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ELECTRIOMAGNETIC RADIATION FROM AN ANTENNA

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Abstract

An antenna or aerial is a conducting wire where the current *I* varies with time and along the length *Z* of the conductor. The alternating current in an element of length of the wire, constitutes a current element which contributes to the magnetic and electric fields at a point away from the conductor. The resultant field at a point is obtained by integrating for all the infinitesimal current elements along the length of the antenna. The interaction between the alternating magnetic and electric fields, in space, gives rise to transverse electromagnetic radiation propagated at the speed of light. This paper treats a half-wave dipole antenna and derived its maximum radiation resistance as 80 ohms and directivity as 1.5.

Keywords: Electric current, magnetic field, magnetic vector potential, potential, radiation

1. Introduction

An antenna or aerial is a device used to transmit or receive radio waves through space, usually at a given frequency. To many students of electronics and telecommunications an antenna is just a piece of straight rigid wire or a rod sticking out of a radio receiver, a length of rigid wire mounted on the ground or suspended by some supports or an array of short conductors attached to a pole. The antenna used as a transmitter for sending or as a receiver for receiving electromagnetic waves across space is the most important device in telecommunications. For an antenna to function there must be an alternating electric current along its length. The frequency of this current may be several hundreds of megahertz. The higher the frequency the shorter and more practicable is the antenna.

This paper will deal with quarter-wave monopole and half-wave dipole antennas, which are most common. The purpose is to give a theoretical treatment of the workings of popular antennas employed in telecommunications at frequencies ranging from long-wave to short-wave a.m. radio (150 kHz to 30 MHz), through very high frequency (30 to 300 MHz) for f.m. radio to ultra-high frequencies (300 MHz to 3GHz) for television. The paper is aimed at students of electromagnetic fields and waves with application in telecommunications.

2. Fields and Radiation from a Short Antenna

An antenna is considered short if its length is less than 1/10th of the wavelength of the signal being received. For a short antenna of length *k* at the origin of spherical coordinates *O*, carrying an oscillating current *I*, in the direction of unit vector $\hat{\mathbf{a}}$, as shown in Fig.1, the magnetic vector potential [1, 2] **a** at *P* a point of position vector **r** is:

$$\mathbf{a} = \frac{\mu_o I k}{4\pi r}\hat{\mathbf{a}} \tag{1}$$

where μ_o is the permeability of vacuum. The current *I* is considered uniform across the length of the antenna, but varying with time. For a signal of wavelength λ let the current *I* vary as:

$$I = I_m \sin\frac{2\pi c}{\lambda}\left(t - \frac{r}{c}\right) \tag{2}$$

where I_m is the amplitude, *c* is the speed of light in vacuum, *t* is the *present time*, with respect to the effective point *P* distance *r* from the current, and the *retarded time* with respect to the causative antenna is $t - s/c$, with s/c as the retardation.

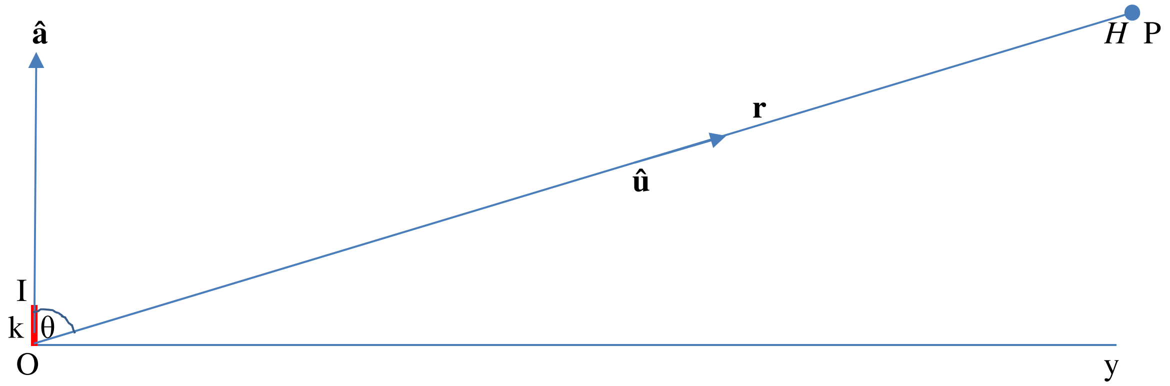

Fig.1 A short antenna of length *k* at origin *O* carrying a current *I* and creating a magnetic field *H* and electric field *E* at a point *P* distance *r* from *O*. The line *OP*, in the direction of unit vector **û**, is at an angle θ to the current direction **â**.

Equations (1) and (2) give:

$$\mathbf{a} = \hat{\mathbf{a}} \frac{k\mu_o I_m}{4\pi r} \sin \frac{2\pi c}{\lambda}\left(t - \frac{r}{c}\right) \tag{3}$$

The magnetic flux intensity, **b** at *P*, is given by equation (3) as:

$$\mathbf{b} = \nabla \times \mathbf{a} = \hat{\mathbf{u}} \times \frac{\partial \mathbf{a}}{\partial r} = -\hat{\mathbf{u}} \times \hat{\mathbf{a}} \frac{k\mu_o I_m}{4\pi r}\left\{\frac{1}{r}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right) + \frac{2\pi}{\lambda}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}$$

The magnetic field intensity **h** is given by:

$$\mathbf{h} = -\hat{\mathbf{u}} \times \hat{\mathbf{a}} \frac{kI_m}{4\pi r}\left\{\frac{1}{r}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right) + \frac{2\pi}{\lambda}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\} \tag{4}$$

For a changing magnetic flux, a reactive electric field **x** is generated, given by equation (4) and Faraday's law of electromagnetic induction [1, 2], thus:

$$\nabla \times \mathbf{x} = -\frac{\partial \mathbf{b}}{\partial t} = -\frac{\partial}{\partial t}\nabla \times \mathbf{a} = -\nabla \times \frac{\partial \mathbf{a}}{\partial t}$$

$$\mathbf{x} = -\frac{\partial \mathbf{a}}{\partial t} = -\hat{\mathbf{a}} \frac{kc\mu_o I_m}{2\lambda r}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right) \tag{5}$$

The Pointing vector **x**×**h** is obtained as:

$$\mathbf{x} \times \mathbf{h} = \hat{\mathbf{a}} \times (\hat{\mathbf{u}} \times \hat{\mathbf{a}}) \frac{c\mu_o k^2 I_m^2}{8\pi\lambda r^2}\left\{\frac{1}{2r}\sin\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right) + \frac{2\pi}{\lambda}\cos^2\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}$$

Power *P* passing across spherical surface **S** surrounding the antenna is the integral:

$$P = \int_S \mathbf{x} \times \mathbf{h}.(d\mathbf{S}) = \int_S \hat{\mathbf{a}} \times (\hat{\mathbf{u}} \times \hat{\mathbf{a}}).\hat{\mathbf{u}} \frac{c\mu_o k^2 I_m^2}{8\pi\lambda r^2}\left\{\frac{1}{2r}\sin\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right) + \frac{2\pi}{\lambda}\cos^2\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}(dS)$$

$$P = \int_S \hat{\mathbf{u}} \times \hat{\mathbf{a}}.(\hat{\mathbf{u}} \times \hat{\mathbf{a}}) \frac{c\mu_o k^2 I_m^2}{8\pi\lambda r^2}\left\{\frac{1}{2r}\sin\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right) + \frac{2\pi}{\lambda}\cos^2\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}(dS)$$

$$P = \int_S \frac{c\mu_o k^2 I_m^2}{8\pi\lambda r^2}\left[\frac{1}{2r}\sin\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right) + \frac{\pi}{\lambda}\left\{1+\cos\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}\right]\sin^2\theta(dS) \tag{6}$$

where **â**×(**û**× **â**).**û** = **û**.**â**×(**û**× **â**) = **û**×**â**.(**û**× **â**) = $\sin^2\theta$.

Power is exchanged between the magnetic field and the electric field. Over one cycle, the energy dissipated, in the fields, is zero. However, some power is radiated into space. In equation (6) there is a steady term, derived from the square of cosine, giving rise to power radiation. The power radiated P_r is:

$$P_r = \int_S \frac{\pi c\mu_o k^2 I_m^2}{8\pi\lambda^2 r^2}\sin^2\theta(dS) = \frac{\pi c\mu_o k^2 I_m^2}{8\pi\lambda^2}\iint\frac{1}{r^2}\sin^2\theta\{\mathrm{r}^2\sin\theta(d\theta)(d\varphi)\}$$

$$P_r = \frac{\pi c \mu_o k^2 I_m^2}{4\lambda^2} \int_{\pi}^{0} \sin^3\theta (d\theta) = \frac{\pi c \mu_o k^2 I_m^2}{3\lambda^2} \quad (7)$$

where $dS = r^2 sin\theta(d\theta)(d\varphi)$ and $\int_{\pi}^{0} \sin^3\theta (d\theta) = \frac{4}{3}$.

If R_r is the radiation resistance of a short antenna $(k < \lambda/10)$ with radiation power given by equation (7), we get:

$$\frac{\pi c \mu_o k^2 I_m^2}{3\lambda^2} = \frac{I_m^2}{2} R_r$$

$$R_r = \frac{\pi c \mu_o k^2}{3\lambda^2} = \frac{\eta \pi k^2}{3\lambda^2} = \frac{377\pi k^2}{3\lambda^2} = 394.84\left(\frac{k}{\lambda}\right)^2 \Omega \quad (8)$$

where $\eta = c\mu_o = 377$ ohms is the characteristic impedance of free space (vacuum).

3. Quarter-wave Monopole Antenna

The monopole antenna consists of a single conductive rod, one quarter of wavelength long, with one side of the input or output signal connected to one end of the rod and the other side connected to some type of ground. Fig. 2 shows a vertical quarter-wave antenna connected above the ground and carrying current I_z. The earth acts as a mirror, with zero field below and radiation field above. The wave is vertically polarised with vertical electric field and horizontal magnetic field, radiated uniformly away from the antenna. There is no radiation in the vertical direction. The radiation from a quarter-wave antenna, also called Marconi Antenna, need only be considered with respect to the conductor above the earth. The image of the antenna below ground, does not take part in radiation. This type of antenna is used in medium wave broadcast where the mast may be as high as 50 metres kept vertical by wire stays. The radiation resistance is 37.5Ω.

One terminal of the current lead is connected to the bottom end of the antenna and the other current terminal properly earthed through a buried wire mesh, called counterpoise, to provide a good earth connection. The earth may be artificial such as the metal chassis of a motor vehicle. In point-to-point mobile communication equipment the metal chassis, or even the human body, may form the connection to the earth.

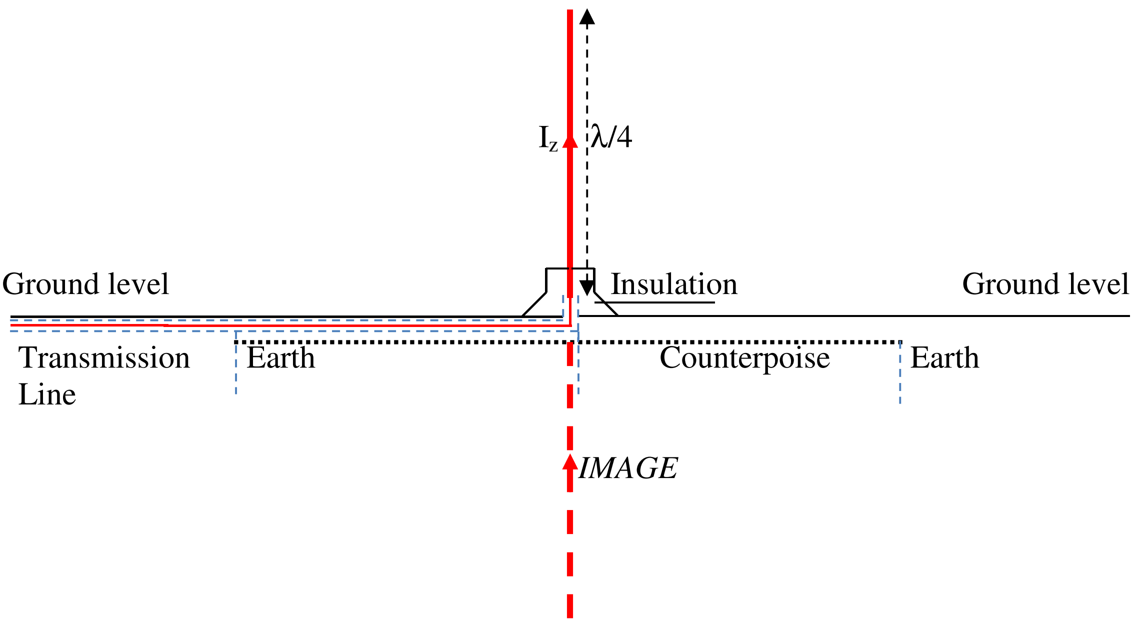

Fig.1. A vertical quarter-wave monopole antenna, radiating above the ground, with the image shown below the ground level.

4. Half-wave Dipole Antenna

The dipole antenna, of length equal to one-half of wavelength, is shown in Fig. 3a. It normally consists of two identical and usually bilaterally symmetrical conductive elements such as metal wires or rods separated by a narrow (insulating) gap. The input or output signal from the transmitter or receiver is applied at the middle between the two halves of the antenna. Each side of the feed to the transmitter or receiver is connected, across the gap, to one end of the conductors.

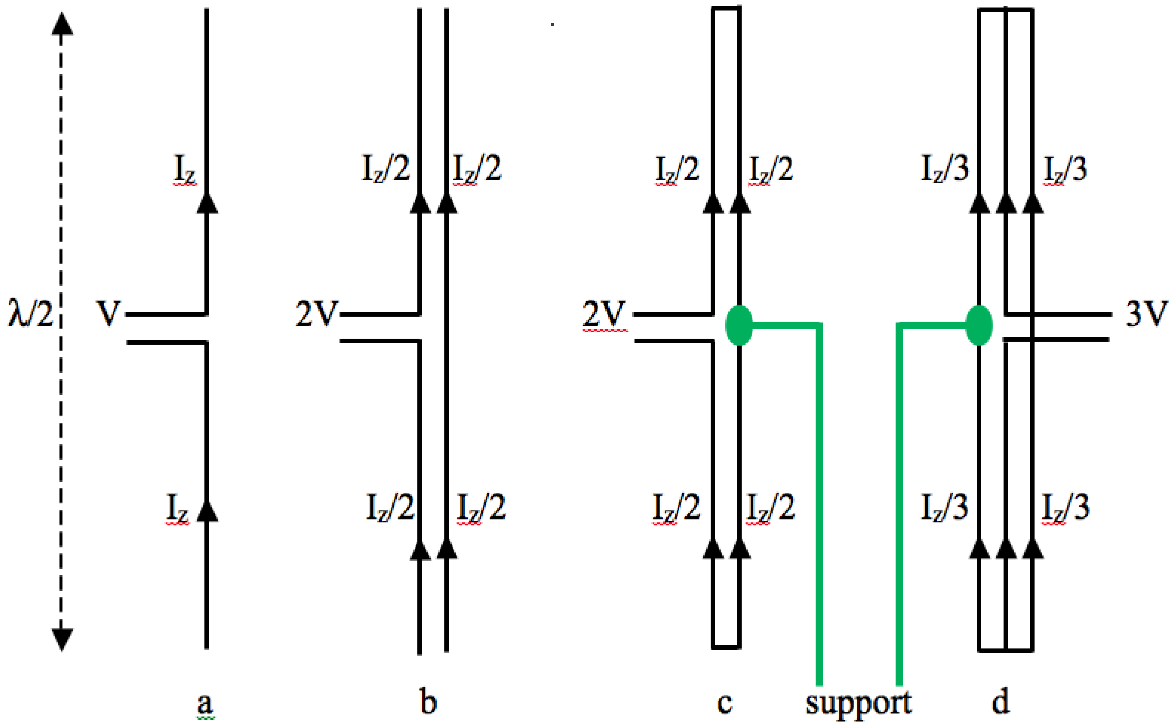

Fig. 3. Single dipole antenna (a) and folded dipole antennas (c) & (d) each radiating the same power.

In Fig. 3a the single half-wave dipole antenna carries a signal current I_z in the **z**-direction. In (b) we have a half-wave conductor very close to the energized dipole as shown. The oscillating magnetic field due to the dipole antenna induces a current in the half-wave conductor of the same phase and magnitude. Thus we have two antennas, carrying equal currents, surrounded by the same magnetic field. The half-wave dipole in (b) develops the same power as in (a) but with one-half of the current.

Since the current at each end of the antenna in Fig. 2b is zero, the top and bottom ends can be joined without altering any electrical effect. This gives a folded dipole antenna of two elements as shown in Fig. 2c. A folded dipole antenna of three elements is shown in Fig. 2d. A folded dipole antenna can have any number n of elements, with current fed in one element.

Let the single dipole antenna in Fig. 2a radiate power P with r.m.s. input current I and equivalent radiation resistance R_1 so that $P = I^2R_1$. The 2-element folded dipole in Fig. 2c, radiating the same power P with I/2 and radiation resistance R_2, gives $P = \frac{1}{4}I^2R_2 = I^2R_1$, making $R_2 = 4R_1$. For the 3-element folded dipole in Fig. 2d, the equivalent radiation resistance is $9R_1$. For an n-element folded dipole, the radiation resistance is n^2R_1. The radiation resistance R_1 of single dipole antenna (Fig. 2a) being about 75 Ω and the radiation resistance of the 2-element folded dipole antenna (Fig 2c) is about 300 Ω. The radiation resistance may be changed by increasing or reducing the width of the gap between the elements. Knowing the radiation resistance of an antenna is important for matching with the characteristic impedance of a transmission line.

One advantage of the folded dipole antenna is that it may provide a continuous path for direct current and can be connected directly to the output of a transmitter without a coupling transformer. The second advantage is that it is rigid and its central point (of maximum current) can be connected to an earthed metal rod/pole to provide a continuous and rigid support. Most aerials are various embellishment of the half-wave dipole antenna.

5. Magnetic and Electric Fields from a Dipole Antenna

Consider a dipole antenna (Fig. 2a) made of two straight and similar metal rods *AO* and *OB* of length *2Z* as shown in Fig. 3. The current I, fed between a narrow gap separating the rods, is not constant along the length of the antenna; it may, at an instant of time, be a maximum at *O* and zero at the open ends *A* and *B*. Let the current be I_z in an element of length δz at a position distance z from *O*. Each current element $I_z(\delta\mathbf{z})$ along the antenna contributes a magnetic field $\delta\mathbf{H}$ at a point *P* distance *r* from from *O*, where **OP** is at an angle θ to **AOB**. The small segment of wire of length δz may be regarded as a short antenna.

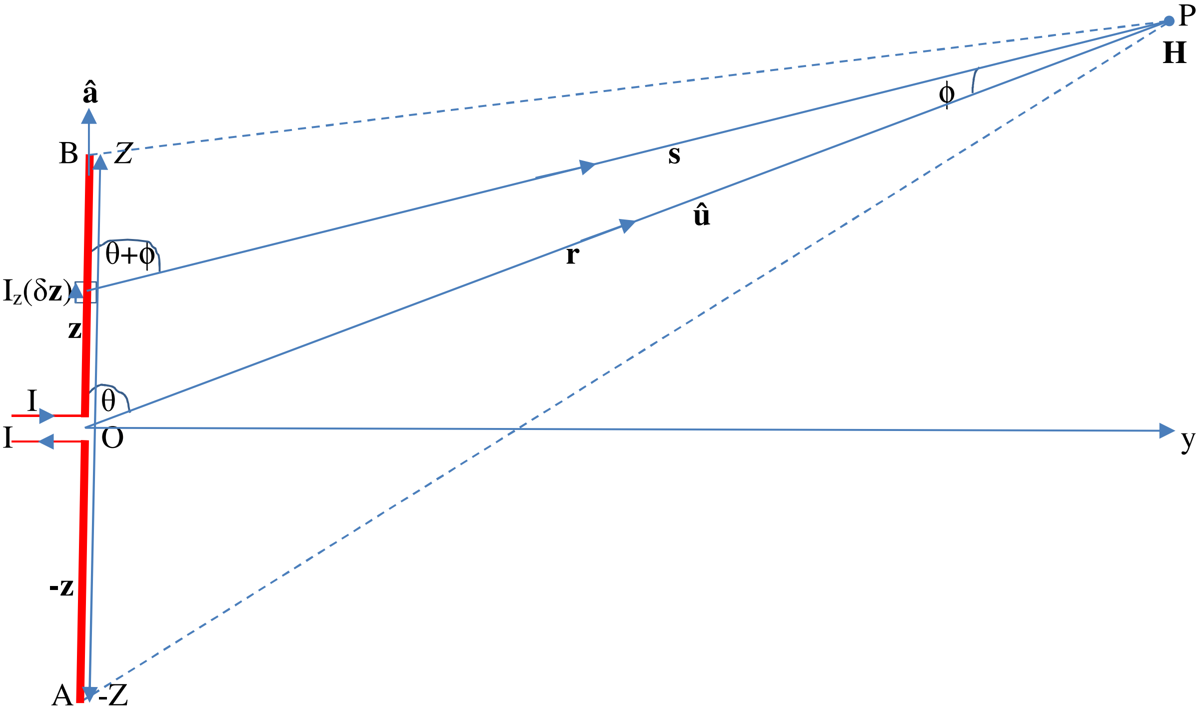

Fig. 3. A half-wave antenna **AOB** of length 2*Z* creating a magnetic field **H** at a far point *P*. A current element $I_z(\delta\mathbf{z})$ of length ($\delta\mathbf{z}$) carrying current I_z at a distance *z* from the midpoint *O*, contributes a magnetic field $\delta\mathbf{H}$ at *P*, a point at an angle *θ* from **AOB** and distance r from *O* and angle (θ+ϕ) at distance *s* from the current element.

5.1 Magnetic Vector Potential

With reference to Figure 3, the differential magnetic vector potential $\delta\mathbf{A}$ at *P*, distance *s* from the current element $I_z\,(\delta\mathbf{z})$, is given by:

$$\delta\mathbf{A} = \frac{\mu_o I_z(\delta\mathbf{z})}{4\pi s} \tag{9}$$

Let the antenna current I_z at a point distance *z* from *O*, for a signal of angular frequency *ω*, propagated with speed *c* and wavelength *λ*, vary with displacement *z* as:

$$I_z = I_o \cos\frac{2\pi z}{\lambda} \tag{10}$$

where $\omega = 2\pi c/\lambda$ and I_o varies with time *t* as:

$$I_o = I_m \sin\frac{2\pi c}{\lambda}\left(t - \frac{s}{c}\right) \tag{11}$$

Equations (10) and (11) give:

$$I_z = I_m \sin\frac{2\pi c}{\lambda}\left(t-\frac{s}{c}\right)\cos\frac{2\pi z}{\lambda} \tag{12}$$

where I_m is the amplitude, c is the speed of light in a vacuum, t is the *present time*, with respect to point P, distance s from the current element, and the *retarded time* with respect to the antenna is $t - s/c$, with s/c as the retardation.

The magnetic vector potential **A** at P (Fig. 3) is given by the line integral:

$$\mathbf{A} = \int_{-Z}^{Z} \frac{\mu_o I_m}{4\pi s}\sin\frac{2\pi c}{\lambda}\left(t-\frac{s}{c}\right)\cos\frac{2\pi z}{\lambda}(d\mathbf{z}) \tag{13}$$

In Fig. 3, the antenna of length $2Z$ is normally very short compared with the distance s, the distances s and r are usually very long and the angle ϕ is small, so that:

$$s \approx r$$

$$z \approx r\phi \text{ and } dz \approx r(d\phi)$$

Equation (13) becomes:

$$\mathbf{A} = \hat{\mathbf{a}}\int_{-Z}^{Z} \frac{\mu_o I_m}{4\pi r}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\cos\frac{2\pi r\phi}{\lambda}r(d\phi)$$

$$\mathbf{A} \approx \hat{\mathbf{a}}\int_{-Z}^{Z} \frac{\mu_o I_m}{4\pi}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\cos\frac{2\pi r\phi}{\lambda}(d\phi) \tag{14}$$

At a given point P (Fig. 3), far away from the antenna equation (14), for a quarter-wave (resonant) antenna of length $2Z = \lambda/2$, is integrated, between $\phi = \lambda/4r$ and $\phi = -\lambda/4r$, to give:

$$\mathbf{A} = \hat{\mathbf{a}}\frac{\lambda\mu_o I_m}{4\pi^2 r}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right) \tag{15}$$

5.2 Magnetic and Electric Fields

The magnetic flux intensity, **B** at P, is given by equation (15) as:

$$\mathbf{B} = \nabla\times\mathbf{A} = \hat{\mathbf{u}}\times\frac{\partial\mathbf{A}}{\partial r} = -\hat{\mathbf{u}}\times\hat{\mathbf{a}}\frac{\lambda\mu_o I_m}{4\pi^2 r}\left\{\frac{1}{r}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)+\frac{2\pi}{\lambda}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}$$

The magnetic field intensity **H** is given by:

$$\mathbf{H} = -\hat{\mathbf{u}}\times\hat{\mathbf{a}}\frac{\lambda I_m}{4\pi^2 r}\left\{\frac{1}{r}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)+\frac{2\pi}{\lambda}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\} \tag{16}$$

For a changing magnetic flux, a reactive electric field **X** is generated, given by equation (15) and Faraday's law of electromagnetic induction [1, 2], thus:

$$\nabla\times\mathbf{X} = -\frac{\partial\mathbf{B}}{\partial t} = -\frac{\partial}{\partial t}\nabla\times\mathbf{A} = -\nabla\times\frac{\partial\mathbf{A}}{\partial t}$$

$$\mathbf{X} = -\frac{\partial\mathbf{A}}{\partial t} \tag{17}$$

Equation (17) and equation (15) give:

$$\mathbf{X} = -\frac{\partial\mathbf{A}}{\partial t} = -\hat{\mathbf{a}}\frac{c\mu_o I_m}{2\pi r}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right) \tag{18}$$

5.3 Radiation from a Half-wave Antenna

Taking the vector (cross) product with **X** (equation 18) and **H** (equation 16) gives the Poynting vector **X**×**H**, power flow across unit area, as:

$$\mathbf{X}\times\mathbf{H} = \hat{\mathbf{a}}\times(\hat{\mathbf{u}}\times\hat{\mathbf{a}})\frac{c\lambda\mu_o I_m^2}{8\pi^3 r^2}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\left\{\frac{1}{r}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)+\frac{2\pi}{\lambda}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\} \tag{19}$$

The total power flow P, across surface area **S** surrounding the antenna, is:

$$P = \int_S \hat{\mathbf{a}}\times(\hat{\mathbf{u}}\times\hat{\mathbf{a}}).\hat{\mathbf{u}}\frac{c\lambda\mu_o I_m^2}{8\pi^3 r^2}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\left\{\frac{1}{r}\sin\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)+\frac{2\pi}{\lambda}\cos\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}(dS)$$

$$P = \int_S \frac{c\lambda\mu_o I_m^2}{8\pi^3 r^2}\left\{\frac{1}{2r}\sin\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right)+\frac{2\pi}{\lambda}\cos^2\frac{2\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}\sin^2\theta(dS)$$

where (Fig. 3) **â×(û× â).û = û.â×(û× â) = û×â.(û× â)** = $\sin^2\theta$

$$P = \int_S \frac{c\lambda\mu_o I_m^2}{8\pi^3 r^2}\left[\frac{1}{2r}\sin\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right)+\frac{\pi}{\lambda}\left\{1+\cos\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}\right]\sin^2\theta(dS) \quad (20)$$

5.4 Power Stored in the Magnetic and Electric Fields of an Antenna

Equation (20) gives the power stored in the magnetic and electric fields as:

$$P_s = \int_S \frac{c\lambda\mu_o I_m^2}{8\pi^3 r^2}\left\{\frac{1}{2r}\sin\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right)+\frac{\pi}{\lambda}\cos\frac{4\pi c}{\lambda}\left(t-\frac{r}{c}\right)\right\}\sin^2\theta(dS) \quad (21)$$

Power is exchanged between the magnetic field and the electric fields. Over one cycle, the energy dissipated in the fields, is zero. However, some power is radiated.

5.5 Power radiated by an Antenna

Equation (20) has a steady term which gives rise to power radiation. The power radiated is:

$$P_r = \int_S \frac{c\mu_o I_m^2}{8\pi^2 r^2}\sin^2\theta(dS) \quad (22)$$

Equation (22) shows that radiation power intensity (radiant power per unit area) is inversely proportional to r^2 and proportional to $\sin^2\theta$. This means that maximum power is transmitted perpendicular to the antenna and no radiation along the line of the antenna.

$$P_r = \frac{c\mu_o I_m^2}{8\pi^2}\int_S \frac{1}{r^2}\sin^2\theta(dS) = \frac{c\mu_o I_m^2}{8\pi^2}\int \frac{1}{r^2}\sin^2\theta\{r^2\sin\theta(d\varphi)(d\theta)\}$$

where the spherical surface area $dS = r^2\sin\theta(d\theta)(d\varphi)$.

$$P_r = \frac{c\mu_o I_m^2}{4\pi}\int \sin^3\theta(d\theta) = \frac{c\mu_o I_m^2}{3\pi} \quad (23)$$

where θ goes from 0 to π radians and $\int_\pi^0 \sin^3\theta(d\theta) = \frac{4}{3}$. Energy radiated, in time T, is TP_r.

Power intensity (power per unit area) R_m in the direction of strongest emission (θ = π/2 radians) is given by equation (22) as:

$$R_m = \frac{c\mu_o I_m^2}{8\pi^2 r^2} \quad (24)$$

5.6 Radiation Resistance

If R_r is the radiation resistance of a dipole antenna with radiation power given by equation (23), we get:

$$\frac{c\mu_o I_m^2}{3\pi} = \frac{I_m^2}{2}R_r$$

$$R_r = \frac{2c\mu_o}{3\pi} = \frac{2\eta}{3\pi} = 80.00\Omega \quad (25)$$

where $\eta = c\mu_o$ = 377 ohms is the characteristic impedance of free space (vacuum).

In practice radiation resistance R_r is reduced to 75 ohms, by a small increase of width of the gap between the conductive elements, to provide for load matching with an appropriate transmission line (coaxial cable). It is as if the whole of free space offers a resistance of R_r

ohms in the dipole antenna through which power is transmitted to other antennas or lost into space. Nothing is said here about the loss or ohmic resistance of the metallic antenna, which is negligible, but significantly affects the bandwidth of the resonant antenna, just like a resonant circuit.

6. Directivity of an Antenna

Directivity is defined as the ratio of power intensity R_m the antenna radiates (at a point) in the direction of its strongest emission, to the power intensity R radiated by an ideal radiator (which emits uniformly in all directions) radiating the same total power. Let R be the power intensity emitted by an ideal isotropic radiator, over a spherical surface area, $4\pi r^2$, surrounding the antenna, so that equation (23) gives:

$$P_r = 4\pi r^2 R = \frac{c\mu_o I_m^2}{3\pi}$$

$$R = \frac{c\mu_o I_m^2}{12\pi^2 R^2} \qquad (26)$$

Equations (24) and (26) give the directivity as:

$$D_R = \frac{R_m}{R} = 1.5 \qquad (27)$$

It can be shown the directivity of short antenna (described in section 2), is also 1.5.

References

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[2] I. S. Grant & W. R. Phillips; *Electromagnetism*, John Wiley & Sons, N, York (2008)