NL-DE BASED MODELING OF NATURAL ORBITAL SYSTEMS WITHOUT RELIANCE ON CONSERVATION OF ENERGY AND ANGULAR MOMENTUM

Slobodan Nedić[1]

[1] University of Novi Sad, Faculty of Technical Sciences
Trg Dimitrija Tucovica 6, 21000 Novi Sad, Serbia
e-mail: nedic.slbdn@gmail.com , nedics@uns.ac.rs

Abstract:

In difference to traditionally established determination of planetary orbits in the "potential force fields" by reliance on the postulated first integrals and with implicit avoidance of direct involvement of the time-parameter *t*, proposed is the orbital motion modeling based on use of a set Non-Linear Differential Equations of motion with provision of explicit centrifugal ('anti-gravitational') force and the implied torque, allowing for the explicit oscillator-like nature of the underlying system of the Kepler-Ermakov type and use of the related exact integration invariants. The untenability of the Kepler-Newtonian invariants has been supported by both analytical derivations and numerical evaluations. Besides by the numerical integration, the previously formulated Termo-Gravitational Oscillator configuration has been evaluated in its integral form. The positive-valued work pertinent to the (quasi-)closed orbital trajectories opens up prospects of the Least Action Principle application as its direct minimization and the awareness of 'precipitativeness' as energy inflow intrinsic feature of the "open" (thermo-)dynamical systems.

Key words: gravity, anti-gravity, orbital motion, non-conservative systems, forces unification

1. Introduction

Following the Newton's fitting of elliptical planetary orbits to the single central force inversely proportional to the square of its distance to the Sun, all natural systems – from atomic to galactic scales – have been treated as non-conservative (based on work over closed loop in the field of potential force equaling to zero). The exclusive reliance on gravitation as the only central force does not allow for the enough formal prediction of the planet's trajectories in accordance with the Kepler's second Law interpreted as the angular momentum, the basic shortcoming of Newton's and other subsequent theories of orbital motion then being the presumed absence of the tangential acceleration component, quite contrary to well established observational results.

In not so distant past and of late, there have been numerous explicit and/or implicit objections regarding both the existence of a conical (elliptical) solution based on particular initial conditions and its uniqueness [1]. While the former can be mostly related to the stability, the latter one is definitely well founded on grounds of the traditional modeling of orbital systems by autonomous differential equations without explicit presence of the time-parameter and unfounded (essentially, just postulated) the so-called invariants of the basic non-linear differential equation(s) integration – the total (sum of kinetic and potential) energy and the angular momentum. Indeed, regarding the exclusivity of the inverse squared distance proportionality of the central force as a cause of the Keplerian elliptic motion, within last several decades (in the context of including the upper-atmosphere caused dragging effect on the low-elevation satellites) there have appeared numerous papers which demonstrate the availability of generally non-central forces resulting in non-constant angular momentum, yet producing the conventional closed elliptical orbits, notably [2]. It has been shown that the truly invariant are the angular momentum with (in Descartes' coordinate system) time-derivatives (*dx/dt* and *dy/dt*) replaced by the area-derivatives (*dx/dA* and *dy/dA*), and the total energy with kinetic energy being evaluated with velocity as time derivative of the arc-length (*ds/dt*) replaced by the ratio *ds/dA*, whereby as the integrals arise ratios of time-varying both the squared angular momentum as well as the total energy and the time-varying Kepler-Newtonian force factor. Furthermore, the related Keplerian-like differential equation(s) can be put in direct correspondence with differential equation(s) having the velocity-dependent, friction term, implicating the steadily decreasing system energy [3] or the needed steadily increasing work of the force(s)-field to preserve the trajectory shape. This in turn urges to look for the source of the related energy supply in the 'operation' of the natural orbital systems (in agreement with impossibility of the "perpetual mobility"), changing the paradigm from the non-conservative systems involving dissipation to those with factual energy in-flow ('precipitation').

Only recently, within explorations of biological molecular systems, as well as in certain domains of particle physics, the need starts arising for looking at such systems as non-conservative, the so-called "open systems", which within the classical formalisms turn out to the "non-integrable" system (inability to be reduced to "cyclic coordinates", the so-called first integrals, be it by even applying the time-varying transformations of coordinate systems). This has led to modifications and specializations of the formalisms of the classical axiomatic mechanics having been developed by Euler, Lagrange, Hamilton, Noether, etc., for essentially conservative systems to be applicable to the non-conservative ones. However, the proper analysis of the matters suggests that all the natural orbital systems (including the planetary, atomic, molecular and galactic ones) are the 'open' ones - with the energy inflow form the Ether for the matter formation and for mediating the dynamical stability and that, in particular for ubiquitously present essentially non-closed orbital trajectories, neither the total energy nor the angular momentum is constant over the time, suggesting that the very basic foundations of orbital mechanics have largely been deficient, being the cause for emergence of quantum-mechanics.

Factually, the Newton's gravitational law was derived in a rather tautological (circular) manner, relying on the 'larger' object's mass also in definition of the gravitational constant, the suite having been followed in domain of electromagnetic and the atom-level phenomena, along the notion of potential, i.e. non-conservative force fields which could not support atoms' energy radiation. The incorporation of his third law of action and reaction, which even Newton himself had been reluctant to rely on explicitly (and despite many objections - notably Leibniz's statement that they cannot simultaneously be applied to the same body) into the theory of orbital motion, has been another misdeed, both with detrimental impact on the further development of physics, and the almost insurmountable difficulties it has been facing, including the forces' unification. In the concept of Thermo-Gravitational Oscillator (TGO) [4] developed by combining Le Sagean gravitational, and thermal (as anti-gravitational) changing of permittivity to the mutual shadowing 'pushing' effect, the central acceleration results in the form of two-components ($-a/r^2 + b/r^3$), that Leibniz had proposed within his critique of the Newton's orbital dynamics, without any reliance on the Newton's third law, and by using M. Milanković's (one over r-squared) law of planetary warming. It should be important to state here that already in the first edition of his "Principia" Newton himself had used the additional, explicit centrifugal force of the form + b/r^3 , in order to analyze and model the precession of the Mercury perihelion (https://en.wikipedia.org/wiki/Newton%27s_theorem_of_revolving_orbits). This, however, has remained largely unknown.

In the TGO-concept the orbital trajectory can in principle be produced by direct minimization of the work (needed to be) done over a 'closed' path, without indispensability of initial conditions (commonly considered as even a part of natural laws in the context of traditional minimization of variation of the Action as time-integral of difference and/or sum of the kinetic and potential energies of an orbital body in Lagrange and/or Hamilton formalisms). Another option is conventional (analytical or numerical) solving of the pertinent non-linear differential equations. While for the latter there has been a wide variety of program modules (as ode45.m in MATLAB), for the former there exists prospect of relating the TGO-like differential equations set-up with the since long known extensions of the Ermakov's system [5] for which the exact integration invariants have been readily available, towards the so-called Kepler-Ermakov systems [6]. While in TGO the gravitational constant (*a* above) is considered as not the "universal" one and basically dependable on actual configurations, the mass get entirely dropped away from the considerations, and in place of it (*b* above) comes the body's thermal capacity (or its specific heat). As further support for righteousness of this approach can be offered that the same form of the central accelerations, i.e. the 'attractive' and 'repulsive' forces are manifested within the toroidal vortex (sub-)atomic-level structures, respectively for the ring (electric field related) and toral (magnetic field related) streaming of the (compressible and viscous - gaseous) Ether-substrate particles [7]. It is unlikely to be a mere coincidence that the vortexes related attractive and repulsive forces, in the context of etherodynamics, along the lines of the pressure/velocity/temperature gradients and their decreases and/or increases, respectively, have exactly the same a/r^2& b/r^3 forms, allowing a wider outlook to commonalities among all the natural orbital systems, and particularly relatedness to the Prigogin's thermodynamics of Open Systems and the Entropy 'issues'.

In the following, firstly (in Section 2) is exposed untenability of the angular momentum constancy by simply 'declaring' co-linearity of the position and the angular momentum vectors and 'suppression' of time variable despite time-varying central force and further supported by analytical and numerical evaluations, along the refutation of the unsupported total energy "invariance". In Section 3 is briefly overviewed the Thermo-Gravitational Oscillator concept of orbital motion based on dynamical equilibrium between the gravitational and the heat-related anti-gravitational central forces, the work over a closed Keplerian elliptic trajectory is evaluated and its optimality is indicated based on its minimal value among all the scaling-like perturbations. Section 4 is devoted to formulation the TGO-related non-linear differential equations for two bodies, accompanied by a number of numerical solutions qualitatively illustrating effects of the added central force and its lateral projection torqueing term, followed by hinting similarity with the Kepler-Ermakov NL-DE system of equations to suggest appropriate formulations of the related "constants of integration" and possibly arrive at the intrinsic quantization and the Golden Ratio implications. In the concluding remarks, relevance to the outstanding problems and anomalies in physics are provided, as is obsolescence of the dark matter and dark energy notions.

2. Untenability of angular momentum and total energy as the Keplerian motion invariants

In the modern vector-analytical context in mechanics textbooks the constancy of both the direction and the absolute value of the angular momentum for the central force field as follows:

"Central forces $\boldsymbol{F(r},t)$ are always directed to a fixed point, wherein we place the origin $\boldsymbol{O}$ of the coordinate system:

$$\boldsymbol{F}(\boldsymbol{r},t)=\frac{\boldsymbol{r}}{|\boldsymbol{r}|}f(\boldsymbol{r},t). \tag{1}$$

Newton's movement equation for a punctual mass *m* then is:

$$m\ddot{\boldsymbol{r}}=\frac{\boldsymbol{r}}{r}f(\boldsymbol{r},t) \qquad \text{with} \qquad r=|\boldsymbol{r}|. \tag{2}$$

Vector multiplication with $\boldsymbol{r}$ gets $\frac{d}{dt}(m\boldsymbol{r}\times\dot{\boldsymbol{r}}) = m\dot{\boldsymbol{r}}\times\dot{\boldsymbol{r}} + m\boldsymbol{r}\times\ddot{\boldsymbol{r}} = \boldsymbol{0}$, (3)

so that $m\boldsymbol{r}\times\dot{\boldsymbol{r}} = \boldsymbol{C}ons\,tan\,t = \boldsymbol{L}$.“ (4)

However, since all the non-circular orbital motion solutions produce time-varying tangential velocity, the stated/alleged/postulated constancy of the angular momentum does not hold in general, since the tangential acceleration turns out to be not co-linear with the radius vector $\boldsymbol{r}$.

On the other hand, by decomposing left-hand side of (1) into its (planar) polar coordinates

$$\ddot{\boldsymbol{r}} = a_{rad}\boldsymbol{e}_{ra} + a_{trv}\boldsymbol{e}_{\varphi}\,;\; a_{rad} = \ddot{r} - r\dot{\varphi}^2\,;\; a_{trv} = r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = \frac{1}{r}\frac{d}{dt}(r^2\dot{\varphi}) = \frac{1}{r}\dot{L} = 0\,, \quad (5)$$

the transverse acceleration is equated by zero due to the definitional absence of the lateral driving force in the central force (field), further (or rather upfront) by identifying the $r^2\dot{\varphi}$ as the rate of change of (half of) the sectorial area, in accordance with the second Kepler’s law. It has largely remained unrecognized that neither one of these motivations for the constancy of *L* does hold: the left-most expression in (5) can be satisfied for non-constant *L* and infinite radius, and the areas of a segments between two different (subsequent, in equal time intervals) radius-lengths (*r* and *r* + *dr*) can be represented by $r^2\dot{\varphi}$ only for rather specific time dependences of radius and polar angle. Yet another claim for conservativeness (in terms of zero wok over a closed path) has been the so-called time-independence of the central - radius-dependent force, although the very radius, de-facto, explicitly depends on time. Indeed, if the force is time dependent, the energy cannot be ‘conserved’ since the work done is path dependent. The traditionally held view that the work (integral over time of the time rate of change of the kinetic energy $dK/dt = \boldsymbol{F}(\boldsymbol{r},t)\cdot\dot{\boldsymbol{r}}$) of the (implicitly) time-dependent ‘central’ force depends only of the velocities at the two time-instants (strictly, the work should be evaluated with absolute value of the sub-integral function)

$$K_1 - K_0 = \left(\frac{m}{2}\dot{\boldsymbol{r}}^2\right)_{|t=t_1} - \left(\frac{m}{2}\dot{\boldsymbol{r}}^2\right)_{|t=t_0} = \int_{t_0}^{t_1} \boldsymbol{F}(\boldsymbol{r},t)\cdot\dot{\boldsymbol{r}}\cdot dt\,, \quad (6)$$

by (inadvertent?!) avoidance of explicitly accounting for the time-dependent central force, in that with (1) and (2) defined acceleration $m\cdot\ddot{\boldsymbol{r}}(t) = \boldsymbol{F}(\boldsymbol{r},t)$, the resulting sub-integral expression $m\cdot\ddot{\boldsymbol{r}}(t)\cdot\dot{\boldsymbol{r}}\cdot dt$ is replaced, w/ $\frac{d}{dt}\left[\dot{\boldsymbol{r}}^2(t)\right] = \frac{d}{d\dot{\boldsymbol{r}}}\left[\dot{\boldsymbol{r}}^2(t)\right]\cdot\frac{d\dot{\boldsymbol{r}}}{dt} = 2\dot{\boldsymbol{r}}(t)\ddot{\boldsymbol{r}}(t)$, by $\frac{1}{2}\frac{d}{dt}\left[\dot{\boldsymbol{r}}^2\right]dt = \frac{1}{2}d\left[\dot{\boldsymbol{r}}^2\right]$. While this seems to be correct, except that the time-variable/ility is fully hidden, it should be noted that the scalar product in the sub-integral function implies only the work over the radial direction. The ‘routinely’ added potential energy, *U*, to compensate for radial energy loss or gain

$$\frac{d}{dt}\left(\frac{m}{2}\dot{\boldsymbol{r}}^2 + U(\boldsymbol{r},t)\right) = 0 \quad \text{(7) and (8)}\downarrow$$

$$dU(\boldsymbol{r},t) = grad\big(U(\boldsymbol{r},t)\big) + \frac{\partial}{\partial t}\big(U(\boldsymbol{r},t)\big);\; F(\boldsymbol{r},t) = -grad\big(U(\boldsymbol{r},t)\big) \Rightarrow \frac{d}{dt}\left(\frac{1}{2}\dot{\boldsymbol{r}}^2 + U(\boldsymbol{r},t)\right) = \frac{\partial U(\boldsymbol{r},t)}{\partial t}$$

also fails by virtue of non-zero time-related gradient of the central force potential[1], per (8) above.

In the case of inverse radius squared central force, the validity of (7) has been supported by both reliance on the angular momentum invariance, as well as on zero-velocities at the perihelion and aphelion positions of the orbital body on an elliptic (Keplerian) orbit. Although the equations of motion (2) in case of central force $f(\mathbf{r},t) = -m\frac{a}{r^2(t)}$ for quite some time in the past can be solved numerically, from the "Feynman's 'trick'" of halving the elementary time-interval for the very first integration step to modern advanced numerical routines for solving non-linear differential equations, despite the critical dependence on suitable initial conditions due to inherent non-oscillatory nature, in wide usage still is the reliance on the alleged constancy (time-invariability) of angular momentum, L and total energy, E, i.e. system of two 1-st order NL-DEs

$$\frac{m}{2}\dot{r}^2 + U_{eff}(r) = E;\ \frac{m}{2}\dot{r}^2 = E + m\frac{a}{r} - \frac{1}{m}\frac{L^2}{r^2} \Rightarrow \frac{d}{dt}r(t) = \sqrt{\frac{2}{m}\left(E + m\frac{a}{r(t)} - \frac{1}{2m}\frac{L^2}{r^2(t)}\right)}$$

$$mr^2(t)\dot{\varphi}(t) = L \quad \Rightarrow \quad \frac{d}{dt}\varphi(t) = \frac{1}{mLr^2(t)};\ \text{with}\ \ \varphi(t) = L\int_0^t \frac{d\tau}{mr^2(t)} + \varphi_0\ \text{as integral form.} \qquad (9)$$

Whereas for the original equation of motion $m\ddot{\mathbf{r}} = -m\frac{a}{r^2(t)}$ the orbiting body mass becomes irrelevant, which is consistent with the independence of gravitational acceleration on mass, in (9) it is put back in the 'play'. Furthermore, although this system of NL-DE can – at least in principle be attempted by a numerical integration of these two equations[2], in order to possibly arrive at some kind of closed solutions the first equation in (10) is looked at as t in function of r, that is $t = \int_{r_0}^{r}\frac{d\rho}{\sqrt{----}}$, but what becomes feasible is nothing more than to calculate orbital period and produce parametric dependence of radius on polar angle with the latter related to t by exactly the Kepler-equation, relating the equidistant time intervals with the true eccentric anomaly (the angle from the center of an ellipse to the point on the large circle vertically above the orbital body position. (The closed form expression becomes available only for $r_0 = r_{max}$ but can't be inversed.)

2.1 Analytical and numerical evaluations to illustrate the issues related to the 'invariants'

For the radius as function of the Eccentric anomaly E, $r(t) = a\cdot[1 - e\cdot cos(E(t))]$, the expression for the radius as function of the True anomaly φ, $r(t) = a\cdot(1-e^2)/[1+e\cdot cos(\varphi(t))]$, the renowned Kepler's equation, $\frac{2\pi}{T}t = E(t) - e\cdot sin(E(t))$, the relation between the True anomaly and the Eccentric anomaly angles $\varphi = 2\cdot arctng\left[\sqrt{\frac{1+e}{1-e}}\cdot tng\left(\frac{E}{2}\right)\right]$, and (to the third equation above)

[1] This fact appears to have been finding expression in the rheonomic potential (V. Vujičić, "Dyn. of Rheon. Sys., Math. Instit. of the Serbian Accad. of Science, Editions Spéciales, Belgrade, 1990.). More generally, it could be argued that the time-variability of a (central) force ensures non-zero Curl feature of the related force-field (non-zero loop integral), and in particular so if there are two (co-linear) potentials involved, as in Whittaker's "A Treatise on the Analytical Dynamics and Rigid Bodies," Article 52 of 4th edition (1937).

[2] In the sequel will be shown the related limitation, in that the solution for fixed E and L tends to a circle.

pertinent time-derivative $\dot{E}=\frac{2\pi}{T}\frac{1}{1-e\cdot\cos(E)}$, the angular momentum (taking a=1+ε to aid the separation of the subsequent plots) becomes

$$L=r^2\cdot\dot{\theta}=(1+e)^2\cdot\left(1-e^2\right)^2\cdot\frac{2\pi}{T}\sqrt{\frac{1+e}{1-e}}\cdot\left[\frac{1}{1+e\cdot cos(\theta)}\right]^2\cdot\left\{cos^2\left(\frac{E}{2}\right)+\frac{1+e}{1-e}\cdot sin^2\left(\frac{E}{2}\right)\right\}^{-1}\cdot\frac{1}{1-e\cdot cos(E)}$$

Although 'visually' only for zero-eccentricity [$e=0\rightarrow a=1$] the angular momentum is a constant ($2\pi/T$), it turns out that it is constant for all eccentricities, as shown in Fig. 1a. (While this holds also for the numerical evaluation by using the 10-terms Lagrange's solution of the involvement of the explicit time-dependence of *E*, its utilization to produce and explicitly use the time-derivative of *E* reveals increased variations of *L* with ε (Fig. 1b).) However, it turns out that the Kepler's equation itself does not satisfy the Kepler's very Second (area-) low, as the results plotted in Fig. 2b reveal. (This might even not be that surprising if one considers the way the Kepler's equation was derived – by explicit removal and insertion of triangular areas pertaining to the large circle and the ellipse, respectively, besides scaling-down the areas under the arcs – see Appendix A.) Similarly, analytical evaluation of the total energy as per the first (upper left) part in (9) reveals

$$K=2\cdot\left(\frac{\pi}{T}\right)^2(1+e)^2\left\{e^2\frac{\left[sin(E(t))\right]^2}{\left[1-e\cdot cos(E(t))\right]^2}+\frac{1+e}{1-e}\cdot\left[cos^2\left(\frac{E(t)}{2}\right)+\frac{1+e}{1-e}\cdot sin^2\left(\frac{E(t)}{2}\right)\right]^{-2}\right\}-\frac{1}{(1+e)\left[1-e\cdot cos(E(t))\right]}$$

and the plots in Fig. 2a reveal its increased time-variability with the increase of the eccentricity.

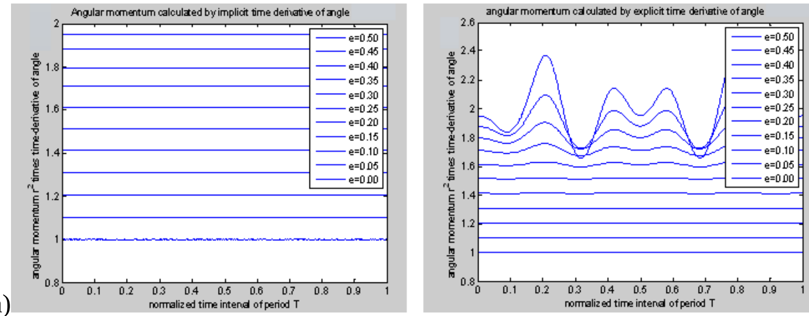
a)  b)

Fig. 1. Analytical evaluations of the angular momentum: w/o - a), & w/ explicit t-derivatives – b).

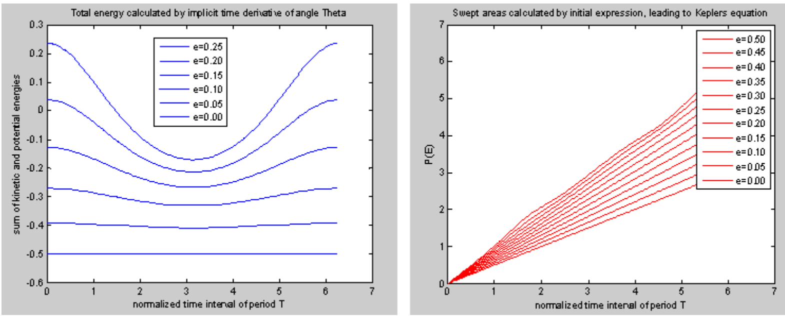
a)  b)

Fig. 2. Analytical evaluations of the total energy- a), and the swept area for Kepler's equation – b).

2.2 Numerical evaluation indicating 'weakness' of the energy and angular momentum integrals

When evaluated numerically on Kepler-ellipse with eccentricity ε=0.25 in polar coordinates (r,φ), with the explicit time-dependence of phase on time produced by numerically inverting the Kepler's equation using the Interpolation Tool inside the MATLAB's plotting routine, the angular momentum and total energy become time-variable with a solid level of regularity and the strong departure from constants (as is the case in actual measurements), shown here in Fig. 3.

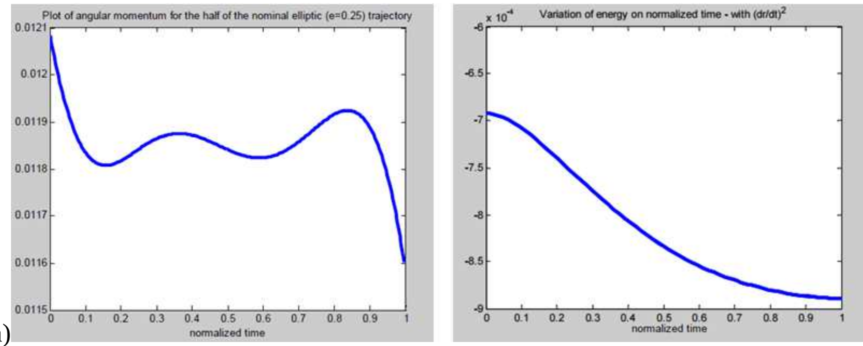
a)  b)

Fig. 3. Dependence of the angular momentum - a) and energy- b) on time (for the first half starting from the perihelion) for Keplerian ellipse with eccentricity factor of e=0.25 and its average value.

This is largely corroborated by significant (starting from lower decimals) and consistent variations replicable by measurements data (http://www.phy.duke.edu/~kolena/comet.html).

Finally, as example for the failure of the corresponding first integrals' differential equations on the right-hand side of (9) to reproduce the elliptic trajectory and the related (non-constant) L and E determined by the set of regular non-linear differential equations (5), that is the left-most part in the two rows of (9), with a=108.0 Nm^2, and m=1kg, particular initial conditions ($r_0 = 1.0; \varphi_0 = 0.0; \upsilon_0 = 6.0 m/s; L_{(0)} = r_0 \upsilon_0$) and optionally solving on $\dot{L}$ instead on $\dot{\varphi}$ as the angular state variable), produced are results shown in Fig. 4.

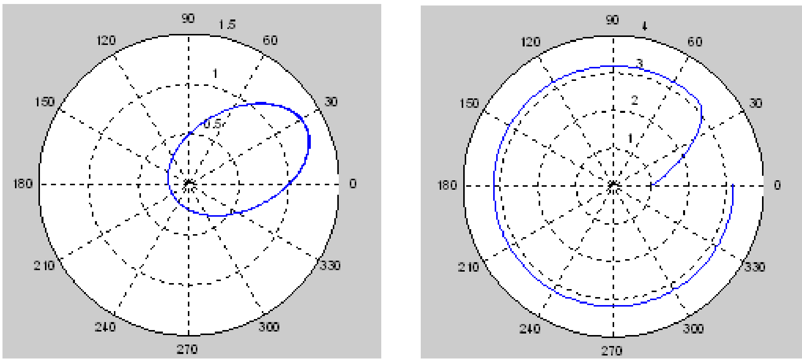
a)  b)

Fig. 4. Comparison of polar plots produced by conventional NL-DE and the traditional invariants.

While for the same elliptical orbit as shown in part a) of Fig. 4 the two conventional nonlinear solutions produce rather different behaviors of the so-called integration invariants (highly time-varying L and E, and constant L and steadily decreasing E, respectively), in both cases the trajectory produced by the reused initial values for L and E tend to produce circular path, part b).

3. Orbital motion as a dynamical equilibrium – Thermo-Gravitational Oscillator approach

The following considerations are based (in the phenomenological sense) on dynamical equilibrium between the Le Sage-like gravitational and the postulated thermal components of the effective 'force' driving the planet around the Sun over certain path (by the second author of [4]). In essence, the gravitational component itself is thermal, and what is exposed here is more like an outline of ultimately thermo-dynamical theory of orbital motion[3].

[3] The truly physical cause of gravitational 'attraction' in the Atshukovsky'setherodynamics [7] being the gradient in pressure in the range of the 'attracted' body induced by the temperature gradient created by the

With the reference to Fig. 5, the work done on an elementary segment dr of a trajectory is the result of two components – gravitational, $dE = m(\gamma / r^2)dr$ (γ representing the gravitational, not necessary "universally valid" Newtonian constant) and the thermal one, $dQ = m \cdot \delta \cdot dT$, ($\delta$ - specfiic heat), with the Milanković's temperature dependence on radius $T = f(r) \propto 1/r^2$, becomes $dQ = -m\xi(1/r^3)dr$, so that work integral takes the form given by the expression

$$\oint \left| \left\{ \left[-\frac{\gamma}{r^2(t)} + \frac{\xi}{r^3(t)} \right] \cdot cos[\alpha(t)] \right\} \frac{dr(t)}{dt} \cdot dt \right| ; \pi/2 - \alpha(t) = \psi(t) = arctng\left(\frac{dr(\varphi(t))/d\varphi(t)}{r(\varphi(t))}\right) \quad (10)$$

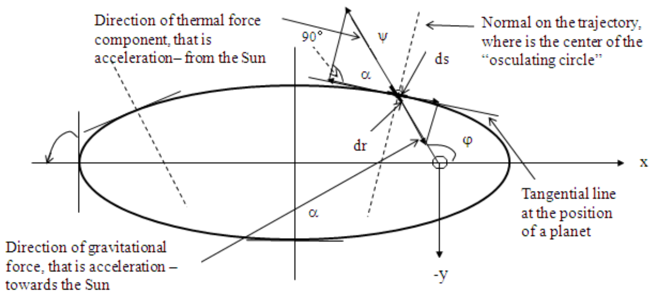


Fig. 5. Illustration of thermo-gravitational equilibrium for motion of a planet around the Sun.

An evaluation of the work done over the closed trajectories produced by vertical scaling of a Keplerian ellipse with eccentricity ε=0.25 reveals (non-negative!) minimum (the positive energy-well!) at the scaling factor one (i.e. the nominal ellipse), as the plot(s) in Fig. 6 show.

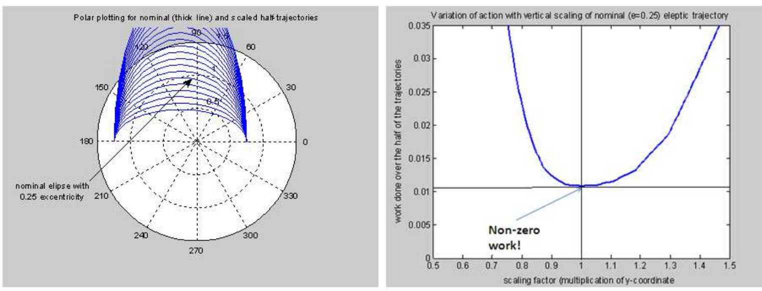


Fig. 6. Scaled nominal elliptic trajectories and the closed-path work scaling-factor dependence.

4. Formulation of orbital motion with direct modeling by non-linear differential equations

With reference to Fig. 5 and the related expressions in (10), the TGO NL-DE set-up becomes:

differential equation for radial direction $$\frac{d^2 r}{dt^2} - r \cdot \left(\frac{d\varphi}{dt}\right)^2 = -\frac{a}{r^2} + \frac{b}{r^3} \quad (11)$$

and differential equation for transverse (lateral) direction

$$\frac{d}{dt}\left(r^2 \cdot \frac{d\varphi}{dt}\right) = r \cdot \left\{ \left(-\frac{a}{r^2} + \frac{b}{r^3}\right) \cdot cos(\psi) \cdot sin(\psi) \right\}, \quad (12)$$

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attracting body, thus reciprocally, the heat received from the central body acts anti-gravitationally in the sense of equalizing the reduced temperature and pressure on the side facing the central body, i.e. the Sun.

the latter one in the form explicating the torque-force as projection of the radial driving force.

For comparison, the traditional Newtonian formulation (presuming alleged constancy of L) is

$$\frac{d^2r}{dt^2} = -\frac{a}{r^2} + \frac{L^2}{r^3} \text{ (for radial[4]), and } r^2 \cdot \frac{d\varphi}{dt} = L \text{ (for lateral, transverse direction)} \qquad (13)$$

In the form which on the left-hand side has the original kinematical form, (12) and (13) are

$$r \cdot \frac{d^2\varphi}{dt^2} + 2 \cdot \left(\frac{dr}{dt}\frac{d\varphi}{dt}\right) = tFlag \cdot \left(-\frac{a}{r^2} + \frac{b}{r^3}\right) \cdot cos(\psi) \cdot sin(\psi), \qquad (14)$$

with the tFlag-parameter equaling 1 and 0 for the TGO and standard forms, respectively.

For qualitative illustration of various orbital motion equations, in Fig. 7 are shown trajectories for the same initial conditions and the four combinations of the parameters a, b and tFlag.

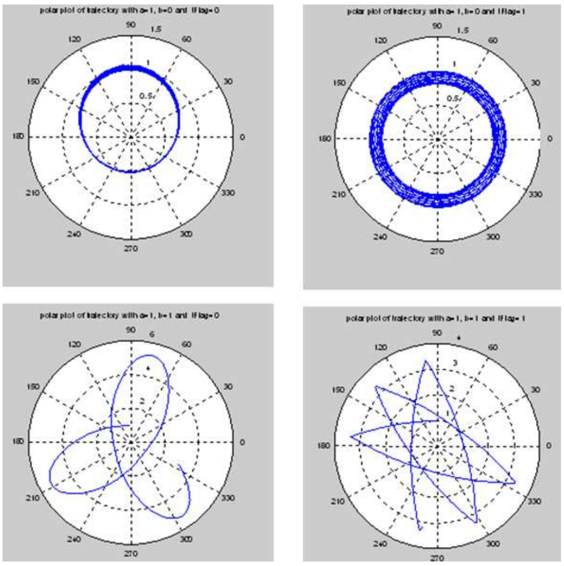

Fig. 7. Trajectories with particular sets of parameters and the resulting $\log_{10}(E)$ values: upper-left – 1,0,0 and 2.00; upper-right - 1,1,0 and 0.25; lower-left - 1,0,1 and 1.30; lower-right – 1,1,1 and 0.50.

These equations of motion bear strong resemblance to the so-called Kepler-Ermakov system of non-linear differential equation, the polar coordinate form of which has the following form, [6]

$$\frac{d^2r}{dt^2} - r \cdot \left(\frac{d\varphi}{dt}\right)^2 = -\frac{F(\varphi)}{r^2} + \frac{G(\varphi)}{r^3} \text{ and } r \cdot \frac{d^2\varphi}{dt^2} + 2 \cdot \left(\frac{dr}{dt}\frac{d\varphi}{dt}\right) = -\frac{dV(\varphi)/d\varphi}{r^3}, \qquad (15)$$

whereby, along the *exact* (Lewis-Ray-Reid) invariant[5] $I = 0.5\left(r^2\dot{\varphi}\right)^2 + V(\varphi)$ and $\eta = 1/r$, the trajectory follows from the solution of the non-homogeneous differential equation $h^2(\varphi; I)\frac{d^2\eta}{d\varphi^2} + h(\varphi; I)\frac{\partial h(\varphi; I)}{\partial\varphi}\frac{d\eta}{d\varphi} + \left(h^2(\varphi; I) + F(\varphi)\right)\eta = G(\varphi)$, with $h(\varphi; I) = \sqrt{2}\left(I - V(\varphi)\right)$.

[4] It should be important to note that in the history of science this inverse proportionality to the distance on cube, commonly denoted as “virtual” centrifugal force (and the related virtual potential as a part of the effective potential) has been mistakenly understood as either the Leibnitz’s or Newton’s explicit forces; this might also have been related to the apparently long-standing controversies over the CF-forces ‘nature’.

[5] There exists, of course, possibility to upfront refute the principles of conservation of energy and angular momentum in the Kepler-Newton system configuration, since the insertion of the equation of motion $\ddot{r} - r\dot{\varphi}^2 = -k/r^2$ into the invariants defining equations (7) and $(d/dt)(r^2\dot{\varphi})$ non-zero result is produced.

This opens the perspective to on one hand bring the physical substantiation to the equations of motion in domain of particle the physics and the quantum mechanics, and on the other, to proceed with specializing and more appropriate parameterization of the TGO equation regarding the explicitly contributing non-central forces – through time-variability of parameters *a* and *b*, etc.

Of particular interest would be to extend and further specialize the pertaining first integrals towards possible revealing the Golden Ratio proportions along the lines of the two-centers configurations and the hyperbolic coordinates considerations and analysis conducted in [8].

5. Conclusions

As its main goal, this paper has brought up questioning of the basic conservation laws of total energy and angular momentum traditionally used in Newtonian dynamics of orbital motion and inherited in other areas of mechanics and physics as GRT, QM and Particles Physics. While in the contemporary mechanics and the mathematical physics the conservativeness of angular momentum and energy have been the foundational principles, their untenability has been demonstrated here by the arguments ranging from the point of view of non-uniqueness of the solution, over the unfulfilling conservation conditions in the context of Keplerian ellipse and elementary analytical evaluation involving the Kepler's equation, to numerical evaluations demonstrating the fact that the produced solutions either do not satisfy the initially formulated invariants or reproduce them by reverting from elliptical to the trivial, circular trajectories. Based on that, and the essentially present non-zero tangential acceleration, the currently ubiquitous characterization of natural orbital systems as conservative (zero-work closed paths) has been refuted, and physically motivated orbital motion formulated in the context of gravitational and anti-gravitational components providing explicit non-central (external) driving forces.

Consequently, the quest for the non-accounted for (outside) forces/effect should be directed towards revealing the hidden resources and the structuring potential features of the very Ether substrate with commonality of the two constituent central forces $-a/r^2$ and b/r^3 with the attracting and repulsive forces related to electric and magnetic phenomena, respectively. Consequently, numerous gravitational anomalies, geostationary satellites "dancing", Lunar paradox and in general three- and many-bodies' problems appear to be readily solvable by adopting the principle formulation of TGO and the implied reliance on the Ether - the so-called Pioneer anomaly becomes solvable, and the conceived "dark matter" and "dark energy" largely obsoleted.

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Appendix A: On the issue that has been revealed in relation to the Kepler's Equation

Here is provided explanation regarding the discrepancy between results of analytical derivation and numerical evaluation of the time-dependence of the angular momentum for the closed (Keplerian) ellipse with the dependence of polar angle (θ, or φ in this paper's body) on the eccentric anomaly angle (E) given by $\theta = 2 \cdot arctng\left[\sqrt{\frac{1+e}{1-e}} \cdot tng\left(\frac{E}{2}\right)\right]$ along the famous Kepler's equation $\frac{2\pi}{T} t = E(t) - e \cdot sin(E(t))$ – please see the Peter Colwell's book "Solving Kepler's Equation over Three Centuries"(Willman-Bell, Inc., 1993 – in Introductory section; available on request from the author of this paper), produced therein by apparent erroneous equating – in (1.2) - of the two forms of an ellipse: the ellipse radius dependence $r(t) = a \cdot [1 - e \cdot cos(E(t))]$ and the conventional expression of ellipse as function of the polar (true anomaly) angle $r(t) = \frac{a \cdot (1-e^2)}{1 + e \cdot cos(\theta(t))}$. By the time-differentiating the Kepler's equation to produce $\frac{2\pi}{T} = \dot{E}(t) - e \cdot cos(E(t)) \cdot \dot{E}(t)$, and further $\dot{E} = \frac{2\pi}{T} \frac{1}{1 - e \cdot \cos(E)}$, along with $\theta = 2 \cdot arctng\left[\sqrt{\frac{1+e}{1-e}} \cdot tng\left(\frac{E}{2}\right)\right]$, the angular momentum (w/ conveniently taking $a = 1 + e$) becomes $L = r^2 \cdot \dot{\theta} = (1+e)^2 \frac{2\pi}{T} \sqrt{\frac{1+e}{1-e}} \cdot \frac{1 - e \cdot cos(E)}{cos^2\left(\frac{E}{2}\right) + \frac{1+e}{1-e} \cdot sin^2\left(\frac{E}{2}\right)}$, revealing the proportionality of the expressions in the numerator and denominator, thus the constant values. However, the incorrectness of the above referenced identity is demonstrated by simple numerical evaluation of the related expressions for the Keplerian ellipse, with the results shown in the following figure.

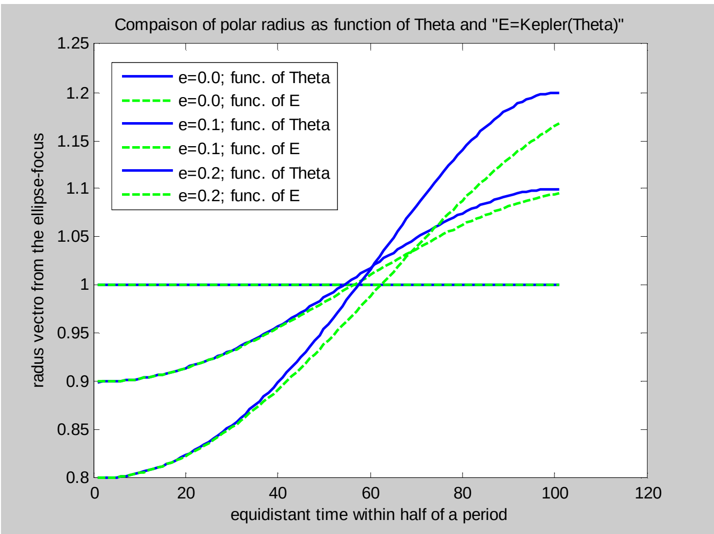

Fig. A.1. Comparison of the radius values for two ellipse expressions for the same angle vlues.