RELATIVITY. — *On the Experiment of Sagnac*.
Note by Mr. **Paul Langevin**.

Messrs. A. Dufour and F. Prunier, after verifying that the result of the Sagnac experiment remains the same when the light source and photographic plate are fixed in the laboratory instead of being connected to the platform in rotation that brings the device interference, have recently published a reasoning ([1]) according to which their experiment fully agrees with the classical theory, whereby difficulties arise when one wishes to interpret the relativistic point of view.

I would like to show here that this conclusion is incorrect and that the forecasts made in accordance with the theory of relativity, for fixed observers or other related to the platform, agree with each other as well as the experiment.

Note first that all observers recognize the independence of the laws of propagation of light and movement of the source or the receiver, as well as the fact that the classical theory which represents in fact, as in relativity, the point of view of observers fixed or more exactly Galilean, the difficulty believed by Messrs. Dufour and Prunier would concern both Sagnac experiments, in its original form and as that which their Note brings and the result discusses. Under this independence, all observers must agree to provide that both experiments give exactly the same phase difference between interfering beams.

It remains to compare the predictions made for this phase difference, in any of the two experiments, for fixed observers and those involved in the movement of the platform. The existence of a disagreement between these predictions seems, *a priori*, unlikely, since the phase of a periodic phenomenon is an invariant transformation of special relativity which corresponds to the passage of a Galilean reference system to another also Galilean.

It is more difficult to verify that the agreement persists when the comparison is between a Galilean system of reference considered fixed and another non Galilean system animated in movement of uniform rotation.

([1]) A. Dufour et F. Prunier, *Comptes Rendus*, **204**, 1937, p. 1925.

For the fixed observer, who thinks in terms of the classical theory, and for which the light propagates in all directions with the same speed c, the strip on which the interfering beams separate and come together, and the mirrors where they reflect, are moving. It follows that the two light beams, each followed by its spread, travel paths of unequal length and therefore uneven in time t_1 and t_2 whose difference is given to first order according to the angular velocity ω which rotates with the platform, by the expression

$$t_1 - t_2 = \frac{4\omega \mathrm{A}}{c^2}, \tag{1}$$

where A represents the interior area of the polygonal journey that the two rays travel in opposite directions.

It should be noted that, from the same point of view, the two rays, although they found at the end of their journey to have the same direction and the same common frequency ν_0 they had before separating, have during the separation, frequencies and therefore wavelengths which differ among themselves at first order in ω. Despite this complication, it is easy to show that the phase difference which the rays interfere is considered equal in number of periods,

$$\nu_0 (t_1 - t_2) = \frac{4\omega \mathrm{A} \nu_0}{c^2}. \tag{2}$$

For the entrained observer, who uses it on the surface of the platform coordinate of space related to it, at a distance r from the center and polar angle θ, for example, I showed ([2]) that it is not possible to associate in this space a uniform time respecting the isotropic propagation of light.

The observer tied to the platform can choose between two simple solutions:

The first consists of adopting a *central* time t, which is of Galilean observers in comparison with whom the center chosen on the platform is stationary. The fundamental invariant ds^2 is present in those conditions in the form

$$ds^2 = \left(c^2 - \omega^2 r^2\right) dt^2 - 2\omega r^2 d\theta dt - \left(dr^2 + r^2 d\theta^2\right).$$

([2]) *Comptes Rendus*, **200**, 1935, p. 49.

The presence of the right-angled term in $d\theta\, dt$ implies anisotropy in the light propagation, of which speed varies with the direction between $c - \omega r$ and $c + \omega r$ with the first order of approximation in ω.

I showed long ago ([3]) that, in adopting this system of reference, we find, by very simple reasoning and general formula (1) for the difference in travel times of the two light beams in the Sagnac experiment. In this system, the courses are equal to the first order, but of unequal length because of the unequal speeds of propagation. For the same reason, the wavelengths are unequal, although the periods are equal, contrary to what was happening for fixed observers.

The second solution, which respects the isotropy in the propagation of light, is to adopt a *local* time τ not uniform, defined in the vicinity through the integration of its partial differential

$$d\tau = \sqrt{1 - \frac{\omega^2 r^2}{c^2}} \left(dt - \frac{\omega r^2 d\theta}{c^2 - \omega^2 r^2} \right)$$

or, to first order,

$$d\tau = dt - \frac{\omega r^2}{c^2} d\theta.$$

In this reference system, routes are equal as are their durations τ_1 and τ_2, the frequencies are equal as are the wavelengths, but as a result of non-uniformity of time τ, the equality between τ_1,

$$\tau_1 = t_1 - \frac{\omega}{c^2} \oint r^2 d\theta = t_1 - \frac{2\omega \mathrm{A}}{c^2},$$

and τ_2,

$$\tau_2 = t_2 + \frac{\omega}{c^2} \oint r^2 d\theta = t_2 + \frac{2\omega \mathrm{A}}{c^2},$$

implies, between the arrivals on the photographic plate rays from the source of light at the same moment taken for the origin, a time difference $t_1 - t_2$ in conformity with the expression (1) and consequently a phase difference consistent with expression (2).

([3]) *Comptes Rendus*, **173**, 1921, p. 831.