{v} = -\frac{E}{c}\mathbf{v}_m\cos\omega\left(t - \frac{r}{c}\right) \quad (29)$$

The magnetic field **H** is given by equation (7) and the reactive electric field $\boldsymbol{E}_a$, due to acceleration, is given by equation (8). The Poynting vector is put as:

$$\mathbf{P}=\left(\mathbf{E}_f+\mathbf{E}_a\right)\times\mathbf{H}=\left\{-\frac{E}{c}\mathbf{v}_m\cos\omega\left(t-\frac{r}{c}\right)+\frac{\mu_o q\omega\mathbf{v}_m}{4\pi r}\sin\omega\left(t-\frac{r}{c}\right)\right\}\times\mathbf{H} \tag{30}$$

$$\mathbf{P}=\mathbf{v}_m\times(\hat{\mathbf{u}}\times\mathbf{v}_m)\left\{-\frac{E}{c}\cos\omega\left(t-\frac{r}{c}\right)+\frac{\mu_o q\omega}{4\pi r}\sin\omega\left(t-\frac{r}{c}\right)\right\}\left\{-\frac{q}{4\pi r^2}\cos\omega\left(t-\frac{r}{c}\right)+\frac{q\omega}{4\pi rc}\sin\omega\left(t-\frac{r}{c}\right)\right\}$$

$$\mathbf{P}=\mathbf{v}_m\times(\hat{\mathbf{u}}\times\mathbf{v}_m)\frac{q}{4\pi r}\left\{\frac{E}{cr}\cos^2\omega\left(t-\frac{r}{c}\right)-\frac{E\omega}{2c^2}\sin 2\omega\left(t-\frac{r}{c}\right)-\frac{\mu_o q\omega}{8\pi r^2}\sin 2\omega\left(t-\frac{r}{c}\right)+\frac{\mu_o q\omega^2}{4\pi rc}\sin^2\omega\left(t-\frac{r}{c}\right)\right\}$$

$$\mathbf{P}=\mathbf{v}_m\times(\hat{\mathbf{u}}\times\mathbf{v}_m)\frac{q}{4\pi r}\left[\frac{E}{2cr}\left\{1+\cos 2\omega\left(t-\frac{r}{c}\right)\right\}+\frac{\mu_o q\omega^2}{8\pi rc}\left\{1-\cos 2\omega\left(t-\frac{r}{c}\right)\right\}-\left(\frac{E\omega}{2c^2}+\frac{\mu_o q\omega}{8\pi rc}\right)\sin 2\omega\left(t-\frac{r}{c}\right)\right] \tag{31}$$

In equation (31), the steady terms responsible for radiation are:

$$\mathbf{P}=\mathbf{v}_m\times(\hat{\mathbf{u}}\times\mathbf{v}_m)\frac{q}{4\pi r}\left(\frac{E}{2cr}+\frac{\mu_o q\omega^2}{8\pi rc}\right) \tag{32}$$

Average power radiated is:

$$P_r=\int_S\mathbf{v}_m\times(\hat{\mathbf{u}}\times\mathbf{v}_m)\frac{q}{4\pi r}\left(\frac{E}{2cr}+\frac{\mu_o q\omega^2}{8\pi rc}\right).\hat{\mathbf{u}}(\mathrm{dS})=\int_S\left(\hat{\mathbf{u}}\times\mathbf{v}_m\right).(\hat{\mathbf{u}}\times\mathbf{v}_m)\frac{q}{4\pi r}\left(\frac{E}{2cr}+\frac{\mu_o q\omega^2}{8\pi rc}\right)(dS)$$

$$P_r=\int_S\left(\hat{\mathbf{u}}\times\mathbf{v}_m\right).(\hat{\mathbf{u}}\times\mathbf{v}_m)\frac{q}{4\pi r}\left(\frac{E}{2cr}+\frac{\mu_o q\omega^2}{8\pi rc}\right)(dS)=\int_S v_m^2\sin^2\theta\frac{q}{4\pi r}\left(\frac{E}{2cr}+\frac{\mu_o q\omega^2}{8\pi rc}\right)(dS) \tag{33}$$

$$P_r=\int_S v_m^2\sin^2\theta\frac{q}{4\pi r}\left(\frac{E}{2cr}+\frac{\mu_o q\omega^2}{8\pi rc}\right)2\pi r^2\sin\theta(d\theta)$$

$$P_r=\int_S v_m^2\frac{q}{4\pi r}\left(\frac{E}{2cr}+\frac{\mu_o q\omega^2}{8\pi rc}\right)2\pi r^2\sin^3\theta(d\theta)$$

$$P_r=\frac{2qv_m^2}{3}\left(\frac{E}{2c}+\frac{\mu_o q\omega^2}{8\pi c}\right) \tag{34}$$

where $\int_\pi^0\sin^3\theta(d\theta)=\frac{4}{3}$

The first tem in equation (34); independent of angular frequency ω, is the power radiated due to resistive or frictional oscillation of a particle of charge q moving in an electric field of magnitude E. The second term, dependent on ω^2, is the power radiated due to oscillation of the charge. In equation (34) there are two types of electromagnetic radiations, different amplitudes and 90^o out of phase. If E is zero, equation (34) reduces to equation (17).

4.6. Directivity of Radiation

Maximum power intensity R_m, is given by equation (33) with θ = π/2 radians, as:

$$R_m = \frac{v_m^2 q}{4\pi r}\left(\frac{E}{2cr} + \frac{\mu_o q\omega^2}{8\pi rc}\right) \tag{35}$$

Power intensity R emitted by an ideal isotropic radiator, over a spherical surface, gives:

$$4\pi r^2 R = \frac{2qv_m^2}{3}\left(\frac{E}{2c} + \frac{\mu_o q\omega^2}{8\pi c}\right) - -R = \frac{qv_m^2}{6\pi r^2}\left(\frac{E}{2c} + \frac{\mu_o q\omega^2}{8\pi c}\right) \tag{36}$$

Directivity is $R_m/R = 1.5$

5. AN ELECTRON REVOLVING ROUND A POSITIVELY CHARGED NUCLEUS

Figure 2 shows an electron of charge $-e$ and mass m revolving, in a radial electric field **E**, due to a stationary nucleus of charge $+Q$ at a point O. The electron, at R with its electric field $\mathbf{E}_o$, sets up a potential φ and magnetic field **H** at a far point P distance s from R and l from O. The electron moves in the Y-Z plane, under attraction of the nucleus, with velocity **v** at R and angular displacement ψ at O.

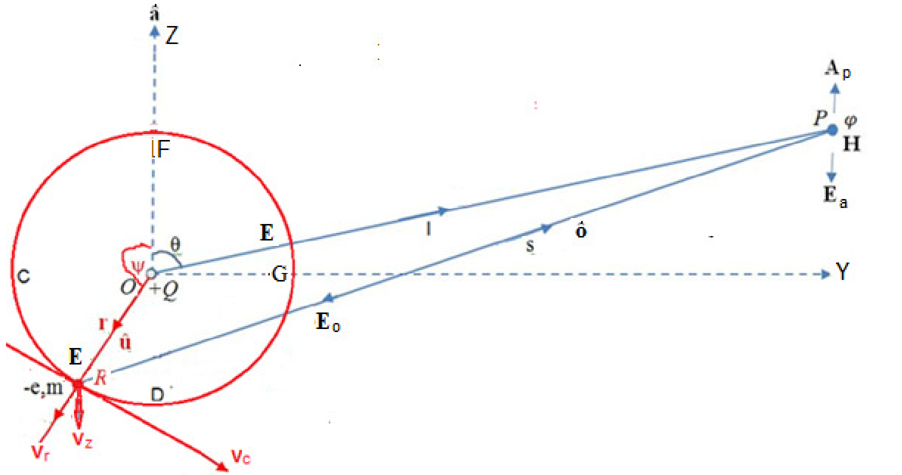

Fig.2 An electron of charge $-e$ and mass m at R evolving with speed v round a nucleus of charge +Q. The electron sets up a potential φ and magnetic field **H** at a far point P, distance s from R, in the direction of unit vector $\hat{\mathbf{o}}$.

Velocity of the electron, at time t, is:

$$\mathbf{v}=\frac{d}{dt}(\mathrm{r}\,\hat{\mathbf{u}})=\hat{\mathbf{u}}\frac{dr}{dt}+r\frac{d\hat{\mathbf{u}}}{dt}=\hat{\mathbf{u}}\frac{dr}{dt}+r\frac{d\psi}{dt}\hat{\mathbf{i}}\times\hat{\mathbf{u}} \tag{37}$$

The velocity $\mathbf{v}$, of the electron at R, has two orthogonal components, $\mathbf{v}_c$ in the peripheral direction and $\mathbf{v}_r$ in the radial direction, thus:

$$\mathbf{v}_c=r\frac{d\psi}{dt}\hat{\mathbf{i}}\times\hat{\mathbf{u}}=r\boldsymbol{\omega}\times\hat{\mathbf{u}} \tag{38}$$

where $\boldsymbol{\omega}$ is the angular velocity, perpendicular to the Y-Z plane (X-axis, out of the page) and:

$$\mathbf{v}_r=\hat{\mathbf{u}}\frac{dr}{dt} \tag{39}$$

The resultant of the two velocit4s, $\mathbf{v}_c$ and $\mathbf{v}_r$, takes the electron into an elliptic orbit. An excited electron revolves in an unclosed elliptic orbit with emission of radiation at the frequency of revolution, before settling into the stable circular orbit. As the electron revolves its velocity in a given direction, the Z axis, becomes:

$$\mathbf{v}_z=\hat{\mathbf{a}}v_r\cos\omega t \tag{40}$$

Let us keep the amplitude, $\hat{\mathbf{a}}v_r$, constant as we investigate emission of radiation from the electron oscillating in its orbit.

5.1. Radiation Reaction Field

In Fig. 2, the velocity along an electric field in the Z direction is maximum and positive at F, zero at C, maximum and negative at D and zero at G. So v_z oscillates at the frequency of revolution of the electron. In this case the radiation reaction field for a revolving electron under force of attraction by an electric field E, is:

$$\mathbf{E}_f=\frac{E}{c}\mathbf{v}_z=\hat{\mathbf{a}}\frac{E}{c}v_r\cos\omega t \tag{41}$$

This is in contrast to equation (2).

5.2. Reactive Electric Field

For a far point P distance s from R, the magnetic vector potential at P, is:

$$\mathbf{A}_p=\mu_o\varepsilon_o\varphi\mathbf{v}_z=-\hat{\mathbf{a}}\mu_o\frac{e}{4\pi s}v_r\cos\omega\left(t-\frac{s}{c}\right)$$

The reactive electric field is:

$$\mathbf{E}_a = -\frac{d\mathbf{A}_p}{dt} = -\hat{\mathbf{a}}\omega\mu_o \frac{e}{4\pi s} v_r \sin\omega\left(t - \frac{s}{c}\right) \tag{42}$$

The centripetal force -e**E**, due to attraction of the nucleus, on the electron, is balanced by the centrifugal force -$e\mathbf{E}_a$, due to acceleration, so that the particle moves in a circle with constant speed and radius. The only remaining radial force is the radiation reaction force $\mathbf{E}_f$.

5.3. Magnetic Field

The magnetic flux intensity **B** at P is given by:

$$\mathbf{B} = \nabla \times \mathbf{A}_p = \hat{\mathbf{o}} \times \frac{d\mathbf{A}_p}{ds} = \mu_o \hat{\mathbf{o}} \times \hat{\mathbf{a}} v_r \frac{e}{4\pi s}\left\{\frac{1}{s}\cos\omega\left(t - \frac{s}{c}\right) + \frac{\omega}{c}\sin\omega\left(t - \frac{s}{c}\right)\right\}$$

where $\hat{\mathbf{o}}$ is a unit vector in the **RP** direction. The magnetic field **H**, is:

$$\mathbf{H} = \hat{\mathbf{o}} \times \hat{\mathbf{a}} v_r \frac{e}{4\pi s}\left\{\frac{1}{s}\cos\omega\left(t - \frac{s}{c}\right) + \frac{\omega}{c}\sin\omega\left(t - \frac{s}{c}\right)\right\} \tag{43}$$

5.4. Poynting Vector

The Poynting vector is:

$$\mathbf{P} = \mathbf{E}_f \times \mathbf{H} = \frac{E}{c} v_r \cos\omega\left(t - \frac{s}{c}\right) \hat{\mathbf{a}} \times (\hat{\mathbf{o}} \times \hat{\mathbf{a}}) v_r \frac{e}{4\pi s}\left\{\frac{1}{s}\cos\omega\left(t - \frac{s}{c}\right) + \frac{\omega}{c}\sin\omega\left(t - \frac{s}{c}\right)\right\}$$

$$\mathbf{P} = \hat{\mathbf{a}} \times (\hat{\mathbf{o}} \times \hat{\mathbf{a}}) \frac{eEv_r^2}{4\pi sc}\left\{\frac{1}{s}\cos^2\omega\left(t - \frac{s}{c}\right) + \frac{\omega}{c}\cos\omega\left(t - \frac{s}{c}\right)\sin\omega\left(t - \frac{s}{c}\right)\right\}$$

$$\mathbf{P} = \hat{\mathbf{a}} \times (\hat{\mathbf{o}} \times \hat{\mathbf{a}}) \frac{eEv_r^2}{4\pi sc}\left[\frac{1}{2s}\left\{1 + \cos 2\omega\left(t - \frac{s}{c}\right) + \frac{\omega}{2c}\sin 2\omega\left(t - \frac{s}{c}\right)\right\}\right] \tag{44}$$

Average power radiation is:

$$P_r = \hat{\mathbf{a}} \times (\hat{\mathbf{o}} \times \hat{\mathbf{a}}) \int_S \frac{eEv_r^2}{8\pi s^2 c} .\hat{\mathbf{o}}(dS) = -\hat{\mathbf{o}} \times \hat{\mathbf{a}}.(\hat{\mathbf{o}} \times \hat{\mathbf{a}}) \int_S \frac{eEv_r^2}{8\pi s^2 c}(dS) = -\int_S \frac{eEv_r^2}{8\pi s^2 c}\sin^2\theta(dS)$$

$$P_r = \int_S \frac{eEv_r^2}{8\pi s^2 c}\sin^2\theta(dS) = -\int \frac{eEv_r^2}{8\pi s^2 c} 2\pi s^2 \sin^3\theta(d\theta)$$

$$P_r = \int \frac{eEv_r^2}{8\pi s^2 c} 2\pi s^2 \sin^3\theta(d\theta) = \frac{eEv_r^2}{3c} \tag{45}$$

where $\int_{\pi}^{0}\sin^{3}\theta(d\theta)=\frac{4}{3}$. What we have, in equation (41), is light radiation from an atomic particle.

5.5. Directivity

The directivity of radiation is 1.5.There is no radiation normal to the plane of the orbit. As the electron revolves in its orbit, the radiation is zero in the radial direction and maximum in the tangential direction.

6. CONCLUSION

In equation (17), for a charged particle under free oscillation, the radiation power is proportional to the square of frequency, as is the case for radiation from a radio transmission antenna. In equation (26) for a charged particle moving along an electric field, the radiation power, for a given frequency, is independent of the frequency, as for power dissipation in a resistor.

In equation (34) power radiation consists of two parts. There is radio radiation, due to oscillation of an electric current, proportional to square of frequency and there is light radiation due to motion of an electric charge in an electric field. In equation (45), there is only light radiation from an agitated electron oscillating round a positively charged nucleus of an atom. Both radio radiation and light radiation are electromagnetic radiations, giving rise to heat radiation, but may suffer differently for transmission in a medium other than vacuum. For example, while light radiation is transmitted through salty seawater, radio radiation is not.

The conclusion here is that, radio radiation due to an oscillating current in an antenna and light radiation from an orbiting electron, are different. This may explain why light is transmissible in seawater but radio waves are not.

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