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Gravitational Deflection of Starlight*:*

The Newtonian Value for Bending of Starlight by the Sun

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Abstract:

The primary objective of the following investigation is to demonstrate, beyond the reasonable doubt, that Johann Georg von Soldner's prediction of *0.876* seconds of arc, for the angle of starlight deflected by the gravitational field of the Sun, as computed within the framework of Newton's theory of universal gravitation, is, mathematically, correct and, physically, valid only in the reference frame, in which the barycenter of the solar system is at rest. And also to show, furthermore, that the relative velocity of deflected starlight, as calculated and measured in the moving reference frame of the planet earth, alters, necessarily, and changes, in a straightforward manner, the angle of gravitational deflection from its aforementioned numerical value of *0.876* arcseconds, as computed within the stationary reference of the solar system, to a new and different numerical value equal to about *1.752* arcseconds, as calculated and measured within the moving reference frame of the planet earth.

Keywords:

Soldner's prediction; gravitational deflection of starlight; hyperbolic orbits; Einstein's prediction; Newton's corpuscular model of light; gravitational lensing; Archimedes' method of exhaustion; gravitational acceleration; relative velocity of starlight.

Introduction:

Although there have been several suggestions, by Isaac Newton and others, that the gravitational fields of celestial bodies (*e.g. the Sun*) can produce measurable amounts of deflection of starlight, Johann Georg von Soldner was, definitely, the first researcher ever, in this field, to publish, in *1801*, a detailed treatise, on this particular topic of celestial mechanics, entitled: "**On the deflection of a light ray from its rectilinear motion, by the attraction of a celestial body at which it nearly passes by**", in Bode's Journal [***Ref. #1***].

In Soldner's *1801* paper, no frame of reference is, explicitly, mentioned or specified; but it's quite obvious that his calculations can be carried out only in the reference frame, in which the barycenter of the solar system is at rest.

And also, in the aforementioned paper, even though the mass of starlight is automatically canceled out, Soldner's calculations imply, necessarily, that light, in general, and starlight, in particular, must have a certain amount of mass; otherwise, it seems, on the face of it, that the force of gravity would not be able to deflect starlight along a hyperbolic trajectory around the Sun.

Nonetheless, even if the assumption of massless starlight is physically true, it would not invalidate the final numerical result of Soldner's calculations, because the predicted value of *0.876* seconds of arc, for the angle of gravitational deflection by the Sun, can still be obtained by simply assuming, as Albert Einstein did in his *1911* paper, that the force of gravity acts on all forms of energy the same way as it does on mass [***Ref. #16***].

But let's assume, for a moment, that starlight, and electromagnetic radiation generally, is, as defined within the framework of the classical wave theory, nothing but periodic oscillations in a carrying medium named 'A*ether*'. What will happen to Soldner's calculations, in this case?

Clearly, if the classical wave theory is applied to the case, under discussion, then Soldner's calculations ought to be altered and re-worked out; but the predicted result of *0.876* seconds of arc will, nevertheless, remain the same.

And that is because, if starlight is assumed to be aetherial vibrations, then gravitational deflection of starlight can only be computed by treating the gravitational field of the Sun as a refractive medium with a varying refractive index [***Ref.* #2 & *Ref.* #9**].

However, although it's more intuitive, the partial differential equation, needed for calculating the variations in a refractive index inversely proportional to the square of distance from the central mass, on the basis of the classical wave theory, is expected, in this case, to be at the same level of complexity as that of Einstein's field equation. And hence, it is going to be virtually impossible to extract any numerical values out of it, without making, first, numerous simplifying and somewhat arbitrary assumptions, in order to solve it. But, since the numerical value of *0.876* arcseconds is known, beforehand, and predicted already by Johann Georg von Soldner, it won't be very difficult to select just the right simplifying assumptions to obtain it.

In any case, the main focus, in the current investigation, is on Soldner's original calculations of gravitational deflection. And since those calculations are based, squarely, upon Newton's theory of universal gravitation as well as Newton's corpuscular theory of light, the assumption that starlight has an amount of mass greater than zero, here, is taken for granted.

But it has to be emphasized, in this regard, that the notion of ponderable light is not just merely a theoretical assumption, within the framework of corpuscular and ballistic theories. On the contrary, and as a matter of fact, there is a considerable number of observations and physical experiments, which either directly support it, or can be easily re-interpreted in its favor, including the photoelectric effect, the black-body radiation, the Compton effect, Eddington's *1919* measurements of deflected starlight [***Ref. #15***], and more importantly the simple fact that all kinds of emitting objects lose mass by emitting light, and all sorts of absorbing objects gain mass by absorbing light. Surely, the widely used label, nowadays, for it is the '*mass-energy equivalence*'; but that, for the most part, is just playing around with words, in order not to let the amounts of momentum and kinetic energy of light, as calculated on the basis of Einstein's theory of special relativity, become infinite; i.e., that widely used label is, mainly, a theoretical peculiarity, within the framework of Einstein's special theory of relativity, nothing more and nothing less.

And finally, it should be pointed out, within the present context, that, in the stationary reference frame of the solar system, every star, located anywhere on the celestial sphere, can have the Sun's surface grazed by its rays, as depicted in Soldner's paper.

While, by contrast, in the moving reference frame of the planet earth, no grazing of the Sun's surface by starlight, as specified in Soldner's paper, is, really, possible, unless it's emitted by stars, located within just one half degree around the ecliptic.

Also, the locations, at which starlight, from stars close to the ecliptic, is observed to be grazing the surface of the Sun, do not change with time very much. For instance, starlight, which was observed by Eddington et al to be grazing the surface of the Sun, on May *29, 1919* [***Ref. #11***], is still doing the same grazing at the same locations up to the present time.

It's true that the orbital motion of the Sun, around the barycenter of the solar system, must change those Eddingtonian locations, but only periodically and not by much.

And since the grazing starlight is always there; and Earth and its Moon just run, occasionally, into it, any theoretical consideration, based upon the travel time of starlight from the Sun to Earth, the length of starlight's path between the Sun and Earth's orbit, . . . etc., is, in this particular case, a nonstarter and out of the question.

But, nevertheless, the fact that the longer the travel time of deflected starlight from the Sun to Earth; the greater the increase, due to the force of gravity, in the transverse component of its velocity, still can be mathematically utilized, with regard to Johann Georg von Soldner's calculations of the angle of deflected starlight by the gravitational field of the Sun.

1. A Brief Review of Soldner's Calculations:

According to Johann Georg von Soldner, if a ray of light arrives at a point at the surface of an attracting body in the horizontal direction, then the straight line, drawn through the place of observation and the center of the attracting body, will be the major axis for the curved trajectory of deflected light, as illustrated in ***Figure #1*** below:

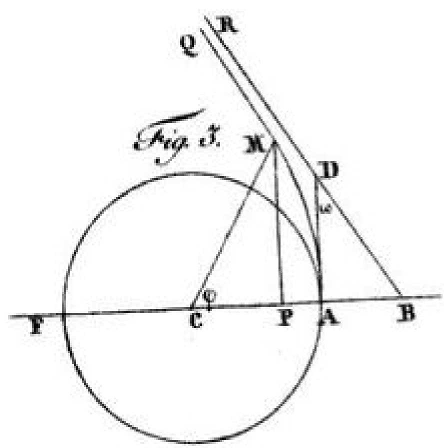

Figure #1: *Soldner's illustration*

And subsequently, the light ray will be forced, by the gravitational field of the celestial body, to travel along the curved line AMQ, instead of the straight line AD [***Ref. #1***].

As it can be deduced, immediately, from the illustrated geometry above, the trajectory of deflected starlight is hyperbolic.

However, right from the very beginning of the quantitative treatment of the hyperbolic orbit of deflected starlight around the Sun, in his *1801* original paper in the Bode's *1804* volume as well as in its *1921* republication by Philipp Eduard Anton von Lenard, Johann Georg von Soldner came up with these two most controversial and hotly disputed formulas in his entire article:

$$\frac{ddx}{dt^2} = -\frac{2g}{r^2}\cos\phi$$

and:

$$\frac{ddy}{dt^2} = -\frac{2g}{r^2}\sin\phi$$

where g is the gravitational acceleration on the surface of a celestial body.

And that is because if they are re-written, in accordance with modern notation:

$$\frac{d^2x}{dt^2} = -\frac{2g}{r^2}\cos\phi$$

and:

$$\frac{d^2y}{dt^2} = -\frac{2g}{r^2}\sin\phi$$

then Soldner's two equations, taken at face value, must lead, mathematically, to this Einstein's equation:

$$\tan\omega = \frac{4GM}{c^2R}$$

where ω is the angle of gravitational deflection; G is the gravitational constant; M is the mass of the deflecting body; and R is the radius of the deflecting body.

Nonetheless, Johann Georg von Soldner obtained, at the end of his calculations, this equation:

$$\tan\omega = \frac{2g}{v\sqrt{v^2 - 4g}}$$

which is equivalent in most respects to the standard Newtonian equation:

$$\tan\omega = \frac{2GM}{c^2 R}$$

where c is the muzzle speed of light in vacuum.

And furthermore, by inserting 18th - century observational data into the above equation, Johann Georg von Soldner obtained the following numerical value:

$$\omega = 0.84 \ \text{ arcseconds}$$

for the angle of starlight deflected by the gravitational field of the Sun.

And so, the big question, now, is this:

Was Johann Georg von Soldner, actually, intending to construct a mathematical formula that gives a maximum numerical value of the Newtonian deflection of starlight, by the Sun's gravitational field, equal to *1.68* arcseconds; but then, unwittingly and accidentally, dropped the factor *2*, and obtained *0.84* arcseconds instead?

According to H. J. Treder & G. Jackisch, despite the presence of the factor *2* at the start of his calculations, "*by no means did Soldner give double the Newtonian amount of the deflection of light, which then would be Einstein's*" [***Ref. #10***].

In the view of these two authors, Johann Georg von Soldner's calculations of the gravitational-deflection angle are based upon these two equations:

$$\frac{d^2x}{dt^2} = -\frac{g}{r^2}\cos\phi$$

and:

$$\frac{d^2 y}{dt^2} = -\frac{g}{r^2}\sin\phi$$

which, naturally, lead to the standard equation for the hyperbolic orbit of deflected starlight.

But, once again, why did Johann Georg von Soldner decide to write his two equations in this form:

$$\frac{ddx}{dt^2} = -\frac{2g}{r^2}\cos\phi$$

and:

$$\frac{ddy}{dt^2} = -\frac{2g}{r^2}\sin\phi$$

where g is the acceleration of gravity on the surface of a celestial body?

To resolve this apparent paradox, H. J. Treder & G. Jackisch have pointed out that the two sets of equations differ by a factor of *2*, mainly because, in the terminology of the German physical literature in the 18th century, the acceleration of gravity was not defined as the differential quotient:

$$g = -\frac{d^2 s}{dt^2}$$

but simply by setting

$$g = st^{-2}$$

and writing the Galilean equations for free fall as:

$$s = gt^2$$

and:

$$v = 2gt$$

where g is the gravitational acceleration as defined by the German physicists and astronomers of the *18th* century, which, according to these two authors was, occasionally, a cause of error with regard to the exact analytic mechanics of Pierre-Simon Laplace and Joseph-Louis Lagrange [***Ref. #10***].

From historical standpoint, the above interpretation, by H. J. Treder and G. Jackisch, on the basis of notations and terminologies, in order to explain away the presence of the *2* factor in Johann Georg von Soldner's *1801* calculations, is very reasonable and quite convincing.

However, it's also possible that Johann Georg von Soldner had tried Archimedes' method of exhaustion first before applying to the case, under investigation, the analytical methods of calculus.

If the trajectory of starlight on both sides of the Sun is divided into small segments, each of which is equal to ΔL, and for each of which the small increase Δv in the transverse component of the velocity of deflected starlight is calculated separately, then the *2* factor ought to be instrumental and very useful in reducing the tremendous amount of required calculations in this regard.

And that is because, for each ΔL with Δv on the far side of the Sun, there is an equal ΔL with equal Δv on the near side of the Sun, with respect to the earth.

And based upon this frequent repetition of the *2* factor, one might expect, at first glance, that the numerical results of calculations, in accordance with the aforementioned Archimedes' method of exhaustion, would fit very nicely into this Einstein's equation:

$$\tan \omega = \frac{4GM}{c^2 R}$$

where ω is the angle of gravitational deflection; G is the gravitational constant; M is the mass of the deflecting body; and R the radius of the deflecting body.

But, believe it or not, within just *10* solar radii, on both sides of the Sun, it becomes abundantly clear that the numerical results of computations, in accordance with Archimedes' method of exhaustion, cannot fit nicely with anything else beside the well-known Newtonian equation:

$$\tan \omega = \frac{2GM}{c^2 R}$$

where c is the muzzle speed of light in vacuum.

Here is a brief list of the essential aspects of Johann Georg von Soldner's calculations that can be, independently, verified through the use of Archimedes' method of exhaustion:

- The radial component of the velocity of deflected starlight increases at the far side of the Sun; but it decreases at the near side of the Sun with respect to Earth. And since most of the increase and decrease, in this velocity component, due to the gravitational field of the Sun, occurs mainly within Earth's orbit, the initial value of the radial velocity component is effectively restored by the time the deflected starlight reaches the earth.

- The transverse component of the velocity of deflected starlight increases at the far side of the Sun, and continues to increase as well at the near side of the Sun with respect to Earth. And since the numerical value of this transverse velocity component does not start to decrease until the deflected starlight crosses the extended radial line from the gravitational center of the Sun, through the center of the solar disc to infinity, about *6* light-days away from the barycenter of the solar system, the value of this velocity component is, effectively, permanent, and deflected starlight is, gravitationally, slingshot. And consequently, the Sun, along with its solar system, would never be able to regain or to take the whole part of its lost angular momentum back from the deflected starlight.

- In order to connect, directly, the physical parameters in Soldner's calculations, as illustrated in ***Figure #1*** above, to the standard Newtonian equation, for calculating the angle of gravitational deflection, in the stationary reference frame of the solar system:

$$\tan \omega = \frac{2GM}{c^2 R}$$

  it's necessary, and more convenient as well, to re-write the above equation, first, in this form:

$$\tan \omega = \frac{v}{c} = \frac{1}{c}\left(\frac{GM}{R^2}\right)\left(\frac{2R}{c}\right)$$

  in which the gravitational acceleration, on the surface of the Sun, g_s is:

$$g_s = \frac{GM}{R^2} = 274.1 \ \text{ms}^{-2}$$

the travel time of starlight, over the diameter of the Sun, T is:

$$T = \frac{2R}{c} = 4.645494 \text{ seconds}$$

and the transverse velocity component of deflected light, v is:

$$v = g_s T = 1273.329846 \ \text{ms}^{-1}$$

and therefore, the angle of gravitational deflection, ω is:

$$\omega = \arctan\left(\frac{v}{c}\right) = 0.876 \text{ arcseconds}$$

- If Johann Georg von Soldner had inserted the following numerical value of the speed of light, c_{OR}, as measured by Ole Roemer in the 18th century:

$$c_{OR} = 214636000 \ \text{ms}^{-1}$$

into the aforementioned Newtonian equation, he would have obtained a numerical result much closer to the observed result, as measured by Arthur S. Eddington in *1919*, than the numerical value of Einstein's prediction, on the basis of his general theory of relativity, for the angle of starlight deflected by the Sun; i.e.,

$$\omega = \arctan\left(\frac{v}{c_{OR}}\right) = 1.70899 \text{ arcseconds}$$

where ω is the angle of gravitational deflection.

- And finally, as comprehensive and truly original as actually is, Johann Georg von Soldner's treatise, on the gravitational deflection of starlight, does not take into account two quantitatively important considerations, which have the potential of changing considerably and altering by a significant amount the numerical result predicted by Newton's theory of universal gravitation, as computed in accordance with this standard equation:

$$\tan \omega = \frac{2GM}{c^2 R}$$

in the case of starlight deflected by the gravitational field of the Sun:

I. Starlight must lose a measurable fraction of its initial velocity c upon climbing out of the gravitational wells of emitting stars.

II. And more importantly, the computed value, within the framework of Newton's theory of universal gravitation, in the case of starlight deflected by the Sun:

$$\omega = \arctan\left(\frac{v}{c}\right) = 0.876 \text{ arcseconds}$$

can be observed and measured only within the reference frame, in which the barycenter of the solar system is at rest. And hence, the above numerical value can never ever, under any circumstances, be obtained or measured anywhere within the reference frame of the moving earth, in which the direction of the relative velocity resultant of the velocity of incident starlight and the orbital velocity of Earth can always play a major role.

2. The Effect of Initial Velocity of Starlight:

As mentioned, in this discussion, already, Johann Georg von Soldner's computation of gravitational deflection, on the basis of Newton's theory of universal gravitation and Newton's corpuscular theory of light, does not take into consideration any potential effects of the gravitational fields of emitting stars on the initial velocity of starlight.

Given the state of observational astronomy at that time, however, the assumption of initial velocity of starlight equal to c at infinity, in Soldner's calculation, is entirely warranted and well justified.

Nonetheless, the assumption of initial velocity of starlight equal to c at ∞ is, definitely, incorrect; since light emitted, by any star, in any direction, must have its muzzle velocity c reduced by an amount directly proportional to the gravitational acceleration, g_{star}, on the surface of the emitting star:

$$g_{star} = \frac{GM_{star}}{R_{star}^2}$$

where G is the gravitational constant; M_{star} is the mass of the emitting star; and R_{star} is the radius of the same emitting star.

However, the expected variations in the initial velocity of starlight, due to the force of gravity, are, in the case of main-sequence stars, relatively small.

And that is mainly because, even though the masses of main-sequence stars can range from about a *0.1* to *200* times the mass of the Sun [***Ref. #14***], the ratio between the radius of a main-sequence star and its mass varies, primarily, in accordance with the following relation:

$$\frac{R_{star}}{M_{star}} = 0.8$$

where R_{star} stands for the radius of the main-sequence star; and M_{star} stands for its mass.

And furthermore, the aforementioned variations in the velocity of starlight, caused by the gravitational field of any emitting star, is automatically included in the radial component of the total velocity of that star relative to the Sun; and accordingly, no separate calculation of those particular variations is needed or required, within the current context.

Let u_s denote the radial component of the velocity of the emitting star relative to the Sun, as measured through the Doppler effect or by any other method.

If the emitting star is approaching the Sun, which is either at rest or receding with a lower velocity, or the Sun is approaching the emitting star, which is either at rest or receding at a lower velocity, or the Sun and the emitting star are approaching each other at the same time, then, in all of these three cases, the radial component of the initial velocity of incident starlight, c_A, remains exactly the same:

$$c_A = c + u_s$$

where u_s is the radial velocity component of the emitting star relative to the Sun.

And likewise, if the emitting star is receding from the Sun, which is either at rest or approaching at a lower velocity, or the Sun is receding from the emitting star, which is either at rest or approaching at a lower velocity, or the Sun and the emitting star are receding from each other at the same time, then, in all of these three cases, the radial component of the initial velocity of incident starlight, c_R, is exactly the same:

$$c_R = c - u_s$$

where c is the muzzle velocity of light in vacuum.

And therefore, in the reference frame, in which the barycenter of the solar system is at rest, this Newtonian equation, for calculating gravitational deflection of starlight, which is assumed to be traveling at a velocity equal to c at infinity:

$$\tan \omega = \frac{2GM}{c^2 R}$$

must take the following two forms, in the case of approach, and in the case of recession, respectively:

A. In the Case of Approach:

In the case of starlight traveling at a velocity of c_A at infinity, the angle of gravitational deflection, ω_A is computed, in the reference frame, in which the gravitational center of the solar system is at rest, by using this equation:

$$\tan \omega_A = \frac{2GM}{c_A^2 R} = \frac{2GM}{c^2 R \left(1 + \frac{u_s^2}{c^2} + 2\frac{u_s}{c} \right)}$$

where G is the gravitational constant; M is the mass of the Sun; R is its radius; and c_A is obtained through the use of the following formula:

$$c_A = c + u_s$$

where u_s is the radial velocity component of the emitting star with respect to the Sun.

For example, the star Sirius (*Alpha Canis Majoris*) is, currently, approaching the Sun with a radial velocity u_s:

$$u_s = 5500 \text{ ms}^{-1}$$

and hence, the radial velocity component of its starlight c_A is:

$$c_A = c + u_s = 299797958 \text{ ms}^{-1}$$

And it follows, therefore, that starlight, emitted by the star Sirius, and deflected by the gravitational field of the Sun, has, in the stationary reference frame of the solar system, an angle of gravitational deflection equal to ω_A:

$$\omega_A = \arctan\left(\frac{2GM}{c_A^2 R}\right) = 0.87597 \text{ arcseconds}$$

where G is the gravitational constant.

_B._ In the Case of Recession:

In the case of starlight traveling at a velocity of c_R at infinity, the angle of gravitational deflection, ω_R is calculated, in the reference frame, in which the barycenter of the solar system is at rest, in accordance with the following equation:

$$\tan\omega_R = \frac{2GM}{c_R^2 R} = \frac{2GM}{c^2 R\left(1 + \frac{u_s^2}{c^2} - 2\frac{u_s}{c}\right)}$$

where G is the gravitational constant; M is the mass of the Sun; R is its radius; and c_R is obtained through the use of the following formula:

$$c_R = c - u_s$$

where u_s is the radial velocity component of the emitting star relative to the Sun.

For instance, the star Aldebaran (*Alpha Tauri*) is, at the present time, receding from the Sun with a radial velocity u_s:

$$u_s = 54260 \ \ \mathrm{ms^{-1}}$$

and accordingly, the radial velocity component of its starlight, c_R is:

$$c_R = c - u_s = 299738198 \ \ \mathrm{ms^{-1}}$$

And it follows, therefore, that starlight, emitted by the star Aldebaran, and deflected by the gravitational field of the Sun, has, as computed in the reference frame, in which the gravitational center of the solar system is at rest, an angle of gravitational deflection equal to ω_R:

$$\omega_R = \arctan\left(\frac{2GM}{c_R^2 R}\right) = 0.87632$$

where c_R is the radial velocity component of deflected starlight

In ***Table #1*** below, the angle ω_c for the gravitational deflection of starlight assumed to be traveling at c at infinity, and the angle $\omega_{c'}$ for the gravitational deflection of starlight assumed to be traveling at $(c \pm u_s)$ at infinity, are computed separately, in the reference frame, in which the barycenter of the solar system is at rest; and then $\omega_{c'}$ is subtracted from ω_c in the case of an approaching star; and ω_c is subtracted from $\omega_{c'}$ in the case of a receding star:

The star	**u_s ms^{-1}**	**ω_c arc-sec.**	**$\omega_{c'}$ arc-sec.**	**$\Delta\omega$**
Regulus (α Leonis)	+5900.0	0.87608319340	0.87642812650	3.44933×10^{-4}
Rigil Kent (α Centauri)	-18600.0	0.87608319340	0.87597449399	1.08699×10^{-4}
Spica (α Virginis)	+1000.0	0.87608319340	0.87689038027	5.84463×10^{-6}
Vega (α Lyrae)	-13900.0	0.87608319340	0.87600159138	8.12343×10^{-5}
Thuban (α Draconis)	-13000.0	0.87608319340	0.87600721857	7.597483×10^{-5}
Rigel (β Orionis)	+17800.0	0.87608319340	0.876187236510	1.04043×10^{-4}
Alpha Cancri (α Cnc)	-13800.0	0.87608319340	0.87600254352	8.06499×10^{-5}
Pollux (β Geminorum)	+3230.0	0.87608319340	0.87610207176	1.88784×10^{-5}
Procyon (α CMi)	-3200.0	0.87608319340	0.87606449610	1.87024×10^{-5}
Canopus (α Carinae)	+20300.0	0.87608319340	0.87620185079	1.18657×10^{-4}

Table #1: *Comparison between ω_c & $\omega_{c'}$ for several stars*

In the above table, $-u_s$ is the radial velocity component of an approaching star; $+u_s$ is the radial velocity component of a receding star; $\omega_{c'}$ is the gravitational-deflection angle of starlight traveling at the velocity of $(c \pm u_s)$ at infinity; ω_c is the gravitational-deflection angle of starlight traveling at the velocity of c at infinity; $\Delta\omega$ is the difference between ω_c & $\omega_{c'}$.

As demonstrated in ***Table #1*** above, for typical radial velocities of milky-way stars relative to the Sun, the numerical differences between the gravitational deflection of starlight assumed to be traveling at c at infinity and the gravitational deflection of starlight assumed to be traveling at $(c \pm u_s)$, at infinity, as calculated in the reference frame, in which the gravitational center of the solar system is at rest, are, in most cases, less than one millisecond of arc.

And therefore, as far as the current observational techniques of the solar deflection of starlight emitted by stars within the galaxy of the milky way are concerned, the assumption of starlight traveling at c at infinity, and the assumption of starlight traveling at $(c \pm u_s)$ at infinity, are practically equivalent to each other and with little or no real impact at all on the calculations of Johann Georg von Soldner or on the calculations of his contemporaries or modern successors.

3. The Effect of Relative Velocity of Deflected Starlight:

As pointed out earlier, in the present investigation, the predicted numerical value, on the basis of Newton's theory of universal gravitation and Newton's corpuscular theory of light, in the case of starlight deflected by the gravitational field of the Sun:

$$\omega = \arctan\left(\frac{v}{c}\right) = 0.876 \text{ arcseconds}$$

can be calculated and measured only in the reference frame, in which the barycenter of the solar system is at rest.

In other words, it's absolutely impossible to measure or to obtain, by any observational means, the above numerical value, anywhere within the reference frame of the moving earth, in which the direction of the vector of the velocity resultant of the velocity of incident starlight and the orbital velocity of Earth, by its very definition, points to a new direction, necessarily, different from the initial direction of incident starlight.

In the moving reference frame of the planet earth, all of the true positions of stars, on the celestial sphere, are shifted, due to the velocity resultant of the orbital velocity of the earth and the velocity of incident starlight, towards the forward direction of the earth's motion, in accordance with James Bradley's equation:

$$\sin\Delta\theta = \sin\left(\theta' - \theta\right) = \frac{v_E}{c}\sin\theta$$

where θ' is the true position of a star, as observed in the stationary reference frame of the solar system; θ is the shifted position of the same star, as observed in the moving reference frame of the earth; v_E is the orbital velocity of the earth; and c is the muzzle velocity of light in vacuum.

Let θ' & θ'_ω denote the true position of the star and the true position of its gravitationally deflected image, as observed in the stationary reference frame of the solar system. respectively.

To compute the shifted position of the star, θ, and the shifted position of its gravitationally deflected image, θ_ω, as observed in the moving reference frame of the earth, we have to rearrange, accordingly, the standard Bradley equation above; in order to obtain these two equations:

$$\tan\theta = \frac{\sin\theta'}{\cos\theta' + v_E/c}$$

and:

$$\tan\theta_\omega = \frac{\sin\theta'_\omega}{\cos\theta'_\omega + \dfrac{v_E + v_g}{c}}$$

in the case of stars west of the Sun;

and these two equations, in the case of stars east of the Sun:

$$\tan\theta = \frac{\sin\theta'}{\cos\theta' + \dfrac{v_E}{c}}$$

and:

$$\tan\theta_\omega = \frac{\sin\theta'_\omega}{\cos\theta'_\omega + \dfrac{v_E - v_g}{c}}$$

where θ'_ω is calculated by using this equation, in the case of a star west of the Sun:

$$\theta'_\omega = \theta' - \omega$$

and by using this equation, in the case of a star located east of the Sun:

$$\theta'_\omega = \theta' + \omega$$

in which ω is computed by using this equation:

$$\omega = \arctan\left(\frac{v}{c}\right) = 0.876 \text{ arcseconds}$$

and where v_E is the orbital velocity of the earth; and v_g is the transverse velocity component of deflected starlight, in the stationary reference frame of the solar system, as computed by using the following equation:

$$v_g = c \tan \omega$$

in the case of starlight assumed to be traveling with c at infinity.

As illustrated in ***Figure #2*** below, the vector of the orbital velocity of the earth is always pointing to the west, on the Sun-facing side of the earth,with respect to the Sun:

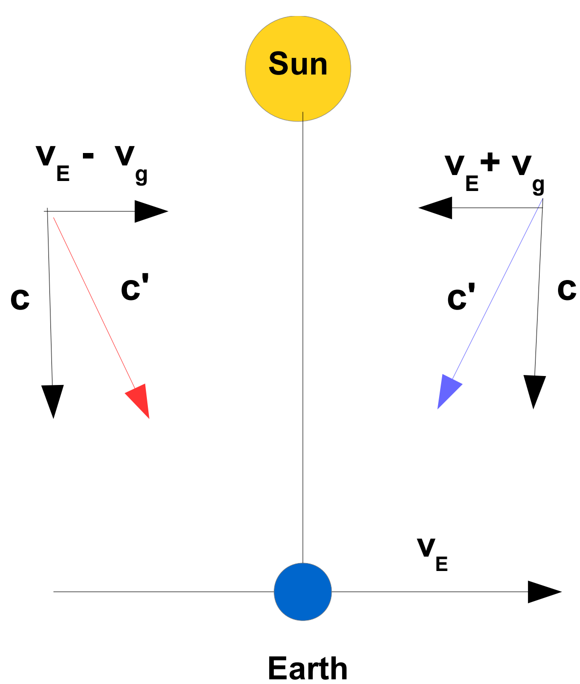

Figure #2: *Relative velocity of deflected starlight*

And therefore, if the star is located west of the Sun, the transverse velocity component of its deflected starlight, v_g, is added to the orbital velocity of the earth, v_E, because the two velocity vectors are pointing to the opposite direction of each other.

While, by contrast, if the star is located east of the Sun, the transverse velocity component of its deflected starlight, v_g, is subtracted from the orbital velocity of the earth, v_E, because the two velocity vectors are pointing to the same direction.

4. The Computed Predictions on the Assumption of c at ∞:

On the assumption of initial velocity of starlight equal to c at infinity, it follows, therefore, that the difference between the shifted position of any star located around the Sun, θ, and the shifted position of its gravitationally deflected image, θ_ω, as measured in the moving reference frame of the planet earth, can be obtained through the use of the following equation:

$$\omega_E = \Delta\theta = \theta - \theta_\omega$$

in the case of a star west of the Sun;

and by using this equation:

$$\omega_E = \Delta\theta = \theta_\omega - \theta$$

in the case of a star east of the Sun;

where ω_E, in both equations, is the angular distance between the shifted position of the star and the shifted position of its deflected image, due to the gravitational field of the Sun, as calculated in the moving reference frame of the earth, in these two cases, respectively:

***A.* In the case of tangential velocity, around Earth's geometrical axis, equal to zero:**

In this case, it's assumed that the tangential velocity of the earth around its geometrical axis is nil, as it's the case at the geographical latitudes of the two poles of the earth.

And correspondingly, if the star is located west of the Sun, then ω_E is calculated in accordance with this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \frac{v_E}{c}}\right) - \arctan\left(\frac{\sin(\theta' - \omega)}{\cos(\theta' - \omega) + \frac{v_E + v_g}{c}}\right)$$

And likewise, if the star is located east of the Sun, then ω_E is calculated through the use of the following equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \frac{v_E - v_g}{c}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \frac{v_E}{c}}\right)$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image, as observed in the moving reference frame of the earth.

And accordingly, if, in the stationary reference frame of the solar system, the given set of observational data is the following:

$$\theta' = 90^o$$
$$\omega = 0''.876$$
$$c = 299792458 \text{ ms}^{-1}$$
$$v_g = c(\tan\omega) = 1273.2089 \text{ ms}^{-1}$$
$$v_E = 29780 \text{ ms}^{-1}$$

then, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \frac{v_E}{c}}\right) - \arctan\left(\frac{\sin(\theta' - \omega)}{\cos(\theta' - \omega) + \frac{v_E + v_g}{c}}\right)$$

for calculating the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star west of the Sun, we can obtain the following numerical result:

$$\omega_E = 1.75199996091 \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image, as observed in the moving reference frame of the earth.

And similarly, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \frac{v_E - v_g}{c}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \frac{v_E}{c}}\right)$$

for computing the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star east of the Sun, we obtain this numerical result:

$$\omega_g = 1.75199996350 \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its deflected image, as observed in the reference frame of the moving earth.

B. **In the case of tangential velocity of Earth, around its axis, greater than zero:**

In this case, the tangential velocity of the earth, around its geometrical axis, is assumed to be greater than zero, as it's the case, in every location on Earth; except at the north pole and the south pole.

Let v_o denote the maximum tangential velocity at the equator, due to Earth's axial rotation.

And hence, the tangential velocity at any latitude on Earth, v_L, can be obtained by using this equation:

$$v_L = v_o \cos\phi$$

where ϕ is the angle of the geographical latitude.

Since, on the Sun-facing side of the earth, the tangential-velocity vector, due to Earth's axial rotation, points always in the opposite direction to the tangential-velocity vector, due to Earth's orbital revolution, the numerical value of the former is subtracted from the numerical value of the latter.

And therefore, if the star is located west of the Sun, then ω_E can be calculated through the use of this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - v_o\cos\phi}{c}}\right) - \arctan\left(\frac{\sin(\theta' - \omega)}{\cos(\theta' - \omega) + \dfrac{v_E + v_g - v_o\cos\phi}{c}}\right)$$

And in the same way, if the star is located east of the Sun, then ω_E can be computed by using the following equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \dfrac{v_E - v_g - v_o\cos\phi}{c}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - v_o\cos\phi}{c}}\right)$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image, as measured in the moving reference frame of the planet earth.

And it follows, therefore, that if, in the reference frame, in which the barycenter of the solar system is at rest:

$$\theta' = 90^o$$
$$\omega = 0''.876$$
$$\phi = 0^o$$
$$c = 299792458 \ \ \text{ms}^{-1}$$
$$v_g = c\left(\tan\omega\right) = 1273.2089 \ \ \text{ms}^{-1}$$
$$v_o = 463.8889 \ \ \ \text{ms}^{-1}$$
$$v_E = 29780 \ \ \ \text{ms}^{-1}$$

then by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \frac{v_E - v_o\cos\phi}{c}}\right) - \arctan\left(\frac{\sin\left(\theta'-\omega\right)}{\cos\left(\theta'-\omega\right) + \frac{v_E + v_g - v_o\cos\phi}{c}}\right)$$

for calculating the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star west of the Sun, we will obtain the following numerical result:

$$\omega_E = 1.75199998195 \ \ \text{arcseconds}$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image, as observed in the moving reference frame of the earth at the equator.

And similarly, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin\left(\theta'+\omega\right)}{\cos\left(\theta'+\omega\right) + \frac{v_E - v_g - v_o\cos\phi}{c}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \frac{v_E - v_o\cos\phi}{c}}\right)$$

for computing the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star east of the Sun, we obtain this numerical result:

$$\omega_E = 1.75199998450 \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image, as observed in the moving reference frame of the earth, at the equator.

5. The Computed Predictions on the Assumption of $(c \pm u_s)$ at ∞:

Based upon the assumption of initial velocity of starlight equal to $(c \pm u_s)$ at infinity, the difference between the shifted position of any star located around the Sun, θ, and the shifted position of its gravitationally deflected image, θ_ω, as measured in the moving reference frame of the planet earth, can be obtained through the use of the following is equation:

$$\omega_E = \Delta\theta = \theta - \theta_\omega$$

in the case of a star located west of the Sun;

and by using this equation:

$$\omega_E = \Delta\theta = \theta_\omega - \theta$$

in the case of a star located east of the Sun;

where ω_E, in the above two equations, is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image, as measured within the reference frame of the moving planet earth, in these two main cases, respectively:

I. **In the case of tangential velocity, around Earth's geometrical axis, equal to zero:**

In this case, it's assumed that the tangential velocity of the earth, around its geometrical axis, is equal to zero, as it's the case at the two poles of the earth.

And correspondingly, if the star is located west of the Sun, then ω_E is calculated through the use of this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E}{c \pm u_s}}\right) - \arctan\left(\frac{\sin(\theta'-\omega)}{\cos(\theta'-\omega) + \dfrac{v_E + v_g}{c \pm u_s}}\right)$$

And likewise, if the star is located east of the Sun, then ω_E is calculated in accordance with the following equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta'+\omega)}{\cos(\theta'+\omega) + \dfrac{v_E - v_g}{c \pm u_s}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E}{c \pm u_s}}\right)$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image, with respect to the moving reference frame of the planet earth.

For example, the star Aldebaran (*Alpha Tauri*) is, currently, receding from the Sun with a radial velocity u_s:

$$u_s = 54260 \;\; \text{ms}^{-1}$$

and subsequently, the radial velocity component of its starlight c_R is:

$$c_R = c - u_s = 299738198 \;\; \text{ms}^{-1}$$

And it follows, therefore, that, in the stationary reference frame of the solar system, starlight, emitted by the star Aldebaran (*α Tauri*), and deflected by the gravitational field of the Sun, has an angle of gravitational deflection equal to ω_R:

$$\omega_R = \arctan\left(\frac{2GM}{c_R^2 R}\right) = 0.87632$$

where c_R is the radial velocity component of deflected starlight

With regard to the star Aldebaran, therefore, if, in the stationary reference frame of the solar system:

$$\begin{aligned}
&\theta' = 90^o \\
&\omega = 0''.87632 \\
&c_R = c - u_s = 299738198 \ \text{ ms}^{-1} \\
&v_E = 29780 \ \text{ ms}^{-1} \\
&v_g = c_R\left(\tan\omega\right) = 1273.4435 \ \text{ ms}^{-1}
\end{aligned}$$

then, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \frac{v_E}{c \pm u_s}}\right) - \arctan\left(\frac{\sin\left(\theta' - \omega\right)}{\cos\left(\theta' - \omega\right) + \frac{v_E + v_g}{c \pm u_s}}\right)$$

for computing the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star west of the Sun, we obtain the following numerical result:

$$\omega_E = 1.75263997845 \ \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star Aldebaran and its deflected image, due to the gravitational field of the Sun, as observed in the moving reference frame of the earth, at the geographical latitude of *+90º*.

And similarly, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \dfrac{v_E - v_g}{c \pm u_s}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E}{c \pm u_s}}\right)$$

for computing the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star east of the Sun, we obtain this numerical result:

$$\omega_E = 1.75263998104 \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star Aldebaran and the shifted position of its gravitationally deflected image, as observed in the moving reference frame of the planet earth, at the north pole.

II. In the case of tangential velocity of Earth, around its axis, greater than zero:

In this case, it's assumed that the tangential velocity of the earth, around its geometrical axis, is greater than zero, as it's the case, in every location on Earth; except on the north pole and the south pole.

Let v_o denote the maximum tangential velocity at the equator, due to Earth's axial rotation.

And accordingly, the tangential velocity at any latitude on Earth, v_L, can be calculated by using this equation:

$$v_L = v_o \cos\phi$$

where ϕ is the angle of the geographical latitude.

And since on the Sun-facing side of the earth, the tangential-velocity vector, due to axial rotation, points always in the opposite direction to the tangential- velocity vector, due to orbital revolution, the former is subtracted from the latter.

And therefore, if the star is located west of the Sun, then ω_E can be calculated in accordance with this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - v_o\cos\phi}{c \pm u_s}}\right) - \arctan\left(\frac{\sin(\theta' - \omega)}{\cos(\theta' - \omega) + \dfrac{v_E + v_g - v_o\cos\phi}{c \pm u_s}}\right)$$

And in like manner, if the star is located east of the Sun, then ω_E can be calculated, in accordance with the following equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \dfrac{v_E - v_g - v_o\cos\phi}{c \pm u_s}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - v_o\cos\phi}{c \pm u_s}}\right)$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image, as observed within the reference frame of the moving planet earth.

And it follows, therefore, that, as in the case of the star (α *Tauri*), for example, if, in the stationary reference frame of the solar system:

$$\begin{aligned}
&\theta' = 90^o \\
&\omega = 0''.87632 \\
&\phi = 0^o \\
&c_R = 299738198 \ \text{ms}^{-1} \\
&v_E = 29780 \ \text{ms}^{-1} \\
&v_g = c_R(\tan\omega) = 1273.4435 \ \text{ms}^{-1} \\
&v_o = 463.8889 \ \text{ms}^{-1}
\end{aligned}$$

then, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - v_o\cos\phi}{c \pm u_s}}\right) - \arctan\left(\frac{\sin(\theta' - \omega)}{\cos(\theta' - \omega) + \dfrac{v_E + v_g - v_o\cos\phi}{c \pm u_s}}\right)$$

for calculating the angular distance between the shifted position of a star and the shifted position of its deflected image, in the case of a star west of the Sun, we obtain the following numerical result:

$$\omega_E = 1.75263997903 \ \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star (*α Tauri*) and the shifted position of its deflected image, as observed in the moving reference frame of the earth, at the equator.

And similarly, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \dfrac{v_E - v_g - v_o\cos\phi}{c \pm u_s}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - v_o\cos\phi}{c \pm u_s}}\right)$$

for computing the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star east of the Sun, we obtain this numerical result:

$$\omega_E = 1.75263998158 \ \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star (*α Tauri*) and the shifted position of its deflected image, due to the gravitational field of the Sun, as observed in the moving reference frame of the planet earth, at the equator.

In the accompanying ***Table #2*** below, the angles of the gravitational deflection for starlight assumed to be traveling at $(c \pm u_s)$ at infinity, and emitted by stars close to the ecliptic, are computed separately, in the moving reference frame of the earth, as observed at the latitude of $\pm 90^o$ and the latitude of 0^o, respectively:

The Star	u_s ms^{-1}	ω_{wp} arcsecs	ω_{ep} arcsecs	ω_{wq} arcsecs	ω_{eq} arcsecs
Rho Leonis	+42000	1.75199998139	1.75199998398	1.75199998194	1.75199998449
Alpha Librae	-24700	1.75199998140	1.75199998400	1.75199998195	1.75199998450
Beta Trianguli	+9900	1.75199998139	1.75199998398	1.75199998197	1.75199998452
Gamma Aquarii	-15000	1.75199998140	1.75199998398	1.75199998198	1.75199998442

Table #2: *Gravitational deflection of starlight traveling at* ($c \pm u_s$)

In the above table, $-u_s$ is the radial velocity component of an approaching star; $+u_s$ is the radial velocity component of a receding star; ω_{wp} is the gravitational-deflection angle for stars west of the Sun as observed from Earth's poles; ω_{ep} is the gravitational-deflection angle for stars east of the Sun as observed from Earth's poles; ω_{wq} is the gravitational-deflection angle for stars west of the Sun as observed from Earth's equator; and ω_{eq} is the gravitational-deflection angle for stars east of the Sun as observed from Earth's equator.

6. An Evaluation of the Effect of Deflected Starlight's Relative Velocity:

The following points have to be expounded further and made clear, with regard to the effect of the relative velocity of starlight on the numerical values of the angle of gravitational deflection, as calculated and measured in the moving reference frame of the planet earth:

1. The relative velocity of incident starlight, with respect to the earth, systematically and consistently, changes and transforms the numerical value of the angle of gravitational deflection of starlight grazing the Sun's surface from the numerical value of *0.876* seconds of arc, in the stationary reference frame of the solar system, as predicted by the Newtonian theory of universal gravitation and the corpuscular theory of light, to a numerical value equal to about *1.75* seconds of arc, as observed and measured anywhere within the reference frame of the moving planet earth.

2. In the reference frame of the moving earth, the numerical values of the angle of deflected starlight, emitted by stars located east of the Sun, are systematically and consistently, larger, by very minute amounts, than the numerical value of the same angle of deflected starlight, emitted by stars located west of the Sun.

3. In the stationary reference frame of the solar system as well as in the moving reference frame of the earth, the differences between the numerical values of the angle of deflected starlight assumed to be traveling with a velocity of c at infinity, and the numerical values of the angle

of deflected starlight assumed to be traveling with a velocity of $(c \pm u_s)$ at infinity, and emitted by stars within the galaxy of the milky way, are extremely tiny and insignificant.

4. The effect of Earth's tangential velocity, around its geometrical axis, on the numerical values of the angle of starlight, deflected by the gravitational field of the Sun, is very small and negligible as well.

5. The effect of the variations in the orbital velocity of the earth, from its maximum value of $30290\ ms^{-1}$ at the perihelion to its minimum value of $29290\ ms^{-1}$ at the aphelion, is also too minute and negligible, as demonstrated in the following table:

Earth's velocity ms^{-1}	ω_{pw} **arcseconds**	ω_{pe} **arcseconds**	ω_{qw} **arcseconds**	ω_{qe} **arcseconds**
(Max): 30290.0	1.75199996030	1.75199996293	1.75199996186	1.75199996345
(Mean): 29780.0	1.75199996092	1.75199996350	1.75199996147	1.75199996402
(Min): 29290.0	1.75199996150	1.75199996405	1.75199996205	1.75199996455

Table #3: *Earth's orbital velocity*

In the table above, ω_{pw} is the gravitational-deflection angle for stars west of the Sun as observed from Earth's poles; ω_{pe} is the gravitational-deflection angle for stars east of the Sun as observed from Earth's poles; ω_{qw} is the gravitational-deflection angle for stars west of the Sun as observed from Earth's equator; and ω_{qe} is the gravitational-deflection angle for stars east of the Sun as observed from Earth's equator.

6. As shown in ***Table #4*** below, even in the reference frame of Mercury with the highest orbital velocity of $47360\ ms^{-1}$ and in the reference frame of Pluto with the lowest orbital velocity of $4670\ ms^{-1}$, the numerical value of the calculated angle of deflected starlight is still equal to about *1.75* seconds of arc:

The Frame of reference	**Mean orbital velocity ms^{-1}**	ω_{pw} **arcseconds**	ω_{pe} **arcseconds**
Mercury's reference frame	*47360.0*	1.75199993372	1.75199993783
Mars' reference frame	*24070.0*	1.75199996716	1.75199996925
Pluto's reference frame	*4670.0*	1.75199997887	1.75199997928

Table #4: *Planetary frames of reference*

In the table above, ω_{pw} is the gravitational-deflection angle for stars west of the Sun; and ω_{pe} is the gravitational-deflection angle for stars east of the Sun, as observed in the reference fame of Mercury; the reference of Mars; and the reference frame of Pluto; respectively.

7. In the stationary reference frame of the solar system as well as in the moving reference frame of the earth, the numerical values of the angle of starlight, deflected by the gravitational field of the Sun, are inversely proportional to the square of distance from the Sun's center of mass; i.e.,

$$\omega \propto \frac{1}{d^2}$$

where ω is the angle of gravitational deflection; and d is the distance between the point of closest approach of starlight and the gravitational center of the Sun. For example, if the point of closest approach is equal to 2 solar radii from the center of the Sun, then the angle of deflected starlight, in this case, is equal to *0.219* seconds of arc.

8. Also, with regard to starlight grazing the surface of the Sun, it should be pointed out, within the current context, that, even though the true position of the emitting star, θ', in the stationary reference frame of the solar system, can have any numerical value within the following range:

$$0^o \leq \theta' \leq 360^o$$

in the moving reference frame of the earth, the true position of any star with respect to the vector of the orbital velocity of the earth, according to Soldner's setup, in the case under investigation, is highly restricted, and can have only values slightly less, by a small amount, or slightly greater, by the same amount, than the following numerical value:

$$\theta' = 90^o$$

And that is because the rays of starlight are, for all practical purposes, incredibly parallel, as if they were emitted right smack in the middle of infinity. And since, in the case under discussion, deflected starlight must have grazed the Sun's surface or have come very close to it; and because the straight line connecting the Sun and the earth deviates only by a very minute amount, due to the small eccentricity of Earth's orbit, from being exactly at right angles with respect to the vector of the orbital velocity of the earth; the initial direction of incident starlight; i.e., the true position of the emitting star, in this particular case, must always make an angle very close to 90^o with the orbital-velocity vector of the earth. Certainly, the vector of Earth's orbital velocity changes its direction continually, during the course of the year, with respect to stars located near the ecliptic; from 0^o and 360^o. But, nevertheless, starlight deflected, by the gravitational field of the Sun, can be observed, here on Earth, only if the initial direction of incident starlight and the direction of the orbital velocity of the earth make an angle very close to 90^o with each other.

7. Concluding Remarks:

As demonstrated in the present investigation, the numerical values of computed predictions of gravitational deflection, within the framework of Newton's theory of universal gravitation, depend upon the state of motion of the selected frame of reference.

In the stationary reference frame of the solar system, Newton's theory of universal gravitation predicts that the angle of starlight, deflected by the gravitational field of the Sun, is equal to about *0,876* seconds of arc, as calculated by using this equation, in the case of approaching stars:

$$\omega = \arctan\left(\frac{2GM}{c^2 R\left(1+\frac{u_s^2}{c^2}+2\frac{u_s}{c}\right)}\right)$$

and in accordance with this equation, in the case of receding stars:

$$\omega = \arctan\left(\frac{2GM}{c^2 R\left(1+\frac{u_s^2}{c^2}-2\frac{u_s}{c}\right)}\right)$$

where ω is the angle of gravitational deflection, as calculated in the stationary reference frame of the solar system; G is the gravitational constant; M is the mass of the Sun; R is its radius; and u_s is the radial component of the relative velocity between the emitting star and the Sun.

By contrast, in the moving reference frame of the earth, the Newtonian theory of universal gravitation predicts that the angle of starlight, deflected by the gravitational field of the Sun, is equal to about 1.*75* seconds of arc, as computed through the use of the following equation, in the case of stars located west of the Sun:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - v_o\cos\phi}{c \pm u_s}}\right) - \arctan\left(\frac{\sin(\theta' - \omega)}{\cos(\theta' - \omega) + \dfrac{v_E + v_g - v_o\cos\phi}{c \pm u_s}}\right)$$

and in accordance with this equation, in the case of stars located east of the Sun:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \dfrac{v_E - v_g - v_o\cos\phi}{c \pm u_s}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - v_o\cos\phi}{c \pm u_s}}\right)$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its gravitationally deflected image; θ' is the true position of the star; ω is the angle of deflected starlight as observed in the stationary reference frame of the solar system; v_E is the orbital velocity of the earth around the barycenter of the solar system; v_o is the tangential velocity of the earth, around its geometrical axis, as measured at the equator; ϕ is the angle of the geographical latitude; u_s is the radial component of the relative velocity between the emitting star and the Sun; and v_g is the transverse component of the velocity of deflected starlight, with respect to the earth, as calculated by using this equation, in the case of approaching stars:

$$v_g = (c + u_s)\tan\omega$$

and by using this equation, in the case of receding stars:

$$v_g = (c - u_s)\tan\omega$$

where u_s is the radial component of the relative velocity between the emitting star and the Sun; and ω is the angle of deflected starlight, in the reference frame, in which the barycenter of the solar system is at rest, as computed by using this equation, in the case of approach:

$$\omega = \arctan\left(\frac{2GM}{c^2 R\left(1+\frac{u_s^2}{c^2}+2\frac{u_s}{c}\right)}\right)$$

and by using this equation, in the case of recession:

$$\omega = \arctan\left(\frac{2GM}{c^2 R\left(1+\frac{u_s^2}{c^2}-2\frac{u_s}{c}\right)}\right)$$

where u_s is the radial component of the relative velocity between the emitting star and the Sun.

As stated earlier, in the current discussion, Johann Georg von Soldner assumed all along, in order to simplify his calculations, that the initial trajectory of starlight is a straight line that makes an angle of 90^o with the orbital-velocity vector of the earth; i.e., the true position of the emitting star is assumed to be always at right angles with respect to the vector of the tangential velocity of the earth in its orbit.

And the question, therefore, is this:

What will happen to the numerical results of gravitational deflection, as predicted by Johann Georg von Soldner, if the true position of the star, under investigation, makes an angle equal to β with the vertical line to the orbital-velocity vector of the earth?

From geometrical standpoint; it should be, immediately, clear from the illustration in ***Figure #3*** below, that, in the reference frame, in which the gravitational center of the solar system is rest, Soldner's predicted value of *0.876* arcseconds must remain, in the aforementioned case, unchanged and exactly the same.

And that is, obviously, because the deviation of starlight's path from the original straight line by the angle β, is, in this case, nothing more than the rotation of the entire co-ordinate system, clockwise in the case of stars west of the Sun, and counterclockwise in the case of stars east of the Sun, by an angle equal to β with respect to the vector of Earth's orbital velocity. Or to put it differently, it's nothing more than the rotation of the vector of Earth's orbital velocity, clockwise in the case of stars east of the Sun, and counterclockwise in the case of stars west of the Sun, by an angle equal to β with respect to the true position of those stars, in the stationary reference of the solar system. And hence, Johann Georg von Soldner's prediction, as computed in the reference frame of the solar system, in which the barycenter of the solar system is at rest, remains unaltered and the same.

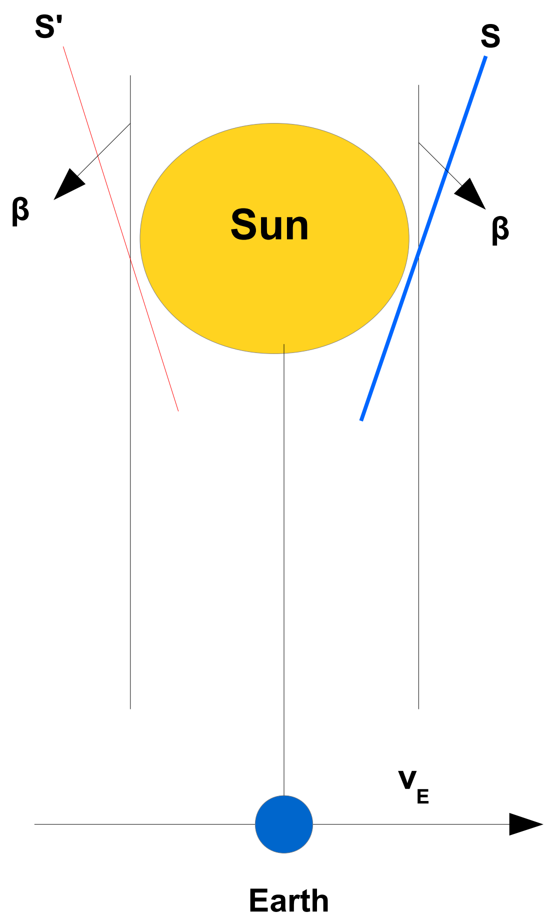

Figure #3: *Initial direction of starlight*

However, as shown in ***Figure #4*** below, in the reference frame of the moving earth, the deviation of starlight's initial trajectory from the straight line of Johann Georg von Soldner's original setup, by an amount equal to the angle β, changes, necessarily, the values of the radial c'_r and the horizontal c'_h components of the relative velocity of deflected starlight, with respect to the orbital-velocity vector of the earth, in these two cases, respectively:

In the case of stars located west of the Sun:

$$c'_r = c\cos\beta$$

and:

$$c_h' = v_E + v_g + c\sin\beta$$

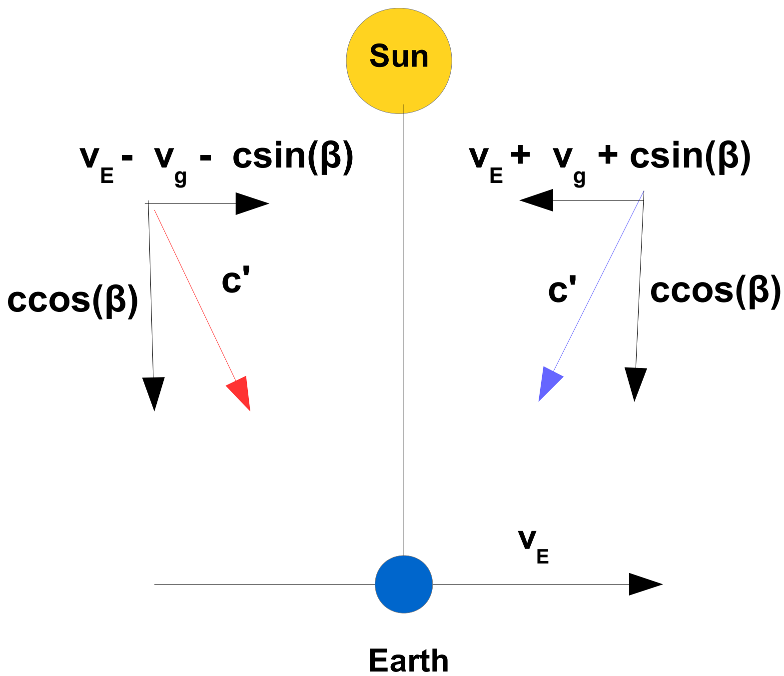

Figure #4: *Relative velocity of deflected starlight*

And in the case of stars located east of the Sun:

$$c_r' = c\cos\beta$$

and:

$$c_h' = v_E - v_g - c\sin\beta$$

where v_E is the orbital velocity of the earth; v_g is the transverse velocity component of starlight deflected by the gravitational field of the Sun; and β is the angle between the initial direction of starlight and the straight line perpendicular to the orbital-velocity vector of the earth.

And accordingly, the two equations, for calculating the gravitational-deflection angle of starlight assumed to be traveling with c at infinity, must, now, be reformulated and re-written in the following form, in the case of stars west of the Sun:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E + c\sin\beta - v_o\cos\phi}{c\cos\beta}}\right) - \arctan\left(\frac{\sin(\theta'-\omega)}{\cos(\theta'-\omega) + \dfrac{v_E + v_g + c\sin\beta - v_o\cos\phi}{c\cos\beta}}\right)$$

and in this form, in the case of stars east of the Sun:

$$\omega_E = \arctan\left(\frac{\sin(\theta'+\omega)}{\cos(\theta'+\omega) + \dfrac{v_E - v_g - c\sin\beta - v_o\cos\phi}{c\cos\beta}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - c\sin\beta - v_o\cos\phi}{c\cos\beta}}\right)$$

where v_o is the tangential velocity, due to the rotation of the earth around its axis as measured at the equator; ϕ is the angle of the geographical latitude; and θ' is defined by this equation, in the case of stars located to the west of the Sun:

$$\theta' = 90^o - \beta$$

and it's defined by this equation:

$$\theta' = 90^o + \beta$$

in the case of stars located to the east of the Sun.

And it follows, therefore, that if, in the stationary reference frame of the solar system:

$$\beta = 0''.5$$
$$\omega = 0''.876$$
$$\phi = 90^o$$
$$c = 299792458 \ \text{ ms}^{-1}$$
$$v_E = 29780 \ \text{ ms}^{-1}$$
$$v_g = c(\tan\omega) = 1273.2089 \ \text{ ms}^{-1}$$
$$v_o = 463.8889 \ \text{ ms}^{-1}$$

then by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E + c\sin\beta - v_o\cos\phi}{c\cos\beta}}\right) - \arctan\left(\frac{\sin(\theta'-\omega)}{\cos(\theta'-\omega) + \dfrac{v_E + v_g + c\sin\beta - v_o\cos\phi}{c\cos\beta}}\right)$$

for calculating the angular distance between the shifted position of the star, under investigation, and the shifted position of its deflected image, in the case of a star west of the Sun, we will obtain the following numerical result:

$$\omega_E = 1.75199995937 \ \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its deflected image, as observed in the moving reference frame of the earth at the poles.

And similarly, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta'+\omega)}{\cos(\theta'+\omega) + \dfrac{v_E - v_g - c\sin\beta - v_o\cos\phi}{c\cos\beta}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - c\sin\beta - v_o\cos\phi}{c\cos\beta}}\right)$$

for computing the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star east of the Sun, we obtain this numerical result:

$$\omega_E = 1.75199996491 \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star and the shifted position of its deflected image, as observed in the moving reference frame of the earth, at the earth's poles.

And moreover, the two equations, for calculating the gravitational-deflection angle of starlight assumed to be traveling with $(c \pm u_s)$ at infinity, must, also, be re-formulated in the following form, in the case of stars west of the Sun:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E + c'\sin\beta - v_o\cos\phi}{c'\cos\beta}}\right) - \arctan\left(\frac{\sin(\theta' - \omega)}{\cos(\theta' - \omega) + \dfrac{v_E + v_g + c'\sin\beta - v_o\cos\phi}{c'\cos\beta}}\right)$$

and in this form, in the case of stars east of the Sun:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \dfrac{v_E - v_g - c'\sin\beta - v_o\cos\phi}{c'\cos\beta}}\right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - c'\sin\beta - v_o\cos\phi}{c'\cos\beta}}\right)$$

where v_o is the tangential velocity, due to the rotation of the earth around its axis as measured at the equator; ϕ is the angle of the geographical latitude; and θ' is defined by this equation, in the case of stars located to the west of the Sun:

$$\theta' = 90^o - \beta$$

and it's defined by this equation:

$$\theta' = 90^o + \beta$$

in the case of stars located to the east of the Sun.

The star Aldebaran (*Alpha Tauri*), for example, is, at the present time, receding from the Sun at a radial velocity of u_s:

$$u_s = 54260 \ \ \text{ms}^{-1}$$

Concerning the star Aldebaran, therefore, if, in the stationary reference frame of the solar system:

$$\begin{aligned}
&\beta = 15''.0 \\
&\omega = 0''.87632 \\
&\phi = 0^0 \\
&c' = c - u_s = 299738198 \ \ \text{ms}^{-1} \\
&v_E = 29780 \ \ \text{ms}^{-1} \\
&v_g = c'(\omega) = 1273.4435 \ \ \text{ms}^{-1} \\
&v_o = 463.8889 \ \ \text{ms}^{-1}
\end{aligned}$$

then by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E + c'\sin\beta - v_o\cos\phi}{c'\cos\beta}}\right) - \arctan\left(\frac{\sin(\theta' - \omega)}{\cos(\theta' - \omega) + \dfrac{v_E + v_g + c'\sin\beta - v_o\cos\phi}{c'\cos\beta}}\right)$$

for calculating the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star west of the Sun, we will obtain the following numerical result:

$$\omega_E = 1.75263991071 \ \ \text{arcseconds}$$

where ω_E is the angular distance between the shifted position of the star Aldebaran and the shifted position of its deflected image, as observed in the moving reference frame of the earth, at the equator.

And in like manner, by inserting the above data into this equation:

$$\omega_E = \arctan\left(\frac{\sin(\theta' + \omega)}{\cos(\theta' + \omega) + \dfrac{v_E - v_g - c' \sin\beta - v_o \cos\phi}{c' \cos\beta}} \right) - \arctan\left(\frac{\sin\theta'}{\cos\theta' + \dfrac{v_E - c' \sin\beta - v_o \cos\phi}{c' \cos\beta}} \right)$$

for computing the angular distance between the shifted position of the star and the shifted position of its deflected image, in the case of a star east of the Sun, we obtain this numerical result:

$$\omega_E = 1.75264000052 \text{ arcseconds}$$

where ω_E is the angular distance between the shifted position of the star Aldebaran and the shifted position of its deflected image, due to the gravitational field of the Sun, as observed in the moving reference frame of the earth, at the earth's equator.

We can conclude, therefore, that, even though within the reference frame of the moving earth, the deviation of starlight's initial trajectory from Soldner's vertical straight line, by an angle β, changes the values of the radial velocity component c'_r and the horizontal velocity component c'_h for starlight deflected by the gravitational field of the Sun, with respect to Earth, the numerical value of the gravitational-deflection angle changes very slightly and essentially remains equal to about 1.75 seconds of arc

As a matter of fact, the value of 1.75 arcseconds, for the gravitational deflection of starlight, caused by the gravitational field of the Sun, as calculated and measured within the moving reference frame of the planet earth, on the basis of Newton's theory of universal gravitation and Newton's corpuscular theory of light, is remarkably stable and unlikely to change substantially, under any circumstances; except in the hypothetical case, in which the gravitational acceleration, on the surface of the Sun, is assumed to vary, somehow, by a significant amount.

Now, at first glance, the numerical value of the Newtonian prediction, 1.75 seconds of arc, as calculated in the reference frame of the moving earth, looks, precisely, the same as the numerical value of the Einsteinian prediction, 1.75 seconds of arc, in the case of starlight grazing the surface of the Sun.

Nonetheless, upon closer examination, it becomes quite clear that there are considerable differences, in this regard, between the Newtonian prediction and the Einsteinian prediction:

I. The numerical value of the Newtonian prediction, in the reference frame of the moving earth, and the numerical value of the Einsteinian prediction, in the case of starlight grazing the surface of the Sun, are equal to each other only to the second decimal.

II. Newton's theory of universal gravitation predicts that the numerical value of gravitational deflection, as computed and measured in the stationary reference frame of the solar system, is equal to about 0.876 seconds of arc.

III. Einstein's theory of general relativity, by comparison, predicts that the numerical value of gravitational deflection, as calculated and measured in the stationary reference frame of the solar system, is equal to about 1.75 seconds of arc.

IV. Newton's theory of universal gravitation uses the notion of force of gravity to generate an angle of deflected starlight equal to about 0.876 seconds of arc, in the frame of reference, in which the barycenter of the solar system is at rest.

V. Einstein's theory of general relativity uses the two notions of curved space-time continuum and the equivalence principle to generate an angle of deflected starlight equal to about 1.75 seconds of arc, in the frame of reference, in which the barycenter of the solar system is at rest.

VI. Newton's theory of universal gravitation uses the notion of relative velocity of starlight, as defined within the framework of ballistic and corpuscular theories of light, to transform the angle of 0.876 seconds of arc, in the stationary reference frame of the solar system, into an angle of 1.75 seconds of arc, in the moving reference frame of the planet earth.

VII. Einstein's theory of general relativity, by contrast, makes no distinction whatsoever between the stationary reference frame of the solar system and the moving reference frame of the earth, and uses its two notions of curved space-time continuum and the equivalence principle to generate the angle of 1.75 seconds of arc, in the stationary reference frame of the solar system as well as in the reference frame of the moving planet earth, at the same time.

Without a shadow of a doubt, in terms of simplicity, intuitive appeal, and predictive power, Newton's theory of universal gravitation wins hands down over Einstein's theory of general relativity, and in particular with regard to the aforementioned deflection of starlight by the gravitational field of the Sun, as measured in the reference frame of the moving earth and in moving frames of reference in general.

Compared to the convoluted methods and the tremendous number of simplifying assumptions that must be made before extracting the $1''.75$ angle from Einstein's theory of general relativity, the Newtonian solution, under discussion, is, refreshingly, intuitive, straightforward, and requiring no more than three very simple steps:

1. Transform the position of the star, in question, from the frame of reference, in which the barycenter of the solar system is at rest, to the frame of reference, in which terrestrial observers

are at rest, by using the direction of the velocity resultant of the non-deflected starlight with respect to observers on Earth.

2. Transform the position of the gravitationally deflected image of the star, in question, from the frame of reference, in which the barycenter of the solar system is at rest, to the frame of reference, in which terrestrial observers are at rest, by using the direction of the velocity resultant of the deflected starlight with respect to observers on Earth.

3. And then just subtract the angle of the shifted position of the star, under investigation, from the angle of the shifted position of its gravitationally deflected image, if the star is located east of the Sun; and conversely, just go ahead and subtract the angle of the shifted position of the deflected image, due to the force of gravity, from the angle of the shifted position of the star, in question, if the star is located west of the Sun.

And that is it.

On the basis of the main results of the current investigation, it follows, therefore, that Albert Einstein's prediction of 1.75 arcseconds, for the angle of starlight deflected by the gravitational field of the Sun, as measured in the moving reference frame of the earth, is, certainly, correct. But his prediction of 1.75 arcseconds, for the angle of starlight deflected by the gravitational field of the Sun, as measured in the stationary reference frame of the solar system, is, completely, wrong.

That is on one hand.

While, on the other hand, Johann Georg von Soldner's prediction of *0.876* arcseconds, for the angle of starlight deflected by the gravitational field of the Sun, as measured in the moving reference frame of the planet earth, is totally wrong. But nevertheless, his prediction of *0.876* arcseconds, for the angle of starlight deflected by the gravitational field of the Sun, as measured in the reference frame, in which the gravitational center of the solar system is at rest, is definitely, correct.

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