THE PROPAGATION OF LIGHT IN ROTATING SYSTEMS

BY
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Some parts of the recent paper on this subject by Silberstein[1] seem to call for comment, partly because of certain features of obscurity or incompleteness, partly because of certain statements whose justice is at least not made clear. In particular may be questioned what seems to be the main conclusion, according to which ether theory is better prepared than relativity theory to adjust itself to a conceivable outcome of Michelson's pending experiment different from the expected. A few minor items may be noticed first, but the following notes refer chiefly to what appears to be the main topic of the paper referred to, a comparison of ether and relativity theories as to their inferences concerning the influence of rotation on optical phenomena.

The suggestion, page 291, that a specification of rotation with respect to some such frame of reference as the fixed stars is necessary also in the relativity theory, "in spite of appearances to the contrary," may be granted readily, but one is left to wonder what those appearances are. Avoidance of reference to a hypothetical set of absolute directions, one natural aim of a theory of relative motion, does not imply rejection of the notion of an angular velocity uniquely determinable with reference to an observable system of bodies. The difficulties connected with the interpretation of the earth's rotation under the generalized theory of Einstein are well known, but there seems to be no known reason why they cannot be ascribed entirely to the insufficiency of data regarding the distribution and motion of cosmic matter. A fair analogy may be found in the case of the general translatory motion of the solar system with respect to the proper motion stars, or the radial velocity stars, this motion being not determinable from Newtonian theory nor yet in conflict with it.

Similarly, the later remark, pages 301–302, that the relativity theory proved unable to "deduce the terrestrial *ds* as a gravita-

[1] Silberstein, J. OPT. SOC. AM., 5, p. 291–307, 1921.

tional effect of the stars . . . ," does not really point out what can be called a flaw in that theory. When a theory is embodied primarily in differential equations an incomplete knowledge of the suitable values to be assigned to the constants of integration is a deficiency in experimental material rather than in the theory.

In this same connection, the reference, page 304, to Thirring's solution for the gravitational field of a rotating body as a "complete failure" seems rather extreme. Thirring's solution is avowedly only an approximation, the exact solution being presumably a difficult matter even if the proper boundary conditions were not uncertain. The apparent strangeness of some of Thirring's results might be reduced by knowledge of a more precise solution. One may be reminded here of some of the early results in celestial mechanics for the values of certain apparently secular variations, which other theories were able to interpret as first approximations to oscillatory variations. At least it seems wiser not to prejudge a theory too firmly in connection with problems where its inferences are not yet definitive.

The remarks at the foot of page 302 concerning the limitations of special relativity are substantially untrue, and seem to be a recrudescence of a mere misconception that formerly had some currency. That theory can use other than inertial frames of reference just as freely as Newtonian mechanics can use rotating axes, by a suitable transformation of variables, and is certainly not "wholly incompetent" to deal with optical problems in rotating systems.

The rule, page 295, regarding convexity of light rays to the left of a person walking in the direction of propagation is to be understood as for the northern hemisphere only, and opposite in the southern. If the intent of the text is equivalent to this the footnote should have "counter-clockwise." As Silberstein in effect points out the curvature is so tiny as to be a mere curiosity of theory. But an indication may be added that the negligibility of this curvature has an important bearing on the feasibility of sufficient accuracy in the construction of Michelson's optical circuit, with reference in this connection to the illustrative diagram on page 300.

The telescope supplies a single beam for incidence on the parting plate; if the transmitted and reflected portions start along the sides of the straight-line triangle and swerve as understood, the other corners of the concave triangle will touch the mirrors below the points B and C, those of the convex triangle above those points, so that the figure will be more like Fig. 1, corresponding to the case where the finally emergent pencils are parallel.

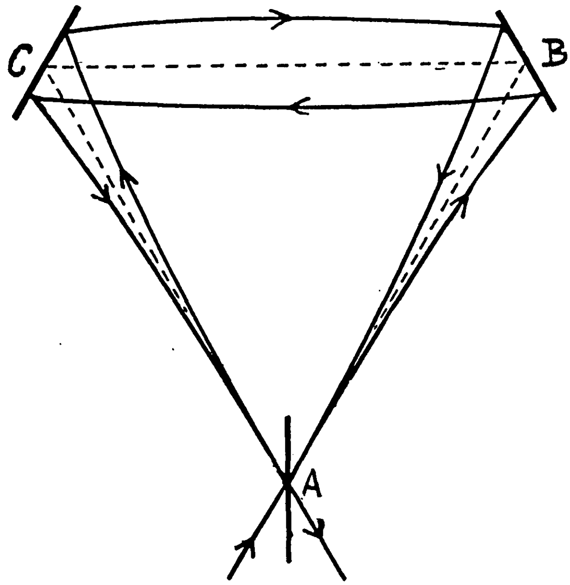

FIG. 1

This is of course merely one ideal arrangement for reference, but departures from it must be understood to be at least partly under instrumental control in order to provide for observable fringes of suitable width. In general, therefore, it appears that the oppositely travelling beams would be reflected at somewhat different portions of the mirrors. While the differences of path thereby introduced are of the first order individually for the successive sides of the circuit, the first order sum is zero because the angles of incidence and reflection are equal to a corresponding approximation. For a circuit of any polygon form in fact this modification does not imply any change in the first order formula for the shift of fringes, but if the swerving were large enough to involve a perceptible portion of the aperture of the beam, so that the two pencils would be reflected at somewhat different portions of the mirror faces, the location of the fringes might be affected by the errors of the optical surfaces and non-homogeneity of intervening medium in a way more difficult to allow for than would be the case where a test by reversal were possible. Actually, however, even with distances of several kilometers the shift along the mirror face is a small part of a wave-length. There appears thus to be no practical error involved in the use of the rectilinear diagram.

In the formula for the line-element, page 303, there is an unnoticed change of notation, in that the letter r, previously denoting a radius in spherical coordinates becomes here the symbol for a radius in cylindrical coordinates. Since the meaning and relation of these two types of coordinates stands in need of definition when the gravitational curvatures are introduced, the foot-note on page 306 wants explanation, to show in what sense it can be considered true that the only modification needed is the additive term mentioned. This is not important for the main discussion since it is agreed that the gravitational terms, including doubtless those due to the rotation of the central mass, are here of insensible magnitude. But it should be noticed that neglecting these terms is equivalent to reducing the general to the special relativity, in spite of the adverse remarks on the latter previously referred to. In other words, the characteristic differences in relevant physical content between Einstein's restricted and extended theories are practically evanescent to the degree of approximation needed for this problem; so that the treatment of optical problems by the method of null geodesics reduces to the elementary case of isotropic rectilinear propagation with respect to a suitable frame of reference, this feature being common to special relativity and to the better developed ether theories.

Moreover, the omission of terms of higher degree in the velocity, again quite sufficient for the purpose, masks the chief features of contrast between the special relativity and such ether theories as are developed without such features as the contraction-factor.

In fact, it is well understood that the Lorentz-Larmor theory, by postulation of the contraction as an effect of motion through the ether, becomes in effect exactly equivalent physically to Einstein's theory over a wide range of phenomena.

For reasons such as these it is apparent that the theories in question are nearly equivalent for the treatment of the problem in hand. It seems to the writer that if the ether theories have any advantage over Einstein's it is likely to lie primarily in two remaining features that still need examination. First, it may be imagined that a theory like Fresnel's or Lorentz', while retaining the notion of a stagnant or rigid ether at least for certain limited regions, might have the greater freedom of choice of the frame of reference with respect to which it is so defined; this seems to be Silberstein's idea, when he uses the fractional factor k for the ether case but allows it to be only unity for Einstein's. Second, the notion of a non-rigid streaming or quasi-fluid ether, possibly even with vortex motion, may seem to offer greater adaptability than relativity allows.

In connection with the first point it is convenient to amplify Silberstein's notation because of ambiguity in the meaning of his S^*, which is said to represent the stellar or other inertial frame; although the partly dragged ether, with fractional coefficient undetermined, is taken to be isotropic in it. This usage seems to blur a distinction intended to be made, since it is probably not meant that a rotating ether should necessarily be an inertial frame in the sense of mechanics.

To indicate distinctions corresponding to rotation only, suppose, then, that S^* is the stellar frame, S^i an inertial frame for either Newtonian or Einstein mechanics since a distinction between these is not needed here, S^e the frame of isotropism of an ether supposed rigid in the neighborhood of the terrestrial experiment, and S the frame of the rotating earth. It will doubtless be agreed that in purely terrestrial experiments, for most mechanical and probably all optical tests hitherto, the distinction between these is obscured and indifferent; because the differential accelerations due to rotation are masked by the effects of inevitable disturbances, and because the rotational velocity is so small compared with the velocity of light. But the deviation of falling bodies speaks for some such distinction and the Foucault pendulum and gyroscope are commonly taken to indicate that S^i is much nearer to S^* than to S, though probably not yet with precision enough to distinguish surely even between siderial and mean solar day. The Michelson experiment seems to be the first real test of the corresponding optical comparison concerning S^e.

Since all these experiments could be performed even if the sky were always clouded everywhere, it seems in a way more suitable to say that these special dynamical tests point to the difference between S^i and S, while the optical test is needed to relate the S^e to the others, with precision enough to make a distinction between them. There are some advantages in this more limited formulation, independent of reference to cosmic phenomena, but the astronomical relations of the problem are clearly vital for a comprehensive theory.

Although diurnal aberration is not directly known, there are in the mode of reduction employed on astronomical observations certain inherent assumptions corresponding to the notion, that if extrapolated as a rigid system to cosmic distances the S^e would fit S^*. Then the planetary motions are to high precision consistent with the coincidence of S^i and S^*. But with the supposition of a cosmic S^e in rotation with respect to S^* there would even be need of inquiry as to the precise meaning of the latter, especially if its determination were understood to include dynamical relations. The notion of a rotational drag extending far out from the surface of the earth would evidently carry with it the need for elaborate re-examination of astronomical observations.

A hypothesis more likely to be entertained is that there could be an ether practically rigid locally and partaking to some extent in the rotation of the earth but connected with a cosmic ether stagnant in S^* by a transitional portion where a sort of fluidity would need to be assumed, and where not even a locally rigid S^e would exist. The varied suppositions that are naturally suggested are, however, special cases of a theory where for no portion of the medium is rigidity initially assumed. Some use of the idea of fluidity seems to be difficult to escape if any rotational drag is observed.

Now it may be asked whether the possibility of a full-fledged theory of a non-rigid ether is at present more than an article of faith. Perhaps an adaptation of the Heaviside-Hertz or Lorentz equations for moving bodies could be made to serve as embodiment of it. But the perplexities that greeted the Stokes theory of aberration in a medium with pure streaming motion are familiar, and there are worse when vortical motion is included. The pending optical experiment, because of the circuital optical path, may in fact be said to be adapted to yield primarily a measure of the difference in curl between the earth's rotation and the ether motion, as measured in the frame of reference used in describing the rotation. Moreover, the determination of optical paths by Huyghens' principle is at best but kinematic, and does not imply the attainment of an ether theory competent to follow the waves with detailed reference also to amplitude and polarization. But granting that such a theory can be made, and for illustration understanding it to be a modified form of Fresnel's or Lorentz', one may still ask whether the theory of relativity could not make a corresponding adaptation within itself. Confidence in such a possibility is certainly encouraged by the previous success of Einstein's theory in absorbing the salient content of earlier theories with only such changes as are permitted by the experimental data. It is quite conceivable that this theory could expect to find such adapting changes possible in the field-equations, of any ether theory at least whose success is connected with terms of orders zero and one in the velocities. But for the present discussion it may suffice to point out the basic feature involved.

The theory of Einstein is like all physical theories using the concepts of space and time, in that it includes a kind of geometry, supplemented by a system of physical notions and postulates which can be developed in harmony with the geometry but which are by no means uniquely determined by any logical considerations alone. It is largely these postulates which are in question in connection with any experimental tests, and they can be changed in detail without destroying the main structure, just as an ether theory could introduce the notion of a locally non-rigid medium.

In the present instance it may be noticed that the postulate primarily concerned is at least as old as modern science, and is very deeply involved in Newtonian astronomy. Its perpetuation in suitable form by Einstein is natural, not because it is inevitable but because there has hitherto been no reason at hand for preferring something different and presumably more complicated. In primitive form this postulate may be roughly said to assert that the straight lines of metric geometry are dynamically and optically straight. Two centuries of celestial mechanics exhibit the remarkable success of this hypothetical identification, in connection with astronomical triangulations and the relation of Newton's first law to planetary motions. In Einstein's theory, using a combination geometry of space and time measurements, and extended in the generalized theory by introduction of the curvatures of the manifold, it is taken as a characteristic of void spaces that the optical geodesics are the null-lines of the dynamical geodesic system and that these are defined by the vanishing variation of the integral space-time separation. This postulate could certainly be changed in various ways if need be, without departing from the natural criteria of a genuine theory of relativity. If the dynamical and optical world-lines of reference do not coincide, their relation has a physical meaning and is a matter of at least partial experimental test, the results of which could be described "covariantively" or impartially, as demanded by such a theory.

This assumed coincidence of reference-lines of two-fold aspect is reflected in the absence of any new arbitrary constant in the computed values of ray-curvature and motion of Mercury's perihelion. The verification of these values suggests that no change in the theory is likely to be required for void spaces. But possibly in the immediate neighborhood of rotating masses, whose theory is still incomplete, and certainly for spaces not void of matter, as the writer expects to show in detail at another opportunity, the Einstein geometry furnishes material for some freedom of choice in modifying the analogous form of the postulate referred to, in such fashion as to fit with the original form of the theory in regions where the tensor of matter is assumed to vanish.

To make the corresponding adaptation of the wave-equations is much the same problem as in the theory of a fluid ether.

The theory of Michelson's apparatus, where source, mirrors and observing instrument alike rotate, may need further study before the interpretation becomes convincing. But it is clearly premature to conclude that any one of the theories is incapable of adapting itself to the result.

A related suggestion may be hazarded. The absence of dynamical symptoms of uniform translation was found to be paralleled by absence of optical and electrical symptoms. The presence of dynamical symptoms of rotation is natural reason for expecting positive optical analogues. But a value for the rotation, less than the expected but not zero, seems quite plausible, in view of the possibility of a region where the portion of ether in rotation merges outwardly into quiescent regions, and this transition part may extend into the body of the earth. The Einstein theory of rotating masses when suitably developed may furnish an analogue, where the internal dynamical-optical geometry merges into that of the external void. The corresponding suggestion is that the angular velocity revealed by Foucault pendulum and gyroscope may not be the siderial value, and might possibly be found to vary with the depth if the experiments could be performed in cavities deep down within the mass. These dynamical experiments also seem to offer renewed interest.

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